BoNuS Proton Momentum Calibration Using Quasi-Elastic Events

BoNuS Technical Note

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May 30, 2006

1 The Spectator Model

In the spectator model we assume that for electron scattering from a neutron in deuterium the proton is on-shell before scattering and there is no final-state interaction between the proton and neutron. Therefore, the measured spectator proton momentum $\vec{p}$ is the proton's internal momentum before scattering as well. The neutron's momentum before scattering is therefore $-\vec{p}$ since the deuteron is at rest.

Let M be the proton mass.
Let $M_d = 2M$ be the deuteron mass.
Let $q = (\nu, \vec{q})$ be the 4-momentum transfer of the electron to the neutron in the lab frame.
Let $Q^2 = -q \cdot q$.
Let $p_p = (E_p, \vec{p})$ be the 4-momentum of the spectator proton in the lab frame.
Let $p = |\vec{p}|$.
Let $p_n = (E_n, -\vec{p})$ be the 4-momentum of the neutron before scattering in the lab frame.
Let $x = \frac{-q \cdot q}{2p_n \cdot q}$ be the true momentum fraction.
Let $x_B = \frac{Q^2}{2M\nu}$.
Let θ be the angle between $\vec{p}$ and $\vec{q}$.
Let $a = 2 - x_B + \frac{2M}{\nu}$.
Let $b = 1 + \frac{2M}{\nu}$.
Let $c = \frac{|\vec{q}|}{\nu} \cos\theta$.
Let $\alpha = \frac{\sqrt{p^2+M^2} - cp}{M}$.

Then,

$$M_d = 2M = E_n + E_p \qquad \text{and} \qquad E_p = \sqrt{p^2 + M^2}.$$

For quasi-elastic scattering

$$M^2 = (p_n + q)^2 = 5M^2 - Q^2 - 4ME_p + 4M\nu - 2E_p\nu + 2\vec{p} \cdot \vec{q}. \tag{1}$$

Dividing Eq. 1 by $2M\nu$ and rearranging the terms yields

$$2 - x_B = \alpha + \frac{2}{\nu}(E_p - M) \tag{2}$$

From the definition of x we find that

$$x = \frac{x_B}{2 - \alpha}. \tag{3}$$

We would expect that $x = 1$ for quasi-elastic scattering, which would imply that $2 - x_B = \alpha$. However, Eq. 2 shows that this is only true in the limit as $\nu \to \infty$

If we assume that we know θ fairly well from the BoNuS detector, we can solve for the spectator proton momentum p using Eq. 2:

$$\frac{p}{M} = \frac{ac \pm \sqrt{b^2(a^2 - b^2 + c^2)}}{b^2 - c^2}. \tag{4}$$

2 Procedure

1) Select quasi-elastic events with a cut $x_B > 0.8$.
2) Determine θ from BoNuS and CLAS.
3) Using $x_B, \nu, |\vec{q}|$ and θ, calculate a, b and c.
4) Calculate p from the positive solution in Eq. 4.
5) Compare p with the value given by BoNuS tracking.

References