Squeezing vacuum with Rb atoms: quantum enhanced magnetometry and competition of spatial modes.

Eugeni E. Mikhailov

CICESE, August 20, 2018
Transition from classical to quantum field

Classical analog
- Field amplitude \( a \)
- Field real part
  \[ X_1 = (a^* + a)/2 \]
- Field imaginary part
  \[ X_2 = i(a^* - a)/2 \]

\[ E(\phi) = |a|e^{-i\phi} = X_1 + iX_2 \]

Quantum approach
- Field operator \( \hat{a} \)
- Amplitude quadrature
  \[ \hat{X}_1 = (\hat{a}^\dagger + \hat{a})/2 \]
- Phase quadrature
  \[ \hat{X}_2 = i(\hat{a}^\dagger - \hat{a})/2 \]

\[ \hat{E}(\phi) = \hat{X}_1 + i\hat{X}_2 \]
Quantum optics summary

Light consists of photons
\[ \hat{N} = a^\dagger a \]

Commutator relationship
\[ [a, a^\dagger] = 1 \]
\[ [X_1, X_2] = i/2 \]

Detectors measure
- number of photons \( \hat{N} \)
- Quadratures \( \hat{X}_1 \) and \( \hat{X}_2 \)

Uncertainty relationship
\[ \Delta X_1 \Delta X_2 \geq 1/4 \]
Heisenberg uncertainty principle and its optics equivalent

Heisenberg uncertainty principle

$$\Delta p \Delta x \geq \frac{\hbar}{2}$$

The more precisely the POSITION is determined, the less precisely the MOMENTUM is known, and vice versa

Optics equivalent

$$\Delta \phi \Delta N \geq 1$$

The more precisely the PHASE is determined, the less precisely the AMPLITUDE is known, and vice versa

Optics equivalent strict definition

$$\Delta X_1 \Delta X_2 \geq \frac{1}{4}$$
Heisenberg uncertainty principle and its optics equivalent

**Heisenberg uncertainty principle**

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Squeezed quantum states zoo

Unsqueezed coherent
Squeezed quantum states zoo

Unsqueezed coherent

Amplitude squeezed

Squeezing with Rb
Squeezed quantum states zoo

Unsqueezed coherent

Amplitude squeezed

Phase squeezed
Squeezed quantum states zoo

Unsqueezed coherent

Amplitude squeezed

Phase squeezed

Vacuum squeezed
Squeezed field generation recipe

Take a vacuum state $|0\rangle$

$$H = \frac{1}{2}$$
Squeezed field generation recipe

Take a vacuum state \( |0> \)

Apply squeezing operator \( |\xi> = \hat{S}(\xi) |0> \)

\[
\hat{S}(\xi) = e^{\frac{1}{2} \xi^* a^2 - \frac{1}{2} \xi a^\dagger^2}
\]

\[
H = \frac{1}{2}
\]
Squeezed field generation recipe

Take a vacuum state $|0\rangle$

Apply squeezing operator $|\xi\rangle = \hat{S}(\xi)|0\rangle$

Apply displacement operator $|\alpha, \xi\rangle = \hat{D}(\alpha)|s\rangle$

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^*a^2 - \frac{1}{2}\xi a^\dagger^2}$$

$$\hat{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$

$H = \frac{1}{2}$

$$\langle \alpha, \xi | X_1 | \alpha, \xi \rangle = Re(\alpha),$$

$$\langle \alpha, \xi | X_2 | \alpha, \xi \rangle = Im(\alpha)$$
Squeezed field generation recipe

Take a vacuum state $|0\rangle$

Apply squeezing operator $|\xi\rangle = \hat{S}(\xi)|0\rangle$

Apply displacement operator $|\alpha, \xi\rangle = \hat{D}(\alpha)|s\rangle$

\[ \hat{S}(\xi) = e^{\frac{1}{2} \xi^* a^2 - \frac{1}{2} \xi a^\dagger^2} \]

\[ \hat{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a} \]

\[ H = \frac{1}{2} \]

Notice $\Delta X_1 \Delta X_2 = \frac{1}{4}$
Tools for squeezing
Tools for squeezing
Tools for squeezing

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Squeezing with Rb
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Two photon squeezing picture

Squeezing operator

\[ \hat{S}(\xi) = e^{\frac{1}{2} \xi^* a^2 - \frac{1}{2} \xi a^\dagger a^\dagger} \]

Parametric down-conversion in crystal

\[ \hat{H} = i\hbar \chi^{(2)} (a^2 b^\dagger - a^\dagger 2 b) \]

Squeezing

maximum squeezing value detected 15 dB at 1064 nm

Henning Vahlbruch, Moritz Mehmet, Karsten Danzmann, and Roman Schnabel

Possible squeezing applications

- shot noise limited optical sensors enhancements
- noiseless signal amplification
- photon pair generation, entanglement, true single photon sources
- interferometers sensitivity boost (for example gravitational wave antennas)
- light free measurements
- quantum memory probe and information carrier
Self-rotation of elliptical polarization in atomic medium

A.B. Matsko et al., PRA 66, 043815 (2002): theoretically prediction of 4-6 dB noise suppression

\[ a_{out} = a_{in} + \frac{igL}{2}(a_{in}^{\dagger} - a_{in}) \]
A.B. Matsko et al., PRA 66, 043815 (2002): theoretically prediction of 4-6 dB noise suppression

\[ a_{out} = a_{in} + \frac{igL}{2}(a_{in}^\dagger - a_{in}) \]
Setup

PBS

RB

LOV .Sq

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Noise contrast vs detuning in hot $^{87}$Rb vacuum cell

$F_g = 2 \rightarrow F_e = 1, 2$

Noise vs detuning

$F_g = 1 \rightarrow F_e = 1, 2$

Noise vs detuning
Observation of reduction of quantum noise below the shot noise limit is corrupted by the excess noise due to atomic interaction with atoms.
Maximally squeezed spectrum with $^{87}\text{Rb}$

W&M team. $^{87}\text{Rb} F_g = 2 \rightarrow F_e = 2$, laser power 7 mW, $T=65^\circ \text{C}$

Lezama et.al report 3 dB squeezing in similar setup

Optical magnetometer based on Faraday effect

$^{87}$Rb D$_1$ line

Susceptibility vs B

![Graph showing susceptibility vs B with detuning on the x-axis and susceptibilities on the y-axis.](image)
Optical magnetometer based on Faraday effect

$^{87}\text{Rb D}_1$ line

Susceptibility vs B

Detuning

Susceptibility values for $\sigma_+$ and $\sigma_-$ transitions.
Optical magnetometer based on Faraday effect

$^{87}\text{Rb D}_1$ line

$F'=2$

$F'=1$

$F=2$

$F=1$

$m=0$

$m=1$

$m'=-1$

$m'=0$

$m'=1$

Susceptibility vs B

\[
\chi''(\pm \Delta), \chi'(-\Delta), \chi'(+\Delta)
\]
Optical magnetometer based on Faraday effect

\[^{87}\text{Rb D}_1\text{ line}\]

\[
\begin{array}{c}
\text{F'=2} \\
\text{F'=1} \\
\text{F=2} \\
\text{F=1}
\end{array}
\]

\[
\begin{array}{c}
\text{m=-1} \\
\text{m=0} \\
\text{m=1}
\end{array}
\]

\[
\begin{array}{c}
\text{\(\sigma_+\)} \\
\text{\(\sigma_-\)}
\end{array}
\]

Susceptibility vs B

\[
\begin{array}{c}
\chi''(+\Delta) \\
\chi''(-\Delta) \\
\chi'(+\Delta) \\
\chi'(-\Delta)
\end{array}
\]

\[
\begin{array}{c}
\text{detuning} \\
\text{B field}
\end{array}
\]
Optical magnetometer based on Faraday effect

$^{87}\text{Rb D}_1$ line

\[ \begin{align*}
F' &= 2 \\
F' &= 1 \\
F &= 2 \\
F &= 1
\end{align*} \]

\[ \begin{align*}
m' &= 0 \\
m &= 0 \\
m &= 1 \\
m &= -1
\end{align*} \]

Susceptibility vs B

\[ \chi''(\pm \Delta) \]

\[ \chi' (\pm \Delta) \]
Optical magnetometer based on Faraday effect

$^{87}$Rb D$_1$ line

$F'=2$

$F'=1$

$F=2$

$F=1$

$m=0$

$m=1$

$m=-1$

$m'=0$

Susceptibility vs B

$\chi''(+\Delta)$

$\chi''(-\Delta)$

$\chi'(+\Delta)$

$\chi'(-\Delta)$

Polarization rotation vs B

$\Delta \chi'$

B field

detuning

susceptibility

Polarization rotation vs B

$\Delta \chi'$

B field

detuning

susceptibility
Optical magnetometer and non-linear Faraday effect

Naive model of rotation

Experiment

\[ \Delta \chi' \]

$B$ field

$\Delta \chi'$

Rb Cell Rb Cell

$\lambda/2$

SMPM FIBER GPLENS MAGNETIC SHIELDING PBS BPD SCOPE LASER Rb$^{87}$

Experiment

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Optical magnetometer and non linear Faraday effect

Naive model of rotation

Experiment

-10 -5 0 5 10
B field

-1
-0.5
0
0.5
1

1 mW
2 mW
4 mW
6 mW
8 mW
12 mW
Rotation response (V)

Magnetic field (G)
Magnetometer response vs atomic density

Atomic density (atoms/cm$^3$)

Slope of rotation signal (V/µT)

Normalized transmission

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Shot noise limit of the magnetometer

\[ S = |E_p + E_v|^2 - |E_p - E_v|^2 \]

\[ S = 4E_pE_v \]

\[ \langle \Delta S \rangle \sim E_p < \Delta E_v > \]
Squeezed enhanced magnetometer setup

Note: Squeezed enhanced magnetometer was first demonstrated by Wolfgramm et. al Phys. Rev. Lett, 105, 053601, 2010.
Magnetometer noise floor improvements

![Graph showing noise spectral density vs. noise frequency](image)

- Noise spectral density (dBm/Hz)
- Noise Frequency (kHz)

(a) and (b) represent two different conditions or settings.

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Magnetometer noise spectra

coherent probe
squeezed probe
SQL
Noise suppression

Response
**Magnetometer with squeezing enhancement**


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**Diagram Description**

- **Components:**
  - Laser
  - Squeezer
  - Magneto-Optical Trap
  - Magnets
  - PBS
  - BPD
  - Scope

- **Annotations:**
  - Sensitivity in pT/√Hz as a function of atomic density (atoms/cm³)
  - Coherent vs. squeezed probe comparison

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**Graph Details**

- **Axes:**
  - *y*-axis: Sensitivity in pT/√Hz
  - *x*-axis: Atomic density (atoms/cm³)

- **Data Points:**
  - Coherent probe
  - Squeezed probe

---

**Equations and Formulas**

- \(\text{Sensitivity} = \frac{\text{pT}}{\sqrt{\text{Hz}}}\)

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**Note:**

- The diagram and graph are used to illustrate the performance of the magnetometer under different conditions.
Self-squeezed magnetometry

20 pT/√Hz self-squeezed magnetometry with 4WM

Why superluminal squeezing?

- Quantum memories
Light group velocity

Group velocity $v_g = \frac{c}{\omega \frac{\partial n}{\partial \omega}}$

Susceptibility

![Graph showing susceptibility as a function of detuning]
Susceptibility and non linear Faraday effect

Naive model of rotation

Experiment

![Graph showing the susceptibility and non-linear Faraday effect](image)

Experiments with different power levels (1 mW to 12 mW) showing the rotation response vs. Magnetic field (G) for different power levels.
Light group velocity

Group velocity $v_g = \frac{c}{\omega \frac{\partial n}{\partial \omega}}$

Delay $\tau = \frac{L}{v_g} \sim \frac{\partial n}{\partial \omega} \sim \frac{\partial R}{\partial B}$
Light group velocity

Group velocity \( v_g = \frac{c}{\omega \frac{\partial n}{\partial \omega}} \)

Delay \( \tau = \frac{L}{v_g} \sim \frac{\partial n}{\partial \omega} \sim \frac{\partial R}{\partial B} \)
Squeezing vs magnetic field

Spectrum analyzer settings: Central frequency = 1 MHz, VBW = 3 MHz, RBW = 100 kHz

Squeezing vs magnetic field

Spectrum analyzer settings: Central frequency = 1 MHz, VBW = 3 MHz, RBW = 100 kHz

Time advancement setup

[Diagram of a squeezer setup with Rb cells, optical fiber, diode laser, and detection module with a balanced photodetector and spectrum analyzer.]
Squeezing modulation and time advancement

(a) 
(b) 

Noise power, dB.

(i) 

Time, ms

−0.4 −0.2 0 0.2 0.4 0.6

−1.6 −1.4 −1.2 −1 −0.8 −0.6 −0.4 −0.2 0
Squeezing modulation and time advancement

![Graphs showing normalized noise power vs. time.](b)
Advancement vs power

![Graph showing the relationship between pump power (mW) and rotation slope (rad/G). The graph indicates a decrease in rotation slope as pump power increases.](image-url)
Advancement vs power

Rotation slope, rad/G

Pump power, mW

0 2 4 6 8 10 12

−0.2 −0.15 −0.1 −0.05 0 0.05 0.1 0.15

Pump power, mW

ΔT, µs

squeezed
advanced

0 2 4 6 8 10 12

−0.2 −0.1 −0.05 0 0.05 0.1 0.15

0 2 4 6 8 10 12

−5 −4 −3 −2 −1 0 1 2 3

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Squeezing advancement vs atomic density

Noise figure and advancement


\[ F = \frac{SNR_{in}}{SNR_{out}} = \frac{1}{T} = e^{2\gamma \Delta t_a} \]
Polarization self-rotation (PSR) squeezing

**Setup**

```
PBS
V.Sq
LO

RB87
```

**Vacuum cell**

```
0  0.5  1  1.5  2
-2  0   2   4   6   8   10  12  14
```

**Coated cell**

```
0.0  0.5  1.0  1.5  2.0
-2   0   2   4   6   8   10  12
```

Beam expansion caused by self-defocusing seems to be decoupled from measured squeezing amount variation.

Beer-Lambert law

\[ dl = -NI\alpha dz \]

\[ I = I_0 \exp(-\tau) \]

where \( \tau \) is optical depth

\[ \tau = \alpha NL \]
Beer-Lambert law

\[ dl = -Nl \alpha \, dz \]

\[ I = I_0 \exp(-\tau) \]

where \( \tau \) is optical depth

\[ \tau = \alpha NL \]

Will we get equivalent result for the following cases?

- double the medium length
  \[ \tau = \alpha N(2L) \]

- double the medium density
  \[ \tau = \alpha (2N)L \]
Multipass setup

[Diagram of the multipass setup]

- SMPM Fiber Diode Laser
- Magnetic Shield Rb Cell
- Double-Pass
- Four-Pass
Optical depth dependence

\[ P = 11 \text{ mW} \]
\[ N = 9.3 \times 10^{11} \text{ cm}^{-3} \]
Squeezing = -2.0 dB

\[ P = 11 \text{ mW} \]
\[ N = 4.3 \times 10^{11} \text{ cm}^{-3} \]
Squeezing = -2.6 dB

\[ P = 11 \text{ mW} \]
\[ N = 2.4 \times 10^{11} \text{ cm}^{-3} \]
Squeezing = -2.2 dB

Squeezing vs effective optical depth

Long cell

- Single Pass
- Double Pass
- Four Pass

Squeezing (dB) vs Effective Atom Density (cm\(^{-3}\) \times 10^{12}):

Single pass -2.1 dB
Double pass -2.6 dB

Long vs short cell

- Cell Length 7.5 cm
- Cell Length 1 cm

Squeezing (dB) vs Effective Atom Density (cm\(^{-3}\) \times 10^{12}):
Squeezing vs effective optical depth

Long cell

- Single Pass
- Double Pass
- Four Pass

Cell length doubled

Reference

Atomic density doubled

Long vs short cell

- Cell Length 7.5 cm
- Cell Length 1 cm

Single pass -2.1 dB
Double pass -2.6 dB
Double cell setup: atomic density optimization

+: combined squeezing
1st cell atomic density
\[ N_1 = 9.3 \times 10^{11} \text{ cm}^{-3} \]

o: the first cell squeezing filtered
1st cell atomic density
\[ N_1 = 4.3 \times 10^{11} \text{ cm}^{-3} \]
Double cell setup: position optimization

atomic densities:

\[ N_1 = 4.3 \times 10^{11} \text{ cm}^{-3} \]
\[ N_2 = 4.3 \times 10^{11} \text{ cm}^{-3} \]

atomic densities:

\[ N_1 = 4.3 \times 10^{11} \text{ cm}^{-3} \]
\[ N_2 = 9.3 \times 10^{11} \text{ cm}^{-3} \]

atomic densities:

\[ N_1 = 9.3 \times 10^{11} \text{ cm}^{-3} \]
\[ N_2 = 9.3 \times 10^{11} \text{ cm}^{-3} \]

+/o: combined squeezing; +/o: the first cell squeezing filtered
Multimode pump output

T = 26 °C

T = 91 °C

Laguerre-Gaussian modes basis
Multimode squeezing

Multimode squeezing decomposition

\[ \hat{S}(\xi) = \exp \left[ \sum_{l,p} \frac{1}{2} (\xi_{l,p}^* \hat{a}_{l,p}^2 - \xi_{l,p} \hat{a}_{l,p}^\dagger \hat{a}_{l,p}) \right] \]

Multimode squeezing decomposition

\[ \hat{S}(\xi) = \exp \left[ \sum_{l,p} \frac{1}{2} (\xi^{*}_{l,p} \hat{a}_{l,p}^{\dagger 2} - \xi_{l,p} \hat{a}_{l,p}^{2}) \right] \]

Quantum imaging effort: from owl to sloth

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Squeezing with Rb

CICESE, August 20, 2018
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Summary

- fully atomic squeezed enhanced magnetometer with sensitivity as low as 1 pT/√Hz
- superluminal squeezing propagation with $v_g \approx -7'000$ m/s $\approx -c/43'000$ or time advancement of 11 µS
- We were able to improve squeezing by multipass configuration
- Our squeezed state is a set of competing multimodes
- We are working on quantum modes extraction and imaging

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