Vector magnetometer: Rb atoms, EIT, and machine learning

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Magnetic fields and magnetometers capabilities

Magnetic field

- Human brain: 0.1 – 1 pT
- Human heart: 100 pT

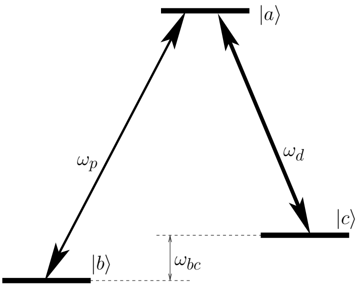
Magnetometers

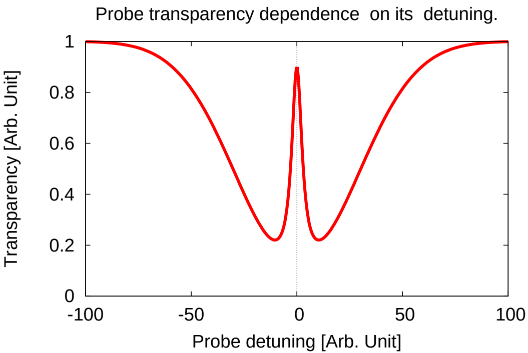
- SQUID: 1 fT
- SERF: 1 fT

Goal

We are looking for compact (< 1 cm^3) **vector** magnetometer with pT precision capable to work at Earth magnetic field (50 μT)

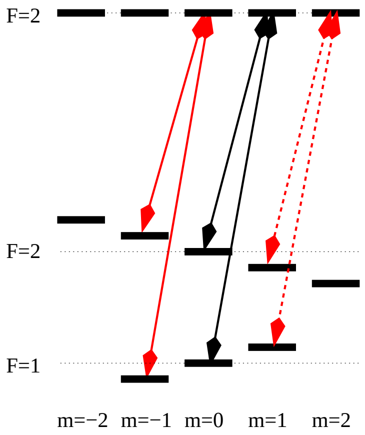
Electromagnetically Induced Transparency (EIT)

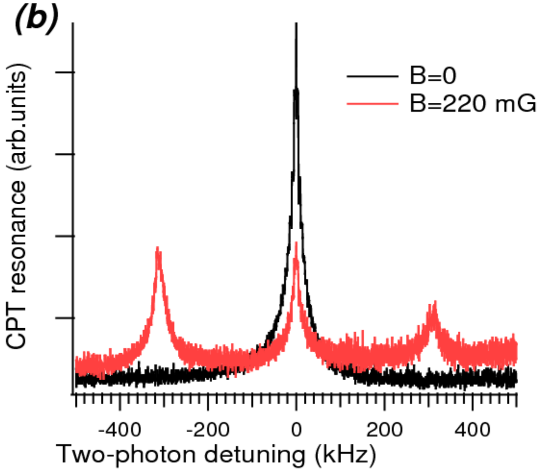




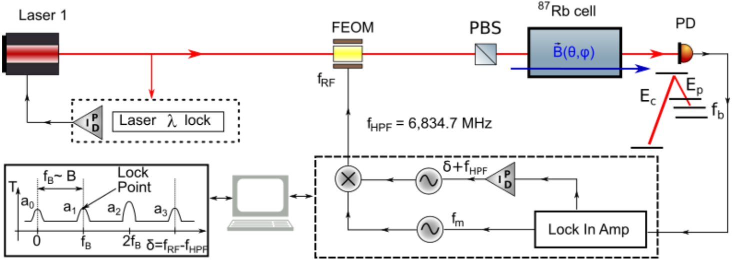
Simple EIT magnetometer

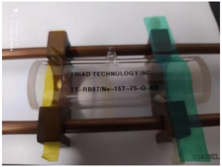
EIT with circularly polarized light

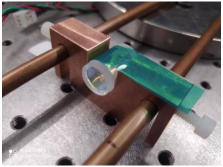


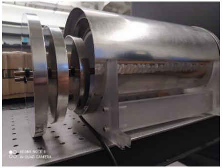


Conceptual design

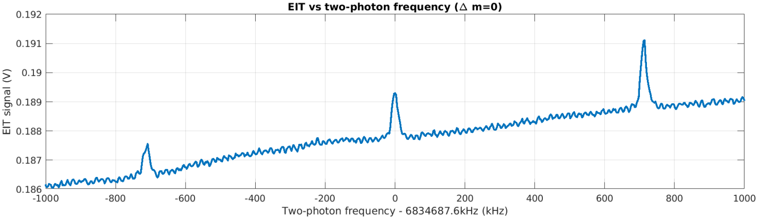








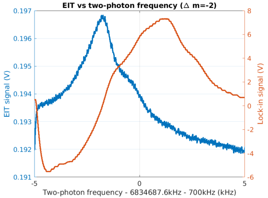
EIT signals vs two-photon detuning. B=50μT, $f_{center} = 6'834'687.6$kHz

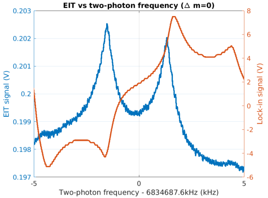


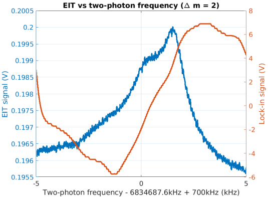
$\Delta m = -2$,
$f_{center} - 700$kHz

$\Delta m = 0$,
f_{center}

$\Delta m = 2$,
$f_{center} + 700$kHz

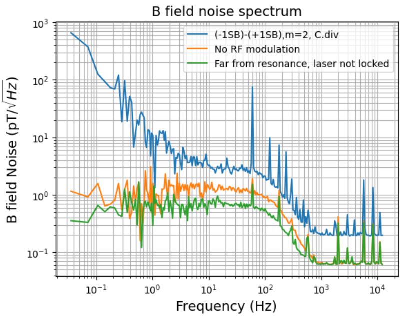




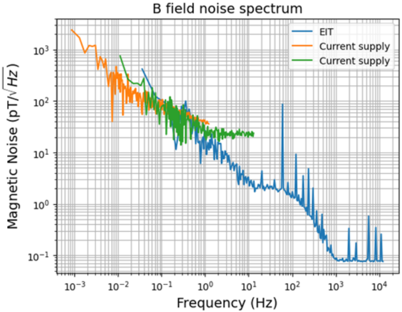


B field sensitivity

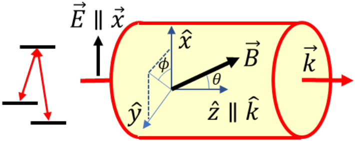
Limiting noise sources

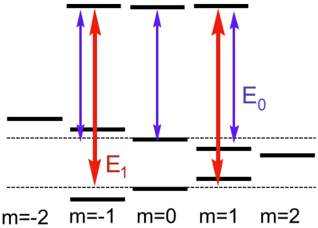


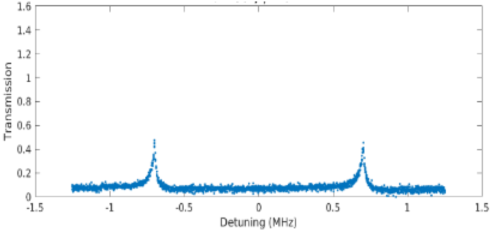
Equivalent current source noise



Compass idea







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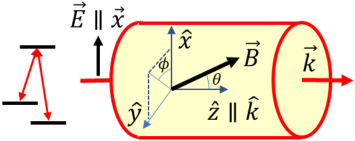
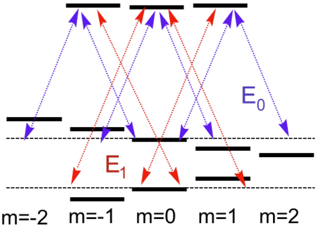
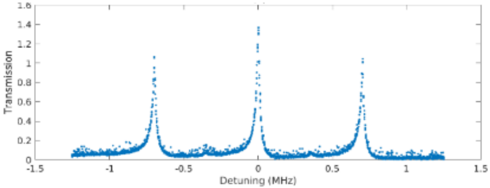
Kevin Cox *et al.* Phys. Rev. A 83, 015801

Compass idea







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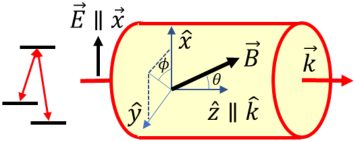

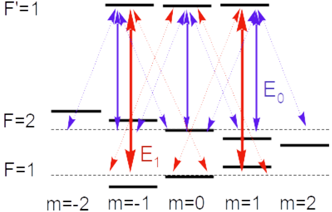

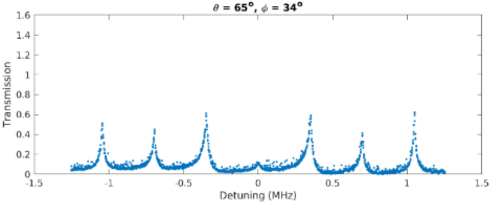

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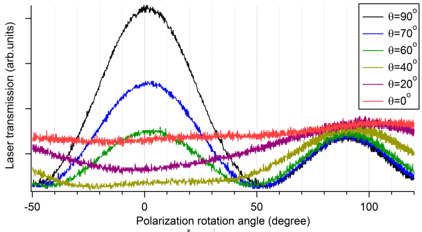
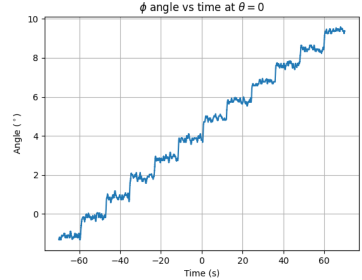
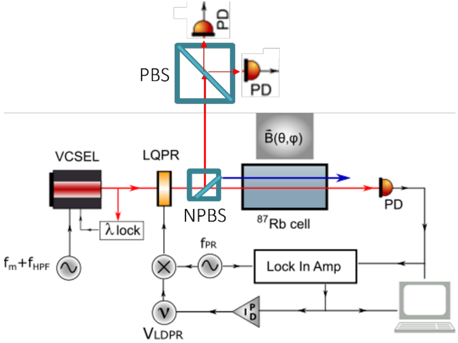
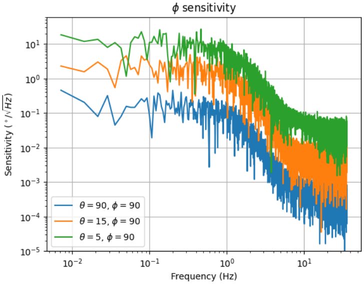
ϕ angle tracking sensitivity

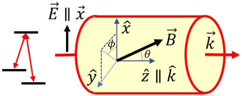


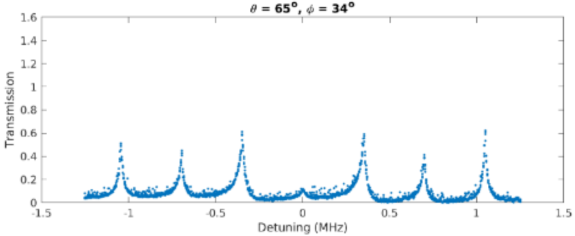






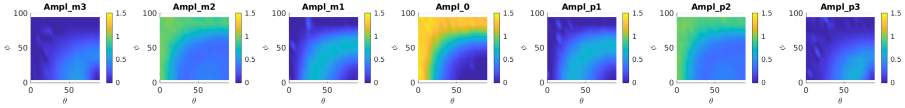
Linear polarization: angular dependence on θ and ϕ





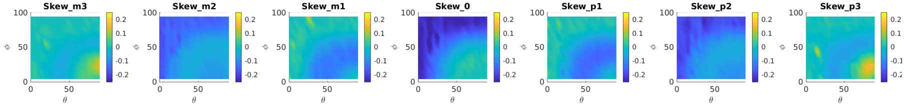
Amplitude

EIT parameter for linear polarization



Skew

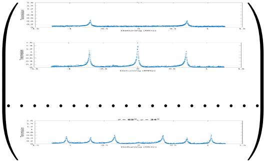
EIT parameter for linear polarization



Principal component analysis: quick intro

We form measurement matrix M

$$\overbrace{\begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} \dots & m_{1,n} \\ m_{2,1} & m_{2,2} & m_{2,3} \dots & m_{2,n} \\ \dots & \dots & \dots & \dots \\ m_{m,1} & m_{m,2} & m_{m,3} \dots & m_{m,n} \end{pmatrix}}^{n \text{ features}} \left.\right\} m \text{ measurements} = M_{m \times n} =$$



and its approximation with product of 2 low ranked matrix W and P

$$\widetilde{M}_{m\times n} = W_{m\times k} \times P_{k\times n}$$

Physical meaning of W and P

$$\begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \\ \cdots & \cdots & \cdots \\ m_{m,1} & m_{m,2} & m_{m,3} \end{pmatrix} = \begin{pmatrix} w_{1,1} & w_{1,2} \\ w_{2,1} & w_{2,2} \\ w_{3,1} & w_{3,2} \\ \cdots & \cdots \\ w_{m,1} & w_{m,2} \end{pmatrix} \times \begin{pmatrix} p_{1,1} & p_{1,2} & p_{1,3} \\ p_{2,1} & p_{2,2} & p_{2,3} \end{pmatrix}$$

Every measurement can be approximated as a superposition of patterns

$$\begin{pmatrix} m_{2,1} & m_{2,2} & m_{2,3} \end{pmatrix} = \begin{matrix} \begin{pmatrix} w_{2,1} \end{pmatrix} \times \begin{pmatrix} p_{1,1} & p_{1,2} & p_{1,3} \end{pmatrix} \\ + \\ \begin{pmatrix} w_{2,2} \end{pmatrix} \times \begin{pmatrix} p_{2,1} & p_{2,2} & p_{2,3} \end{pmatrix} \end{matrix}$$

- P - is the set of patterns or templates or directions
- W - is the set of weights or scores or components

Principal components analysis/decomposition (PCA)

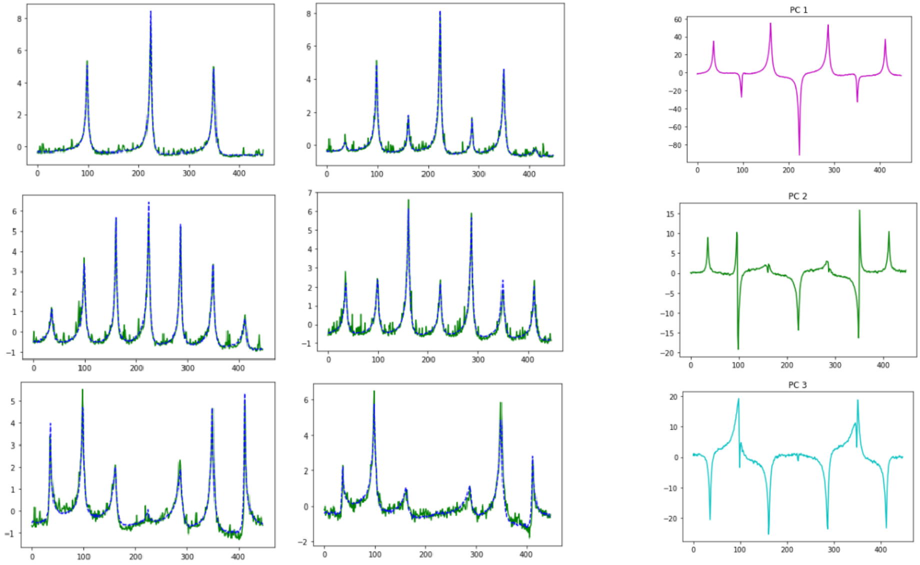
$$\begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \\ \cdots & \cdots & \cdots \\ m_{m,1} & m_{m,2} & m_{m,3} \end{pmatrix} = \begin{pmatrix} w_{1,1} & w_{1,2} \\ w_{2,1} & w_{2,2} \\ w_{3,1} & w_{3,2} \\ \cdots & \cdots \\ w_{m,1} & w_{m,2} \end{pmatrix} \times \begin{pmatrix} p_{1,1} & p_{1,2} & p_{1,3} \\ p_{2,1} & p_{2,2} & p_{2,3} \end{pmatrix}$$

There are many possible realizations of W and P matrices.
PCA requires

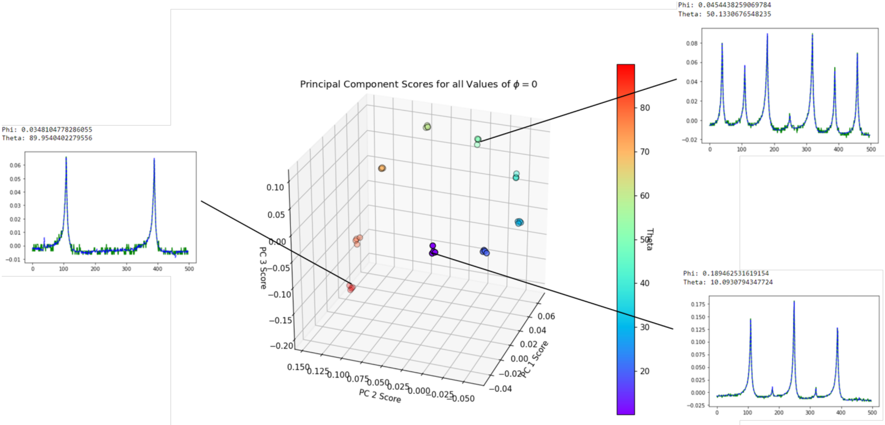
- minimizes $||M_{m\times n} - W_{m\times k} \times P_{k\times n}|| = \epsilon_k$
- $\vec{P}_i \times \vec{P}_j = \delta_{ij}$, orthonormality of patterns
- $\epsilon_{k_1} < \epsilon_{k_2}$, if $k_1 < k_2$

Many practical data sets can be described with truncated $k < \min(m, n)$
Use the single value decomposition (SVD) to get W and P.

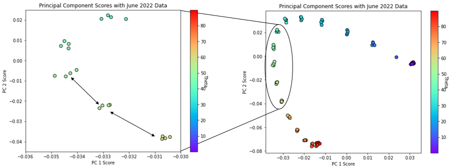
Principal component analysis: get components

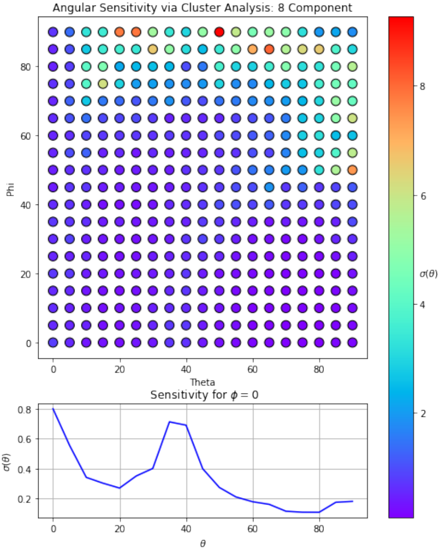


Principal component analysis: use components space

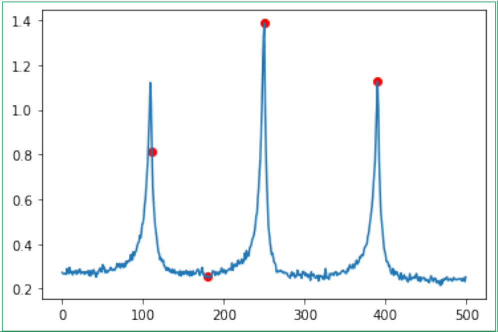


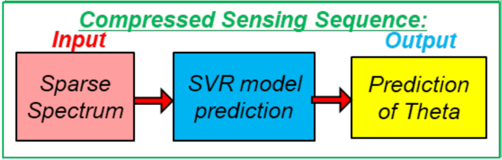
PCA precision

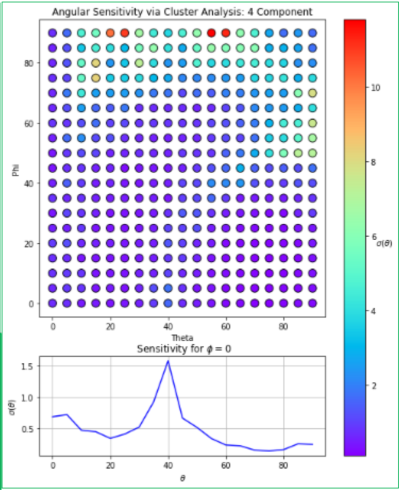




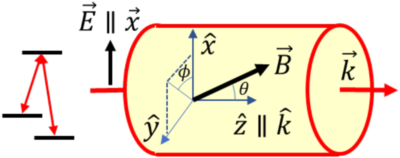
Compressed sensing

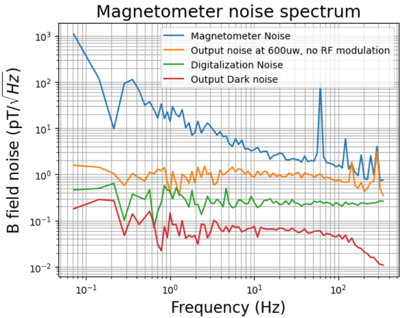






Compass summary





- pT sensitivity
- measures B-field vector
- operates at the Earth magnetic field

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