Earth like magnetic field vector magnetometry: Rb atoms, EIT, and machine learning

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Magnetic fields and magnetometers capabilities

Magnetic field

- Human brain: 0.1 – 1 pT
- Human heart: 100 pT

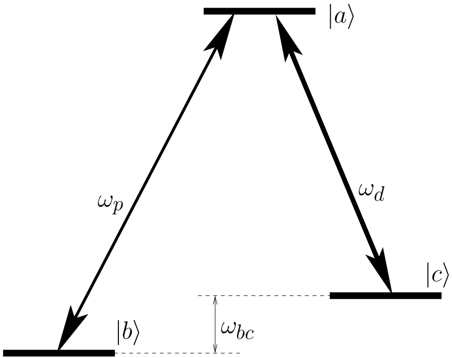
Magnetometers

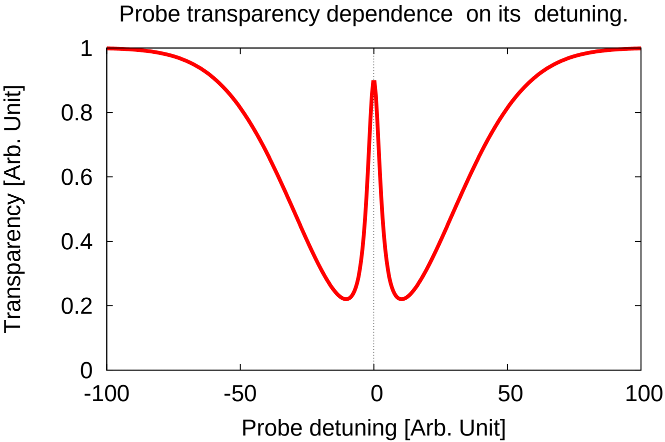
- SQUID: 1 fT
- SERF: 1 fT

Goal

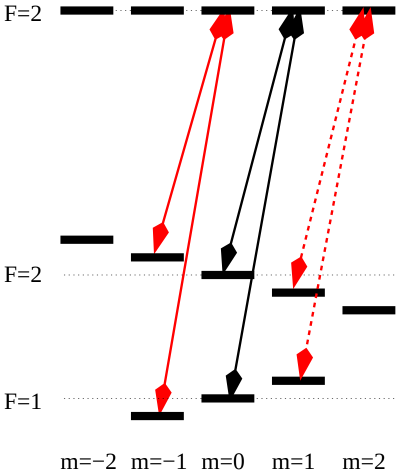
compact (< 1 cm^3) **vector** magnetometer with pT precision capable to work at Earth magnetic field (50 μT)

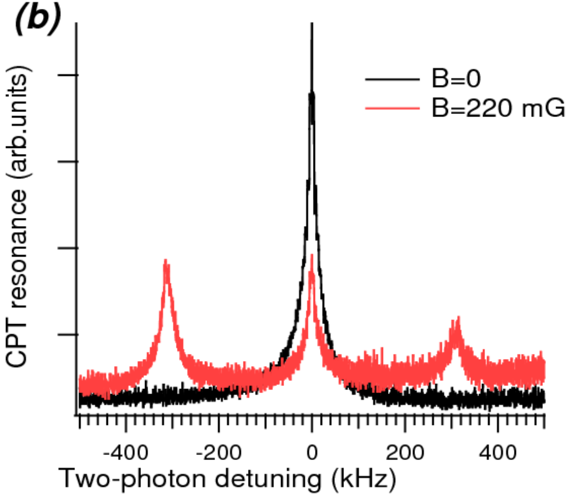
Electromagnetically Induced Transparency (EIT)



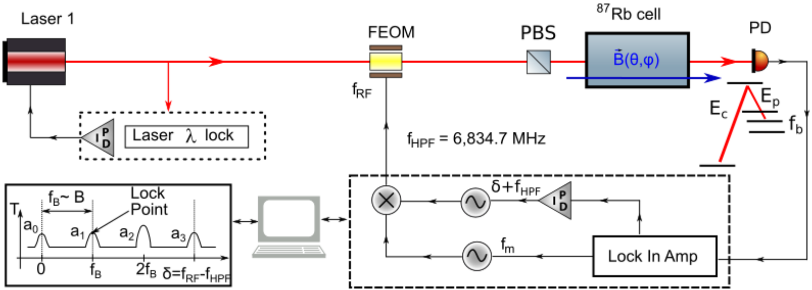


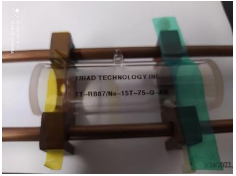
EIT with circularly polarized light

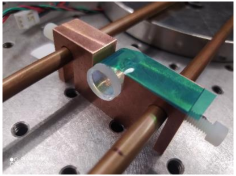


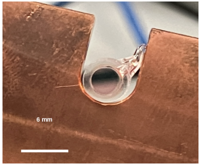


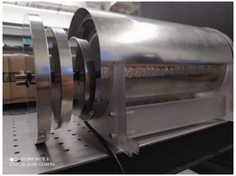
Conceptual design



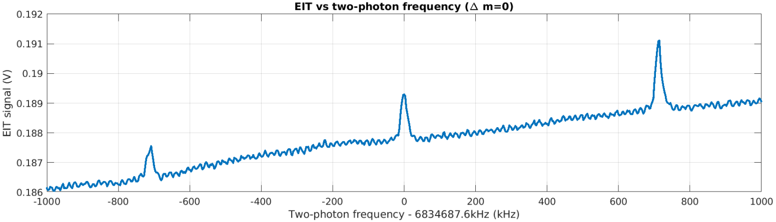








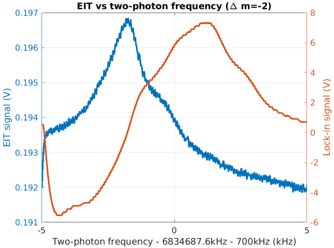
EIT signals vs two-photon detuning. B=50μT, $f_c = 6'834'687.6$kHz

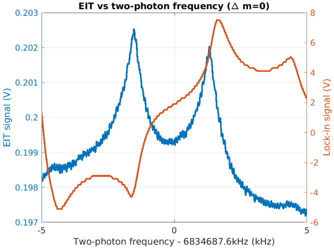


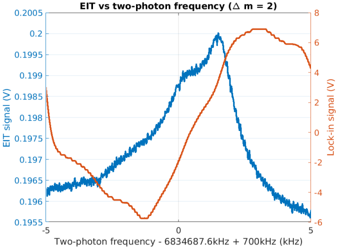
$\Delta m = -2$, $f_{center} - 700$kHz

$\Delta m = 0$, $f_{center} = 0$

$\Delta m = 2$, $f_{center} + 700$kHz

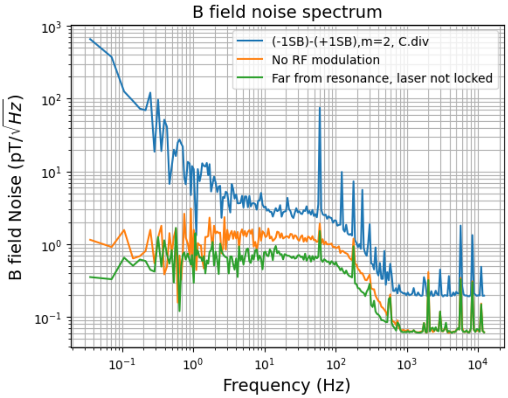




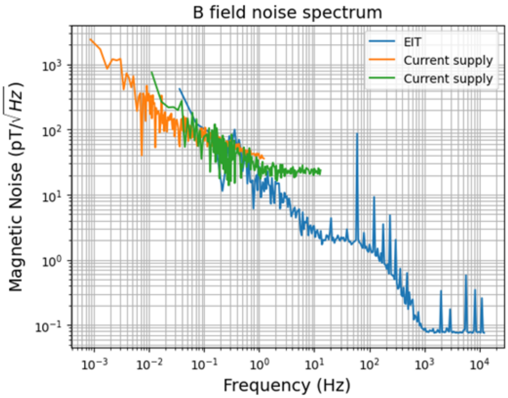


B field sensitivity

Limiting noise sources

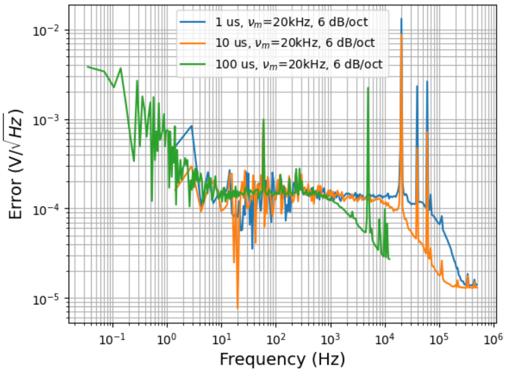


Equivalent current source noise

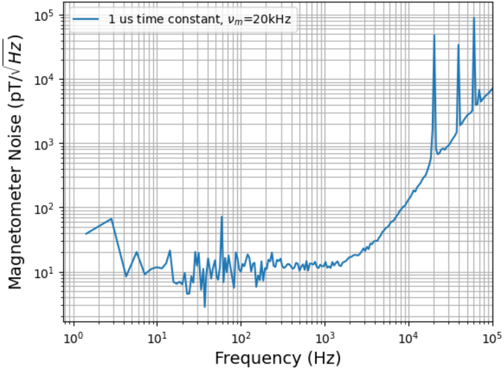


B field sensitivity normalized response

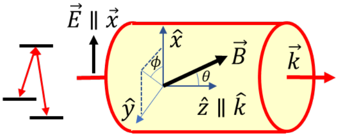
Lock-in time constant limiting response

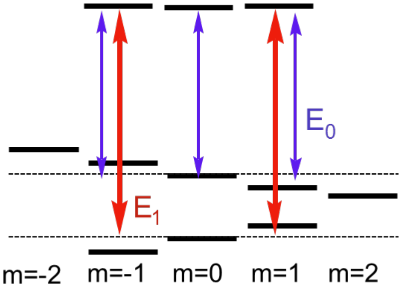


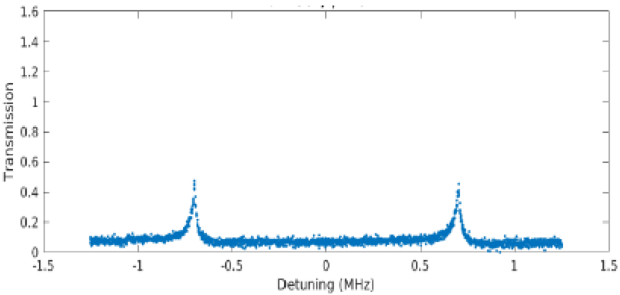
True magnetometer sensitivity



Compass idea

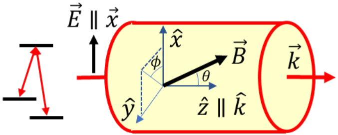


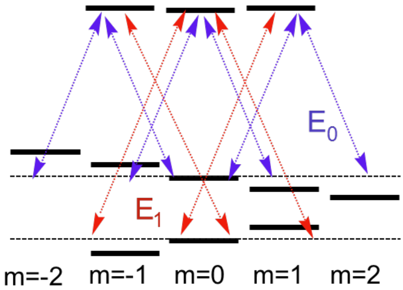


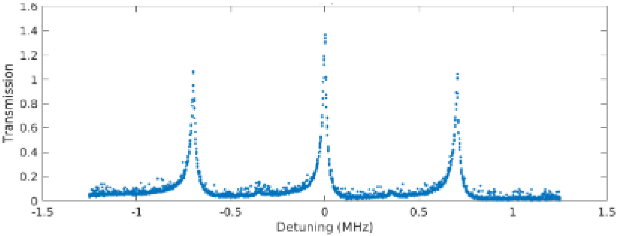


V. I. Yudin *et al.* Phys. Rev. A 82, 033807
Kevin Cox *et al.* Phys. Rev. A 83, 015801

Compass idea







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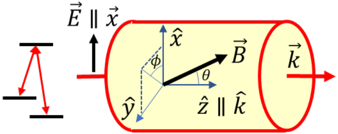
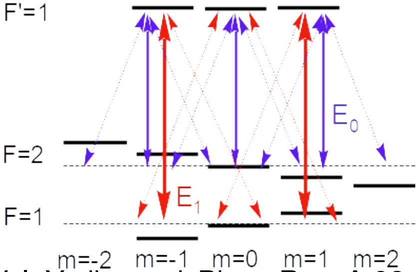
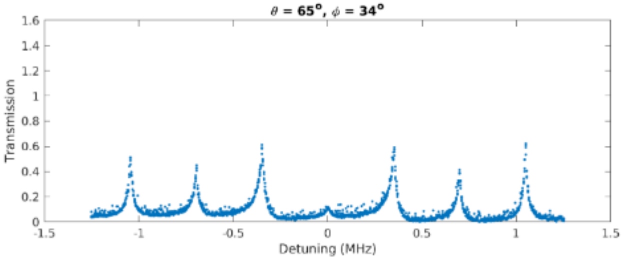
Compass idea







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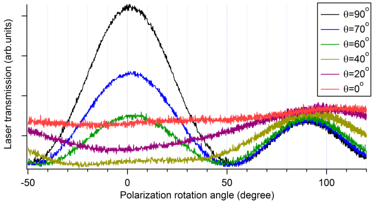
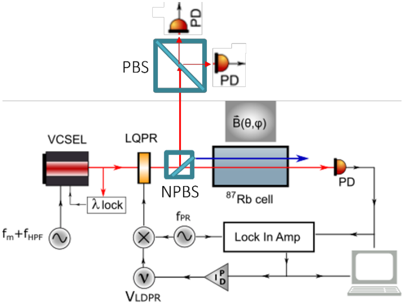
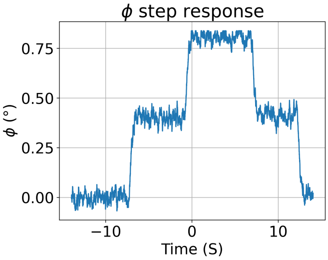
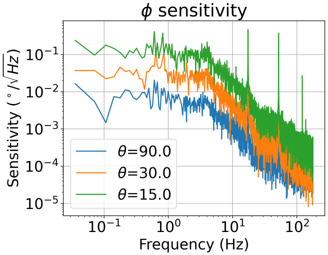
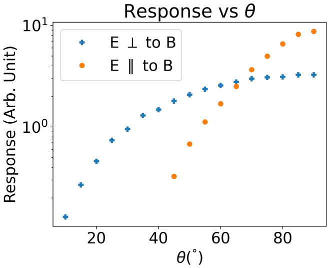
ϕ angle tracking sensitivity

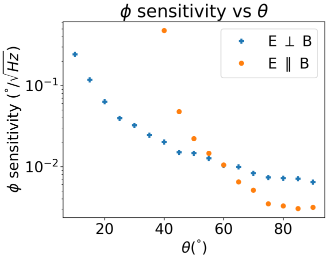


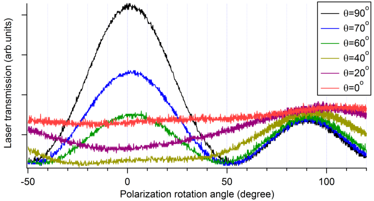


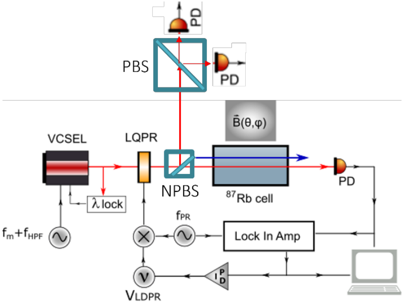


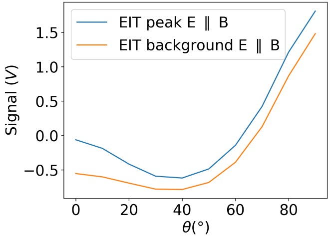


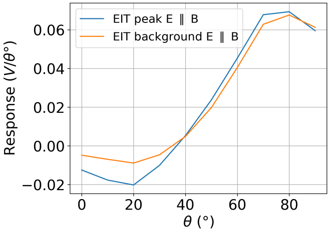


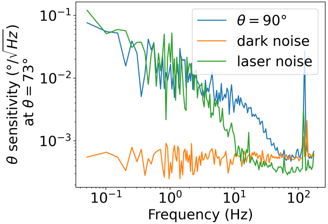


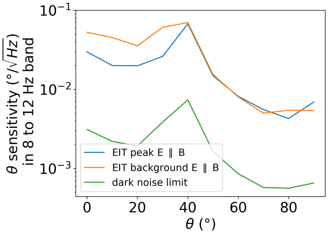

θ=90°
θ=70°
θ=60°
θ=40°
θ=20°
θ=0°
Laser transmission (arb.units)
-50
0
50
100
Polarization rotation angle (degree)


PD
PBS
PD
B(θ,φ)
VCSEL
LQPR
NPBS
87Rb cell
PD
λ lock
fm+fHPF
fPR
Lock In Amp
VLDPR

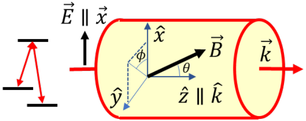

EIT peak E ∥ B
EIT background E ∥ B
Signal (V)
θ(°)

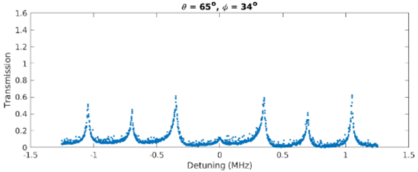

EIT peak E ∥ B
EIT background E ∥ B
Response (V/θ°)
θ (°)


θ = 90°
dark noise
laser noise
θ sensitivity (°/√Hz)
at θ = 73°
Frequency (Hz)

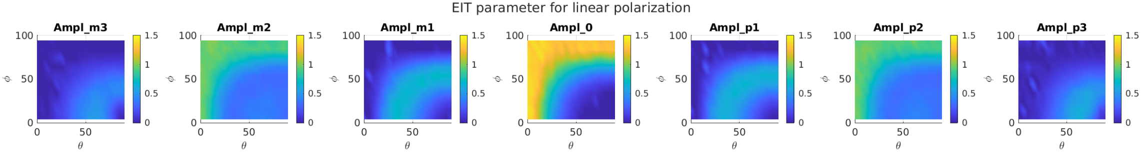

EIT peak E ∥ B
EIT background E ∥ B
dark noise limit
θ sensitivity (°/√Hz)
in 8 to 12 Hz band
θ (°)

Linear polarization: angular dependence on θ and ϕ

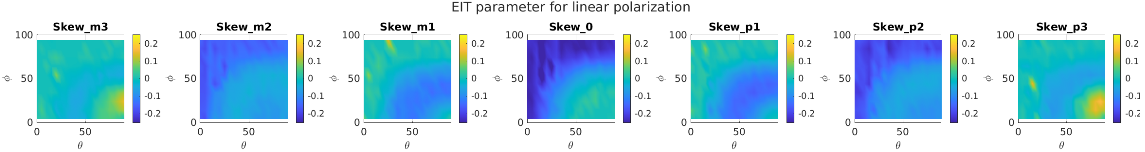




Amplitude



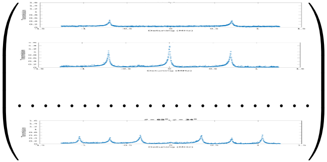
Skew



Principal component analysis: quick intro

We form measurement matrix M

$$\begin{array}{cc} & n \text{ features} \\ m \text{ measurements} & \begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} \ldots & m_{1,n} \\ m_{2,1} & m_{2,2} & m_{2,3} \ldots & m_{2,n} \\ \ldots & \ldots & \ldots & \ldots \\ m_{m,1} & m_{m,2} & m_{m,3} \ldots & m_{m,n} \end{pmatrix} \end{array} = M_{m \times n} =$$



and its approximation with product of 2 low ranked matrix W and P

$$\widetilde{M}_{m \times n} = W_{m \times k} \times P_{k \times n}$$

Physical meaning of W and P

$$\begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \\ \cdots & \cdots & \cdots \\ m_{m,1} & m_{m,2} & m_{m,3} \end{pmatrix} = \begin{pmatrix} w_{1,1} & w_{1,2} \\ w_{2,1} & w_{2,2} \\ w_{3,1} & w_{3,2} \\ \cdots & \cdots \\ w_{m,1} & w_{m,2} \end{pmatrix} \times \begin{pmatrix} p_{1,1} & p_{1,2} & p_{1,3} \\ p_{2,1} & p_{2,2} & p_{2,3} \end{pmatrix}$$

Every measurement can be approximated as a superposition of patterns

$$\begin{pmatrix} m_{2,1} & m_{2,2} & m_{2,3} \end{pmatrix} = \begin{matrix} \begin{pmatrix} w_{2,1} \end{pmatrix} \times \begin{pmatrix} p_{1,1} & p_{1,2} & p_{1,3} \end{pmatrix} \\ + \\ \begin{pmatrix} w_{2,2} \end{pmatrix} \times \begin{pmatrix} p_{2,1} & p_{2,2} & p_{2,3} \end{pmatrix} \end{matrix}$$

- P - is the set of patterns or templates or directions
- W - is the set of weights or scores or components

Principal components analysis/decomposition (PCA)

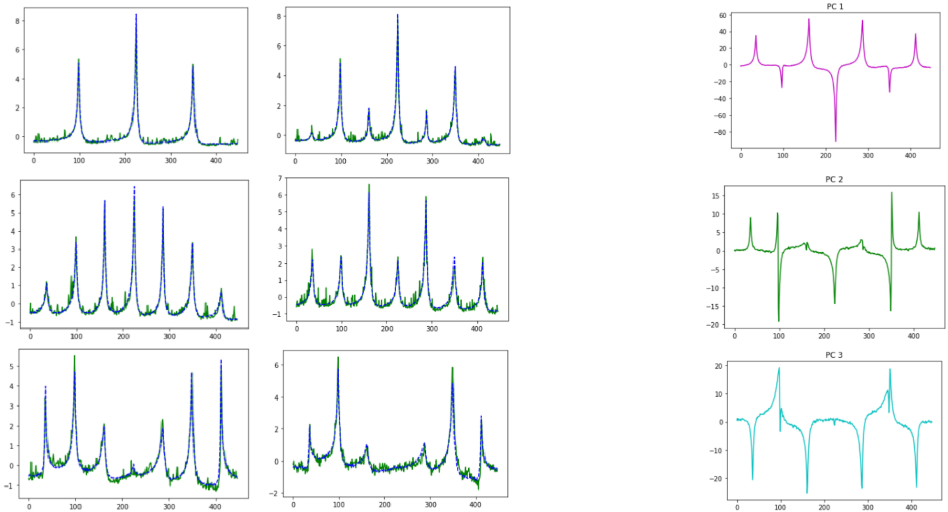
$$\begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \\ \dots & \dots & \dots \\ m_{m,1} & m_{m,2} & m_{m,3} \end{pmatrix} = \begin{pmatrix} w_{1,1} & w_{1,2} \\ w_{2,1} & w_{2,2} \\ w_{3,1} & w_{3,2} \\ \dots & \dots \\ w_{m,1} & w_{m,2} \end{pmatrix} \times \begin{pmatrix} p_{1,1} & p_{1,2} & p_{1,3} \\ p_{2,1} & p_{2,2} & p_{2,3} \end{pmatrix}$$

There are many possible realizations of W and P matrices.
PCA requires

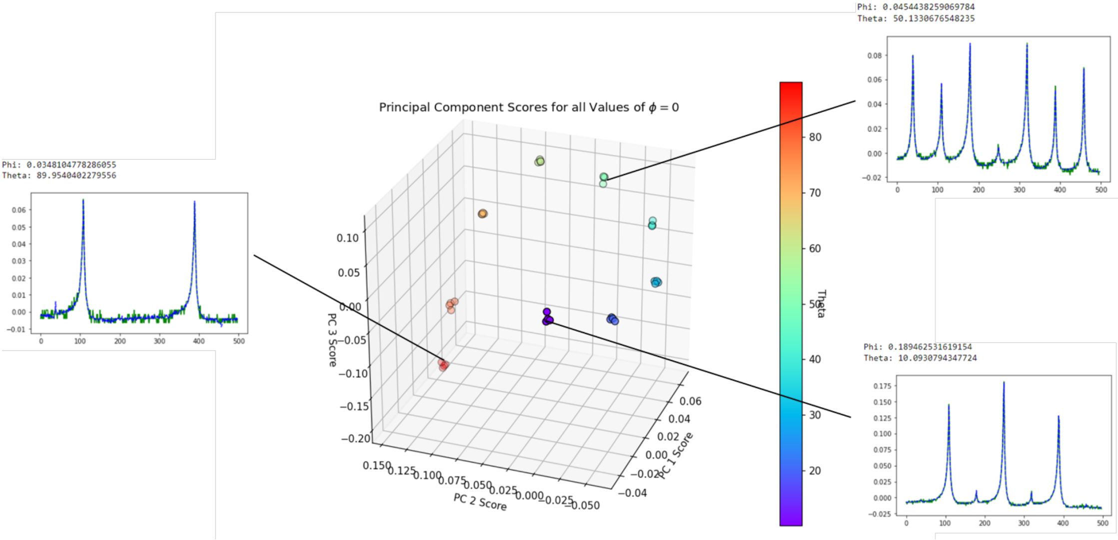
- minimizes $||M_{m\times n} - W_{m\times k} \times P_{k\times n}|| = \epsilon_k$
- $\vec{P}_i \times \vec{P}_j = \delta_{ij}$, orthonormality of patterns
- $\epsilon_{k_1} < \epsilon_{k_2}$, if $k_1 < k_2$

Many practical data sets can be described with truncated $k < \min(m, n)$
Use the single value decomposition (SVD) to get W and P.

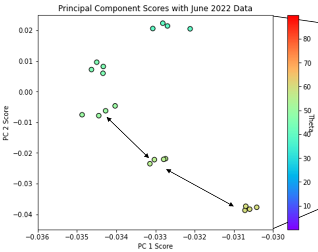
Principal component analysis: get components

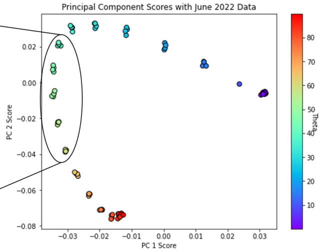


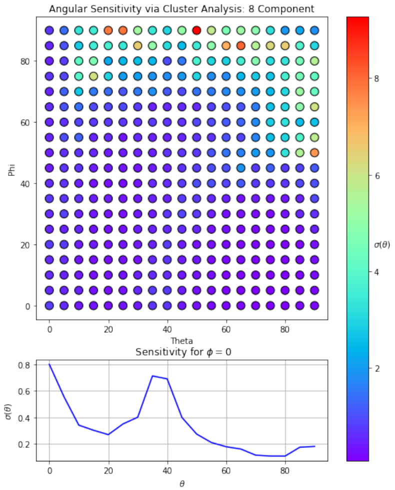
Principal component analysis: use components space



PCA precision

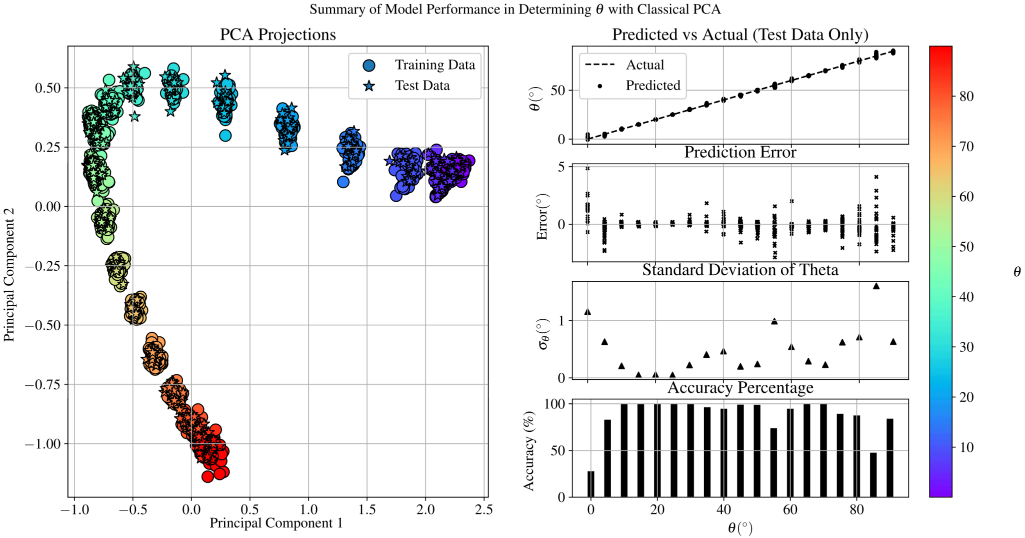




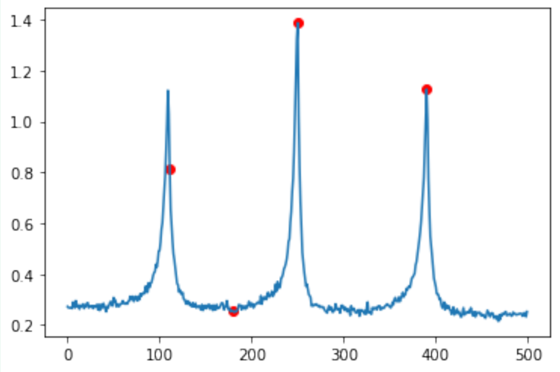


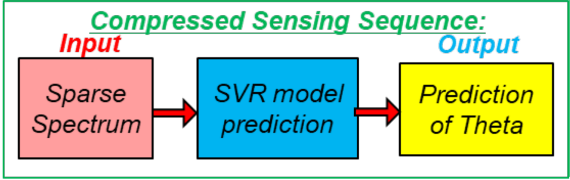
McKelvy *et al* Mach. Learn.: Sci. Technol. 4 (2023) 045048

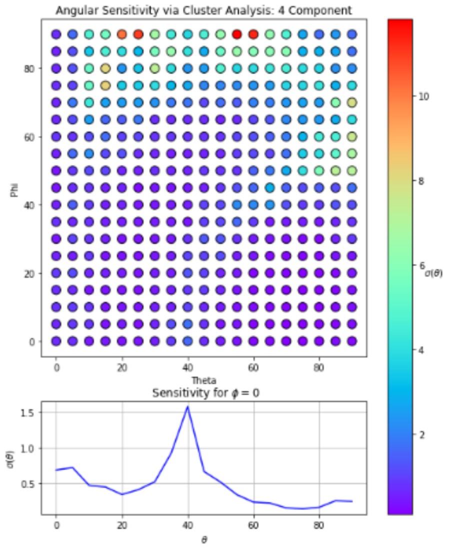
PCA precision



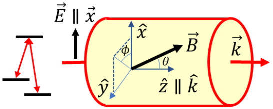
Smart sensing

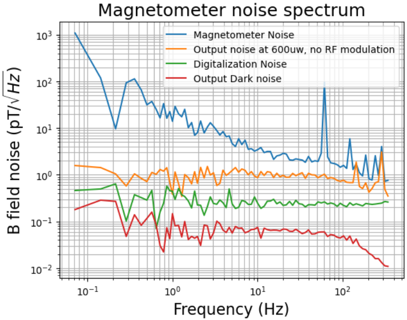






Compass summary





- 2-3 pT$/\sqrt{Hz}$ sensitivity
- measures B-field vector $(10^{-2})^\circ/\sqrt{Hz}$ at optimum
- operates at the Earth magnetic field

McKelvy *et al* "Technical limits of sensitivity for EIT magnetometry" Applied Optics Vol. 62, No. 24, 6518
McKelvy *et al* "Application of kernel principal component analysis for optical vector atomic magnetometry" Mach. Learn.: Sci. Technol. 4 (2023) 045048

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