

Nonlinear light-atom interactions: squeezed states of light and fast laser gyroscopes

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WILLIAM
& MARY

CHARTERED 1693

Tulane, March 9, 2015



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Rochester Scientific: Dmitry Budker and Simon Rochester

LSU: Dr. Jonathan P. Dowling, R. Nicholas Lanning

From ray optics to semiclassical optics

Classical/Geometrical optics

- light is a ray
- which propagates straight
- cannot explain diffraction and interference



From ray optics to semiclassical optics

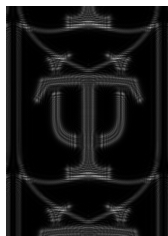
Classical/Geometrical optics

- light is a ray
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- cannot explain diffraction and interference



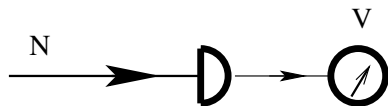
Semiclassical optics

- light is a wave
- color (wavelength/frequency) is important
- amplitude (a) and phase are important, $E(t) = ae^{i(kz - \omega t)}$
- cannot explain residual measurements noise



Detector quantum noise

Simple photodetector

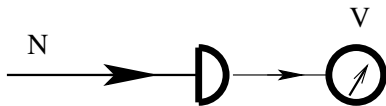


$$V \sim N$$

$$\Delta V \sim \sqrt{N}$$

Detector quantum noise

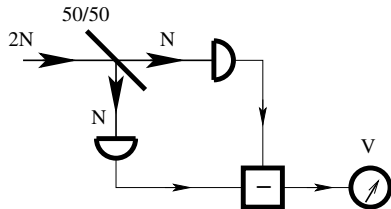
Simple photodetector



$$V \sim N$$

$$\Delta V \sim \sqrt{N}$$

Balanced photodetector



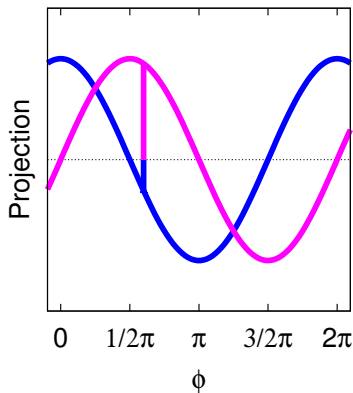
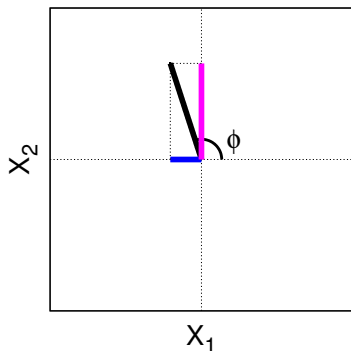
$$V = 0$$

$$\Delta V \sim \sqrt{N}$$

Classical field

$$E(\phi) = |a|e^{-i\phi} = |a|\cos(\phi) + i|a|\sin(\phi) = X_1 + iX_2, \quad \phi = \omega t - kz$$

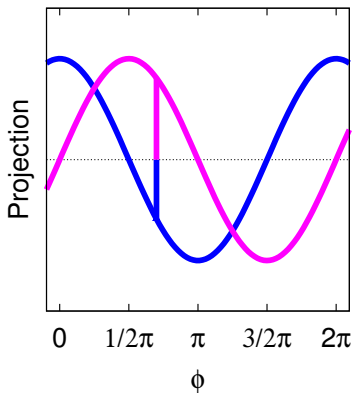
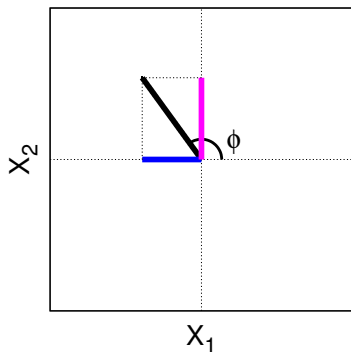
Detectors sense the **real** part of the field (X_1)
but there is a way to see X_2 as well



Classical field

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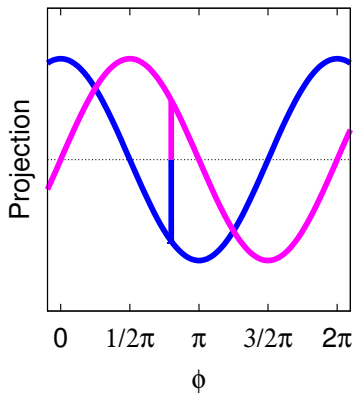
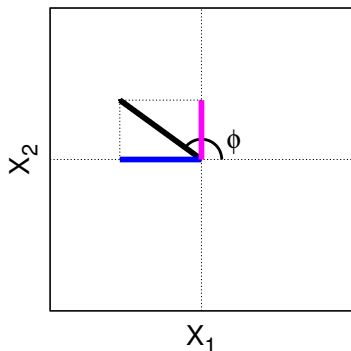
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Classical field

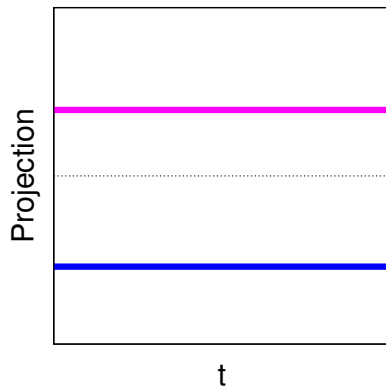
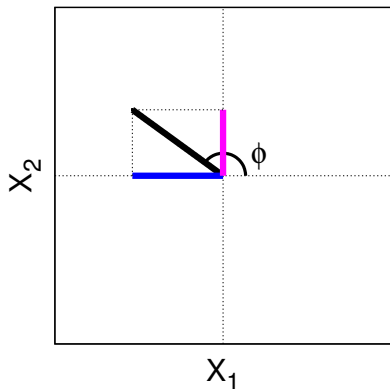
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Detectors sense the **real** part of the field (X_1)
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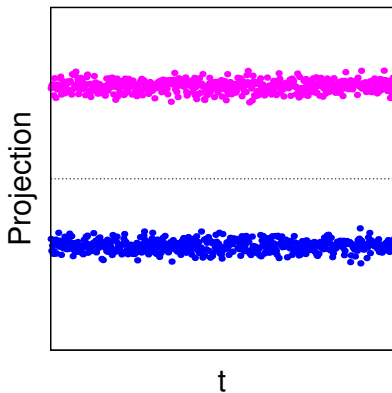
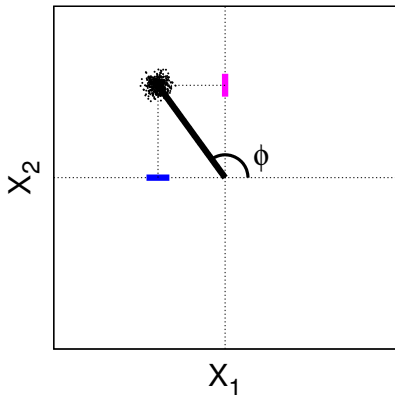
Classical quadratures vs time in a rotating frame

$$E(\phi) = |a|e^{-i\phi} = |a|\cos(\phi) + i|a|\sin(\phi) = X_1 + iX_2, \quad \phi = \omega t - kz$$



Reality check quadratures vs time

$$E(\phi) = |a|e^{-i\phi} = |a|\cos(\phi) + i|a|\sin(\phi) = X_1 + iX_2, \quad \phi = \omega t - kz$$

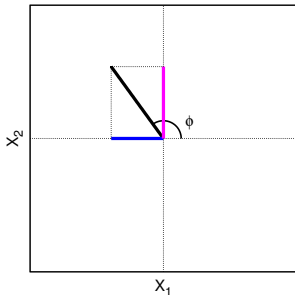


Transition from classical to quantum field

Classical analog

- Field amplitude a
- Field real part
 $X_1 = (a^* + a)/2$
- Field imaginary part
 $X_2 = i(a^* - a)/2$

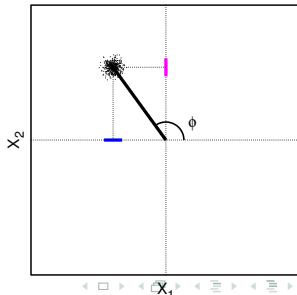
$$E(\phi) = |a|e^{-i\phi} = X_1 + iX_2$$



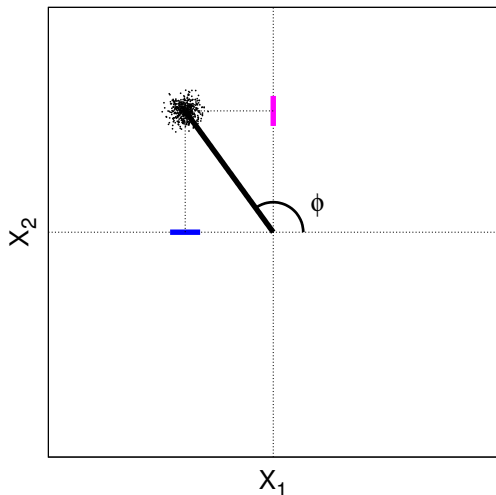
Quantum approach

- Field operator \hat{a}
- Amplitude quadrature
 $\hat{X}_1 = (\hat{a}^\dagger + \hat{a})/2$
- Phase quadrature
 $\hat{X}_2 = i(\hat{a}^\dagger - \hat{a})/2$

$$\hat{E}(\phi) = \hat{X}_1 + i\hat{X}_2$$



Quantum optics summary



Light consist of photons

- $\hat{N} = a^\dagger a$

Commutator relationship

- $[a, a^\dagger] = 1$

- $[X_1, X_2] = i/2$

Detectors measure

- number of photons \hat{N}
- Quadratures \hat{X}_1 and \hat{X}_2

Uncertainty relationship

- $\Delta X_1 \Delta X_2 \geq 1/4$

Heisenberg uncertainty principle and its optics equivalent



Heisenberg uncertainty principle

$$\Delta p \Delta x \geq \hbar/2$$

The more precisely the POSITION is determined, the less precisely the MOMENTUM is known, and vice versa

Heisenberg uncertainty principle and its optics equivalent



Heisenberg uncertainty principle

$$\Delta p \Delta x \geq \hbar/2$$

The more precisely the POSITION is determined, the less precisely the MOMENTUM is known, and vice versa

Optics equivalent

$$\Delta \phi \Delta N \geq 1$$

The more precisely the PHASE is determined, the less precisely the AMPLITUDE is known, and vice versa

Heisenberg uncertainty principle and its optics equivalent



Heisenberg uncertainty principle

$$\Delta p \Delta x \geq \hbar/2$$

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Optics equivalent

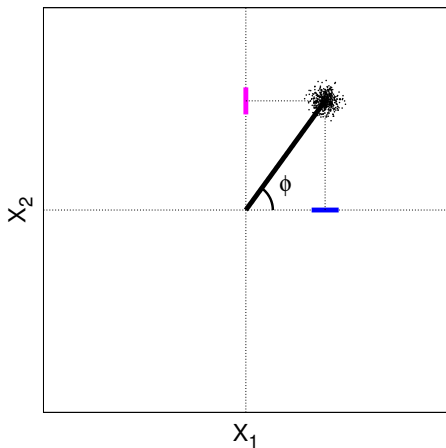
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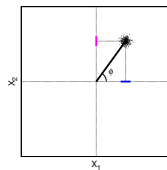
Optics equivalent strict definition

$$\Delta X_1 \Delta X_2 \geq 1/4$$

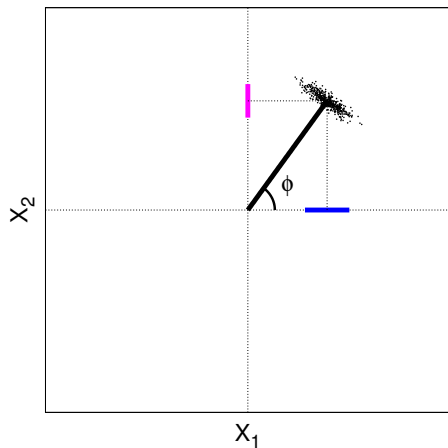
Squeezed quantum states zoo



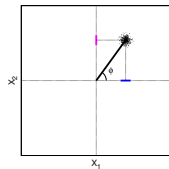
Unsqueezed
coherent



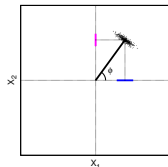
Squeezed quantum states zoo



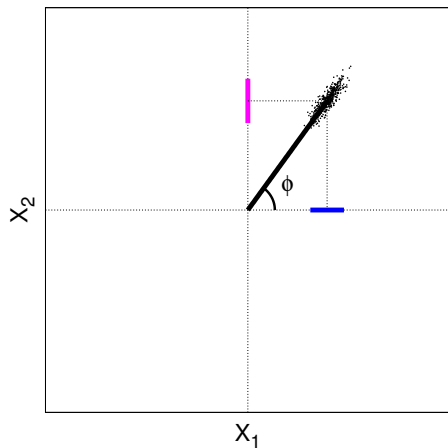
Unsqueezed
coherent



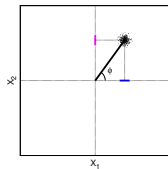
Amplitude
squeezed



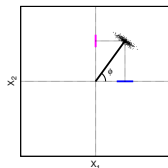
Squeezed quantum states zoo



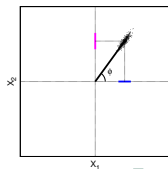
Unsqueezed
coherent



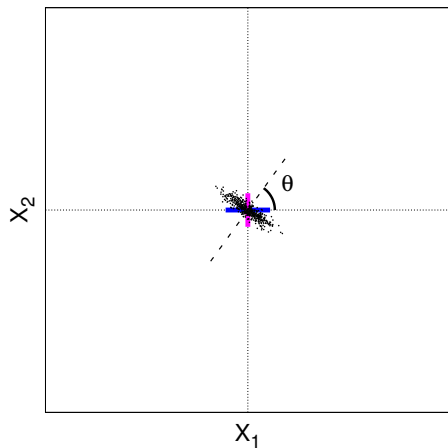
Amplitude
squeezed



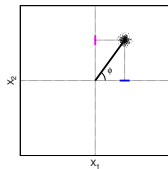
Phase
squeezed



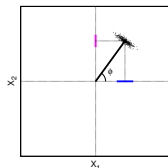
Squeezed quantum states zoo



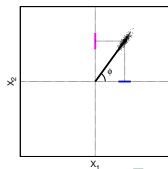
Unsqueezed
coherent



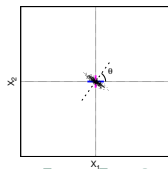
Amplitude
squeezed



Phase
squeezed

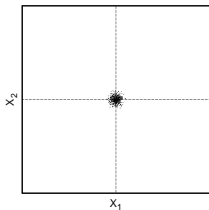


Vacuum
squeezed



Squeezed field generation recipe

Take a vacuum
state $|0\rangle$

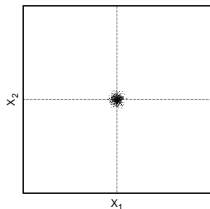


$$H = \frac{1}{2}$$

Squeezed field generation recipe

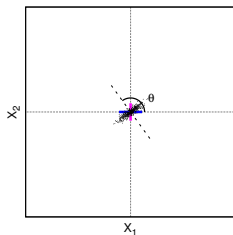
Take a vacuum state $|0\rangle$

Apply squeezing operator $|\xi\rangle = \hat{S}(\xi)|0\rangle$



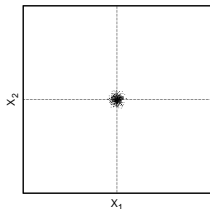
$$H = \frac{1}{2}$$

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$



Squeezed field generation recipe

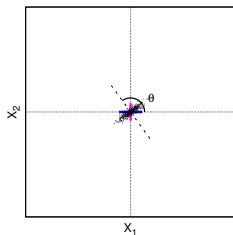
Take a vacuum state $|0\rangle$



$$H = \frac{1}{2}$$

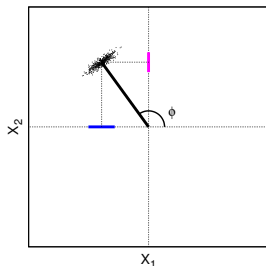
Apply squeezing operator $|\xi\rangle = \hat{S}(\xi)|0\rangle$

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$



Apply displacement operator $|\alpha, \xi\rangle = \hat{D}(\alpha)|\xi\rangle$

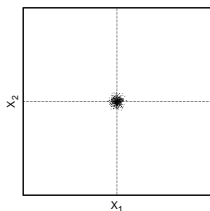
$$\hat{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$



$$\begin{aligned}\langle \alpha, \xi | X_1 | \alpha, \xi \rangle &= \text{Re}(\alpha), \\ \langle \alpha, \xi | X_2 | \alpha, \xi \rangle &= \text{Im}(\alpha)\end{aligned}$$

Squeezed field generation recipe

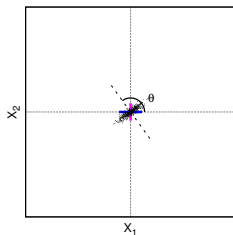
Take a vacuum state $|0\rangle$



$$H = \frac{1}{2}$$

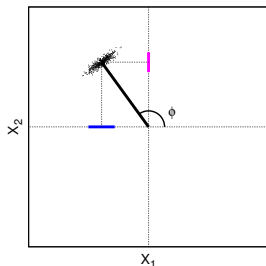
Apply squeezing operator $|\xi\rangle = \hat{S}(\xi)|0\rangle$

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$



Apply displacement operator $|\alpha, \xi\rangle = \hat{D}(\alpha)|\xi\rangle$

$$\hat{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$

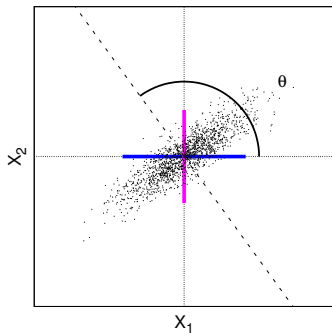


$$\langle \alpha, \xi | X_1 | \alpha, \xi \rangle = \text{Re}(\alpha),$$

$$\langle \alpha, \xi | X_2 | \alpha, \xi \rangle = \text{Im}(\alpha)$$

Notice $\Delta X_1 \Delta X_2 = \frac{1}{4}$

Squeezed state $|\xi\rangle = \hat{S}(\xi)|0\rangle$ properties



$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}, \xi = r e^{i\theta}$$

If $\theta = 0$

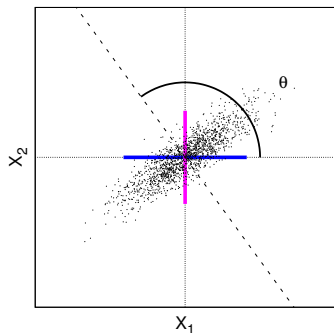
$$\langle \xi | (\Delta X_1)^2 | \xi \rangle = \frac{1}{4} e^{-2r}$$

$$\langle \xi | (\Delta X_2)^2 | \xi \rangle = \frac{1}{4} e^{2r}$$

$$\langle \xi | (\Delta X_1)^2 | \xi \rangle = \frac{1}{4} (\cosh^2 r + \sinh^2 r - 2 \sinh r \cosh r \cos \theta)$$

$$\langle \xi | (\Delta X_2)^2 | \xi \rangle = \frac{1}{4} (\cosh^2 r + \sinh^2 r + 2 \sinh r \cosh r \cos \theta)$$

Photon number of squeezed state $|\xi\rangle$



Probability to detect given number of photons $C = \langle n | \xi \rangle$ for squeezed vacuum

- even

$$C_{2m} = (-1)^m \frac{\sqrt{(2m)!}}{2^m m!} \frac{(e^{i\theta} \tanh r)^m}{\sqrt{\cosh r}}$$

- odd

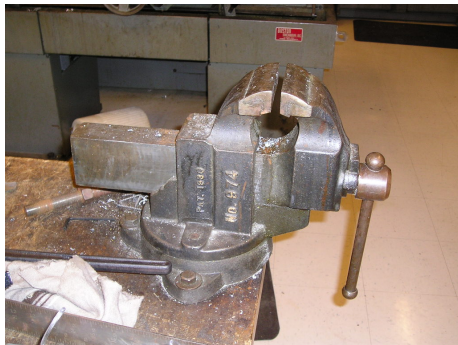
$$C_{2m+1} = 0$$

Average number of photons in general squeezed state

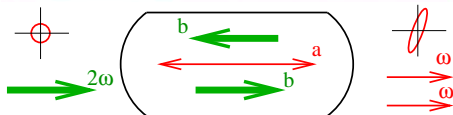
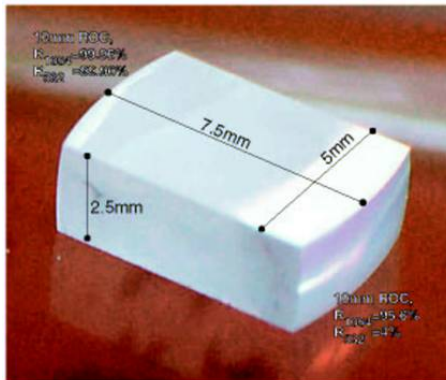
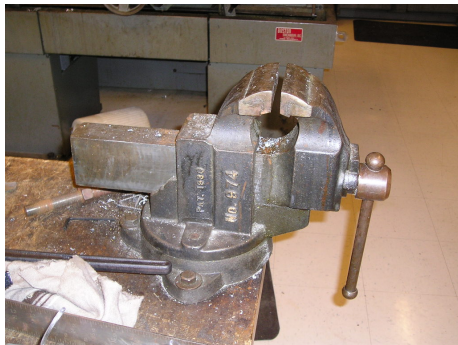
$$\langle \alpha, \xi | a^\dagger a | \alpha, \xi \rangle = \alpha + \sinh^2 r$$

Tools for squeezing

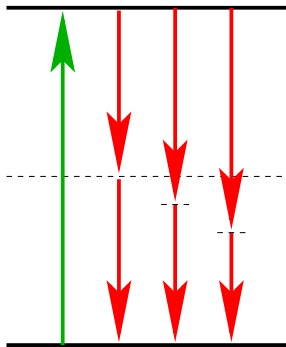
Tools for squeezing



Tools for squeezing



Two photon squeezing picture

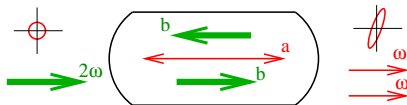


Squeezing operator

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$

Parametric down-conversion in crystal

$$\hat{H} = i\hbar\chi^{(2)}(a^2 b^\dagger - a^{\dagger 2} b)$$



Squeezing

maximum squeezing value detected **11.5 dB at 1064 nm**

Moritz Mehmet, Henning Vahlbruch, Nico Lastzka, Karsten Danzmann, and Roman Schnabel, Phys. Rev. A **81**, 013814 (2010)

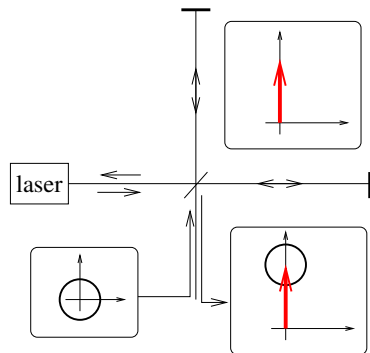
Possible squeezing applications

- shot noise limited optical sensors enhancements
- noiseless signal amplification
- photon pair generation, entanglement, true single photon sources
- interferometers sensitivity boost (for example gravitational wave antennas)
- light free measurements
- quantum memory probe and information carrier

Squeezing and interferometer

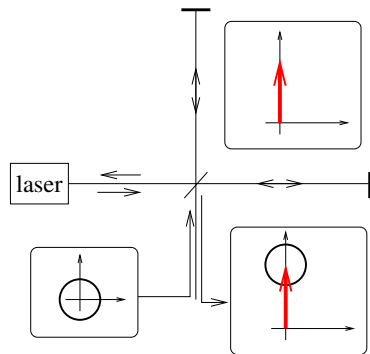
Squeezing and interferometer

Vacuum input

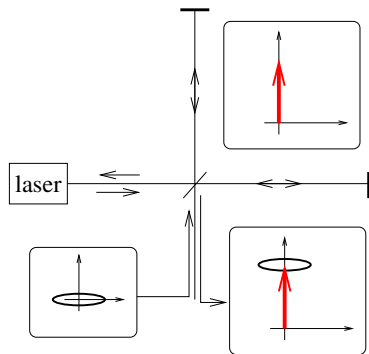


Squeezing and interferometer

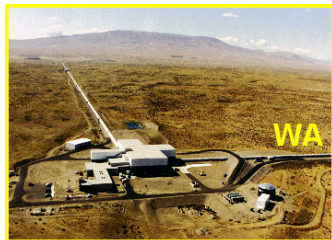
Vacuum input



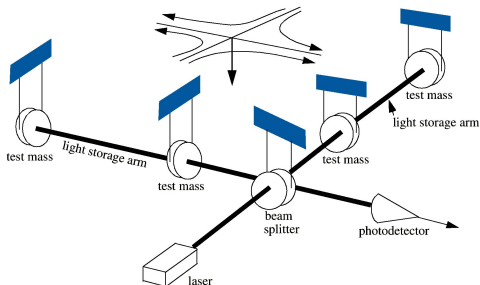
Squeezed input



Laser Interferometer Gravitational-wave Observatory



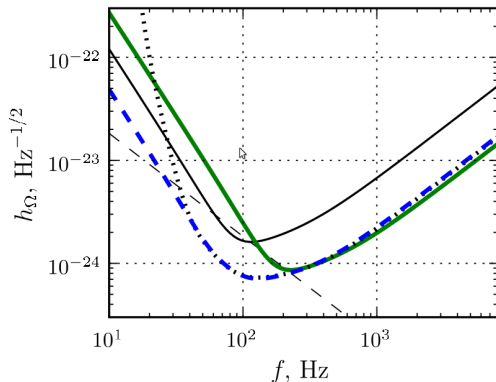
- $L = 4 \text{ km}$
- $h \sim 2 \times 10^{-23}$
- $\Delta L \sim 10^{-20} \text{ m}$



Interferometer sensitivity improvement with squeezing

F. Ya. Khalili Phys. Rev. D 81, 122002 (2010)

Projected advanced LIGO sensitivity



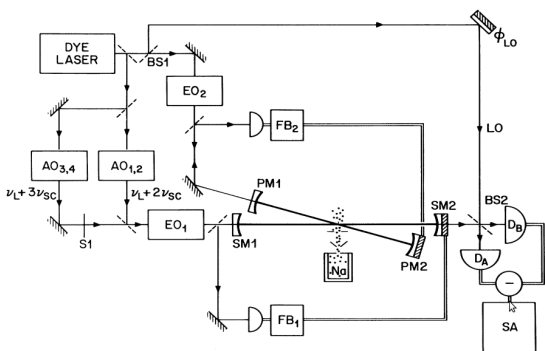
Experimental demonstration with LIGO detectors

Nature Physics, **4**, 472-476, (2008)

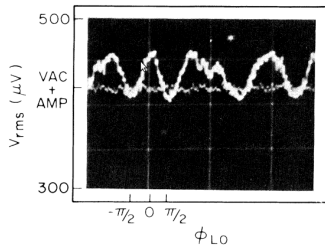
Nature Photonics **7**, 613-619 (2013)

Squeezed States Generated by Four-Wave Mixing

R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley.
Phys. Rev. Lett. 55, 2409-2412 (1985)

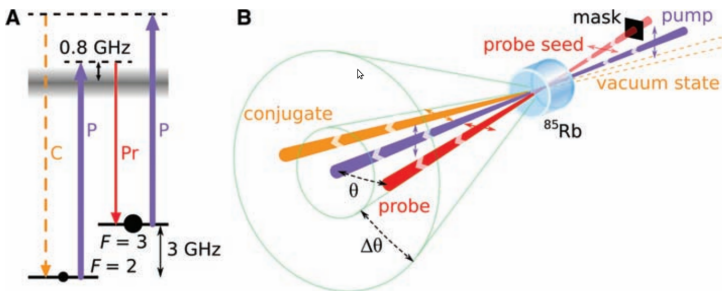


Analysis frequency = 422 MHz



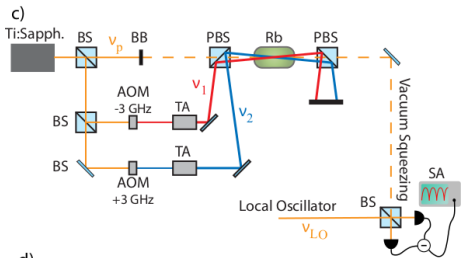
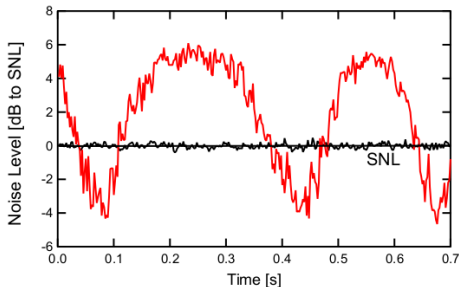
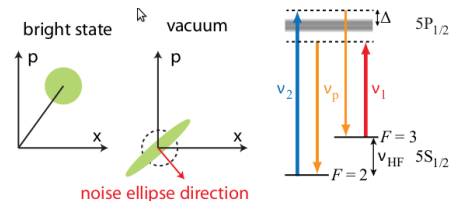
Four-wave-mixing induced squeezing

Vincent Boyer, Alberto M. Marino, Raphael C. Pooser and Paul D. Lett
Science, Vol. 321 no. 5888 pp. 544-547 (2008)

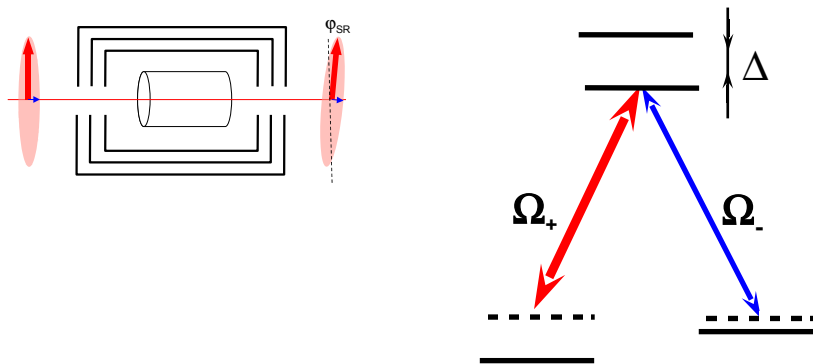


Degenerate vacuum squeezing via four-wave-mixing

Neil V. Corzo, Quentin Glorieux, Alberto M. Marino, Jeremy B. Clark, Ryan T. Glasser, and Paul D. Lett Phys. Rev. A 88, 043836 (2013)



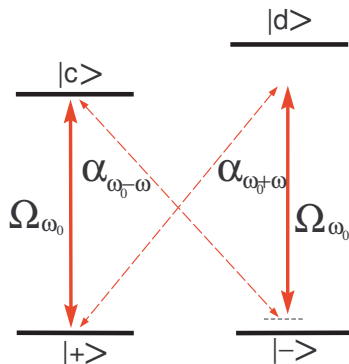
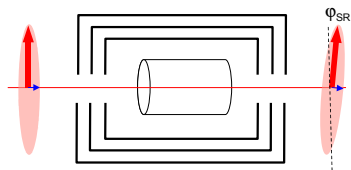
Self-rotation of elliptical polarization in atomic medium



A.B. Matsko et al., PRA 66, 043815 (2002): theoretically prediction of 4-6 dB noise suppression

$$a_{out} = a_{in} + \frac{igL}{2}(a_{in}^{\dagger} - a_{in})$$

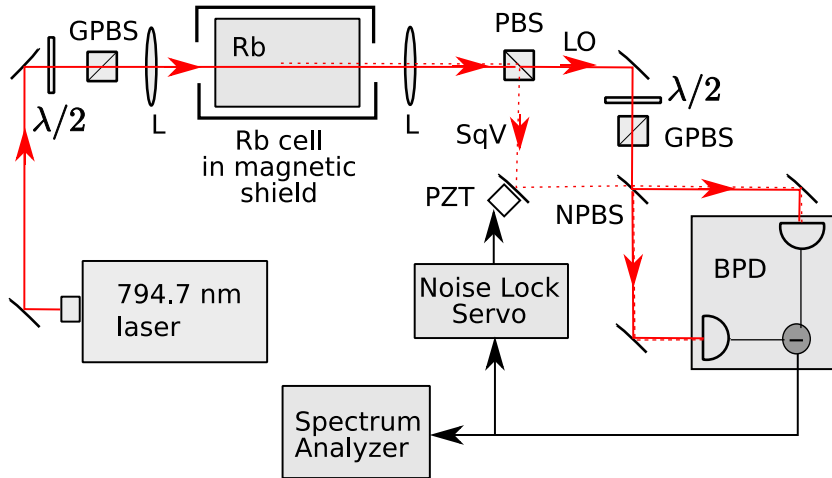
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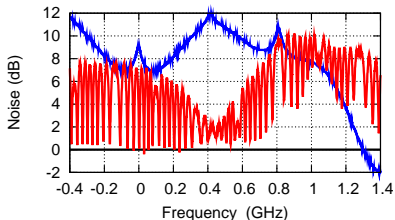
Setup



Noise contrast vs detuning in hot ^{87}Rb vacuum cell

$$F_g = 2 \rightarrow F_e = 1, 2$$

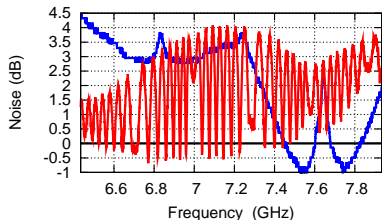
Noise vs detuning



Transmission — PSR noise

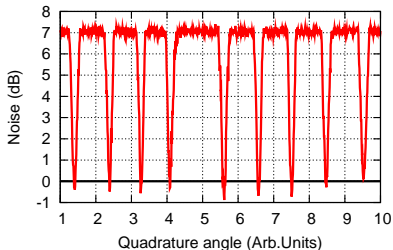
$$F_g = 1 \rightarrow F_e = 1, 2$$

Noise vs detuning



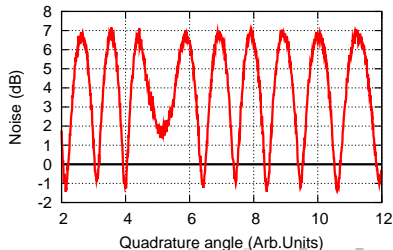
Transmission — PSR noise

Noise vs quadrature angle



Quadrature angle (Arb.Units)

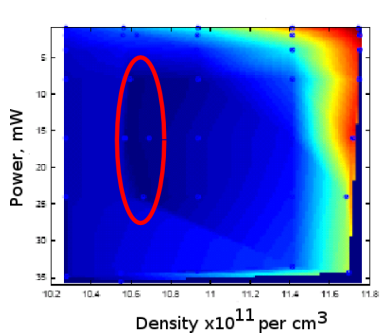
Noise vs quadrature angle



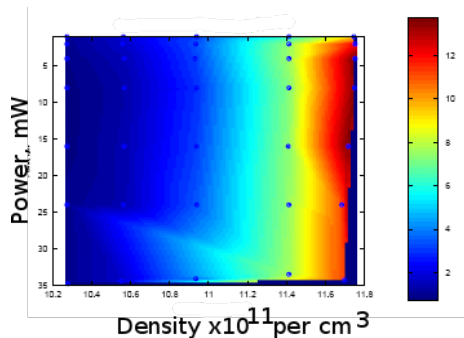
Quadrature angle (Arb.Units)

Squeezing region

Squeezing



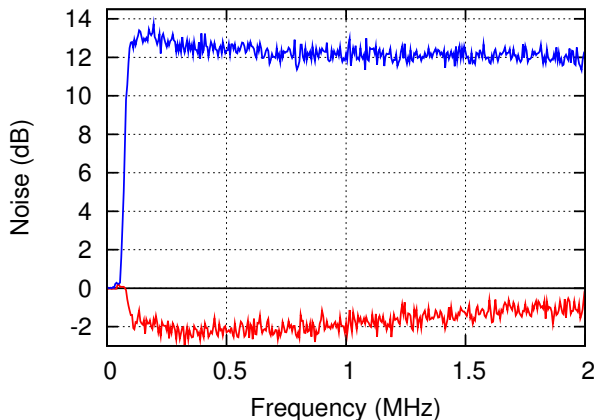
Anti-squeezing



Observation of reduction of quantum noise below the shot noise limit is corrupted by the excess noise due to atomic interaction with atoms.

Maximally squeezed spectrum with ^{87}Rb

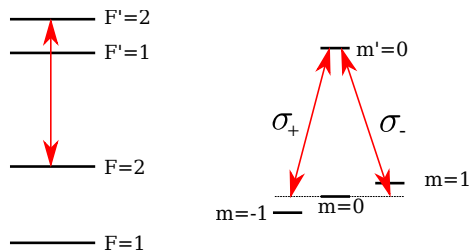
W&M team. ^{87}Rb $F_g = 2 \rightarrow F_e = 2$, laser power 7 mW, $T=65^\circ\text{C}$



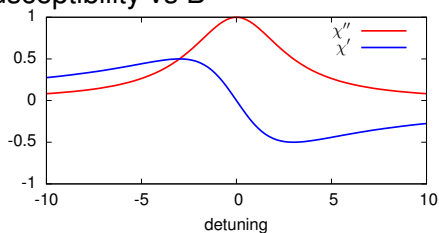
Lezama et.al report 3 dB squeezing in similar setup
Phys. Rev. A 84, 033851 (2011)

Optical magnetometer based on Faraday effect

^{87}Rb D₁ line

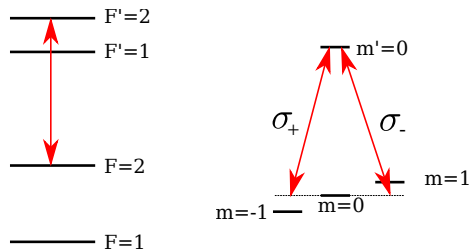


Susceptibility vs B

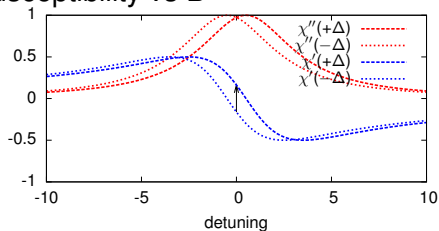


Optical magnetometer based on Faraday effect

^{87}Rb D₁ line

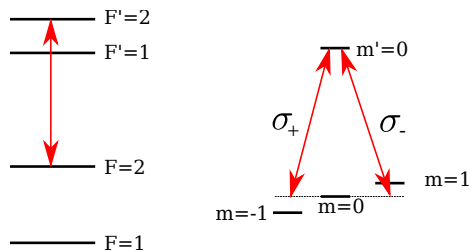


Susceptibility vs B

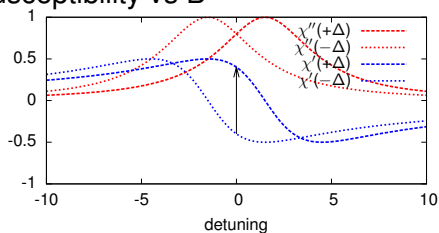


Optical magnetometer based on Faraday effect

^{87}Rb D₁ line

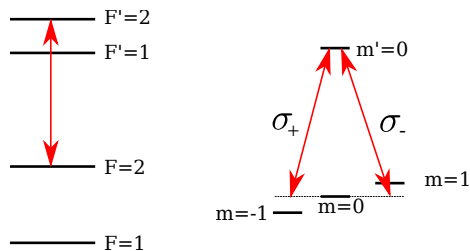


Susceptibility vs B

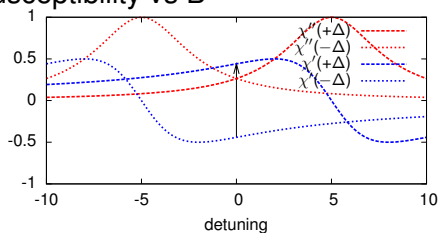


Optical magnetometer based on Faraday effect

^{87}Rb D₁ line

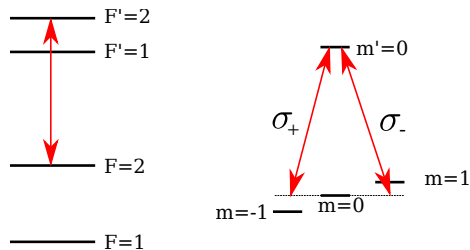


Susceptibility vs B

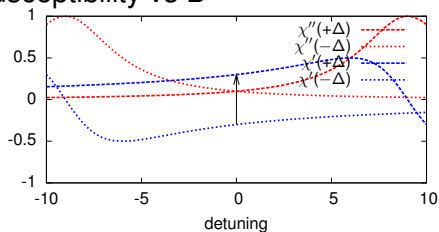


Optical magnetometer based on Faraday effect

^{87}Rb D₁ line

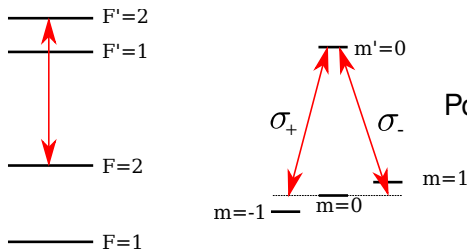


Susceptibility vs B

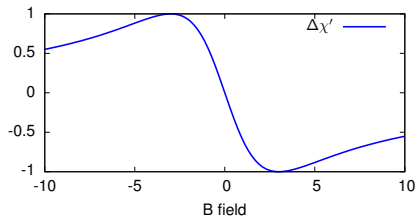


Optical magnetometer based on Faraday effect

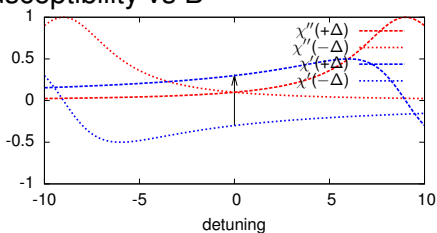
^{87}Rb D₁ line



Polarization rotation vs B

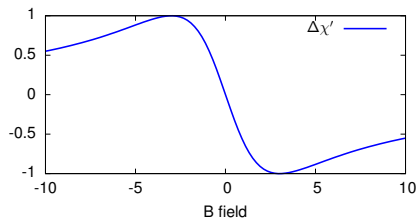


Susceptibility vs B

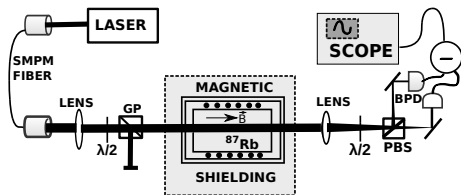


Optical magnetometer and non linear Faraday effect

Naive model of rotation

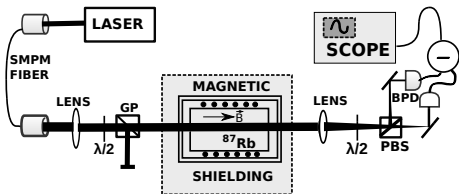
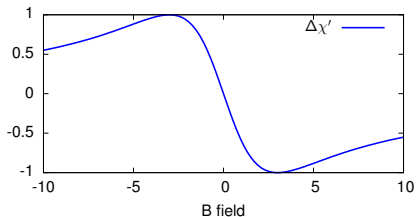


Experiment

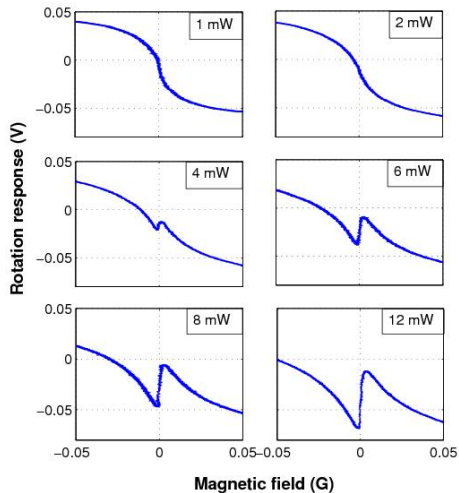


Optical magnetometer and non linear Faraday effect

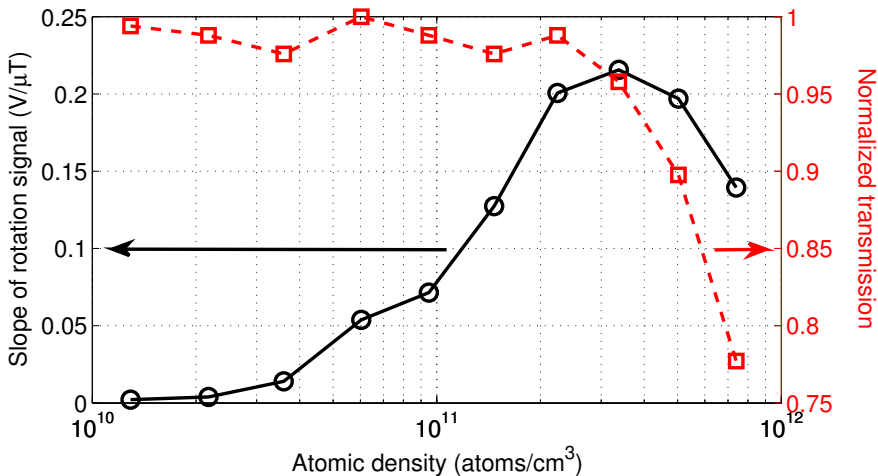
Naive model of rotation



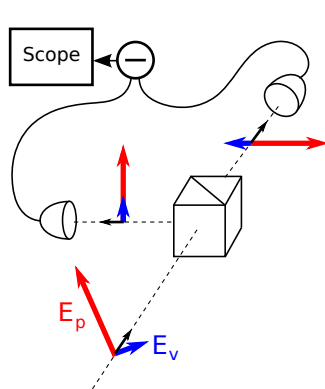
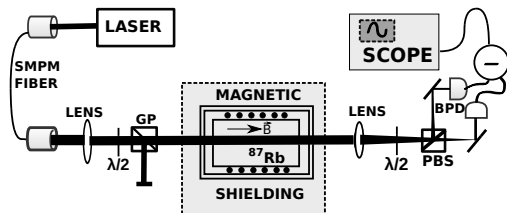
Experiment



Magnetometer response vs atomic density



Shot noise limit of the magnetometer

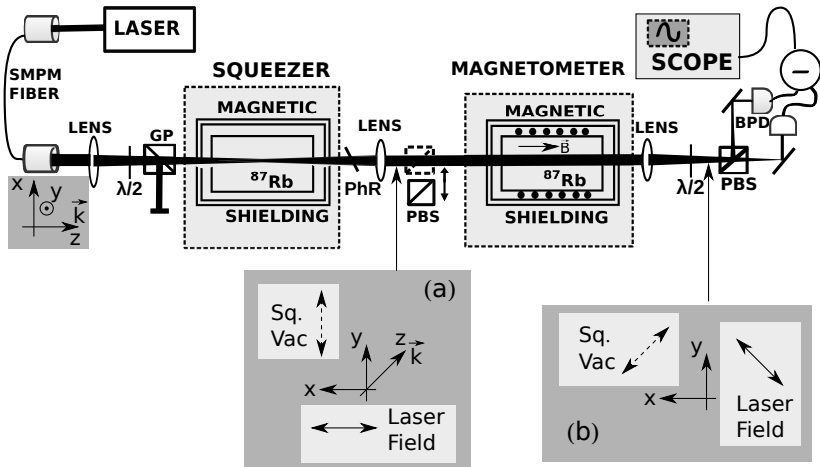


$$S = |E_p + E_v|^2 - |E_p - E_v|^2$$

$$S = 4E_p E_v$$

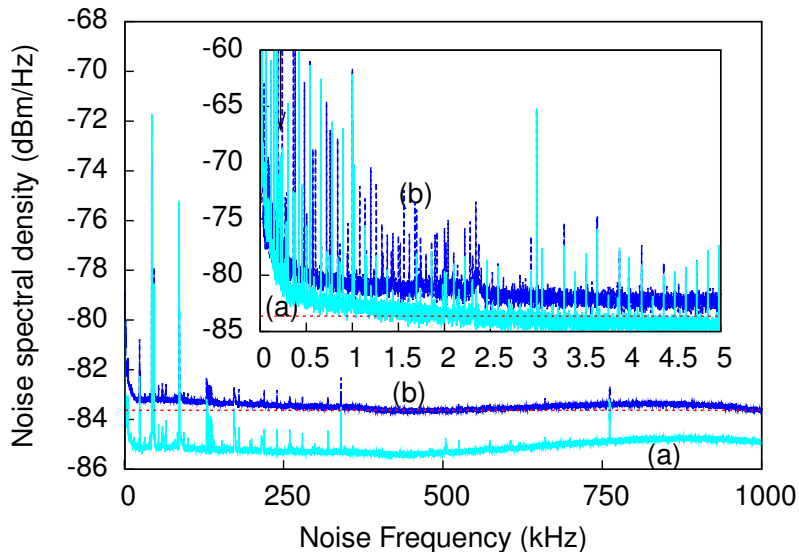
$$\langle \Delta S \rangle \sim E_p \langle \Delta E_v \rangle$$

Squeezed enhanced magnetometer setup

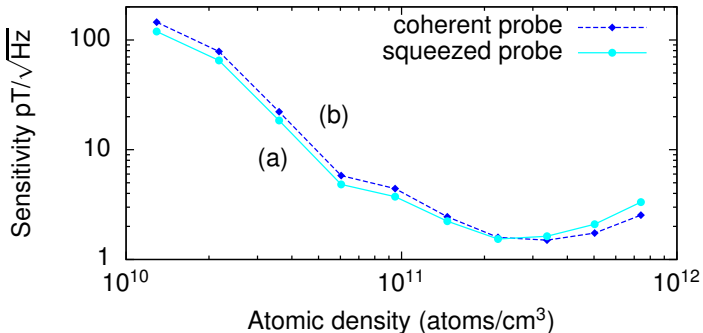
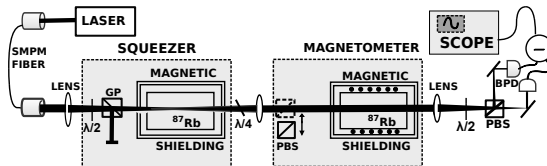


Note: Squeezed enhanced magnetometer was first demonstrated by Wolfgramm *et. al*/ Phys. Rev. Lett, **105**, 053601, 2010.

Magnetometer noise floor improvements

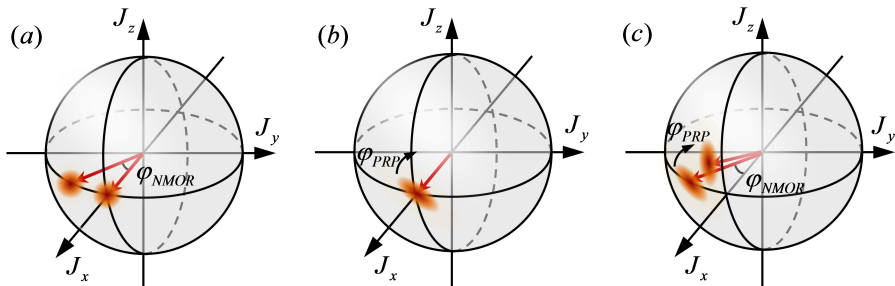
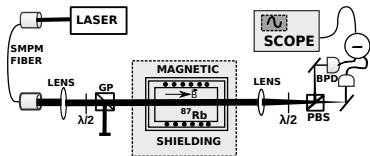


Magnetometer with squeezing enhancement



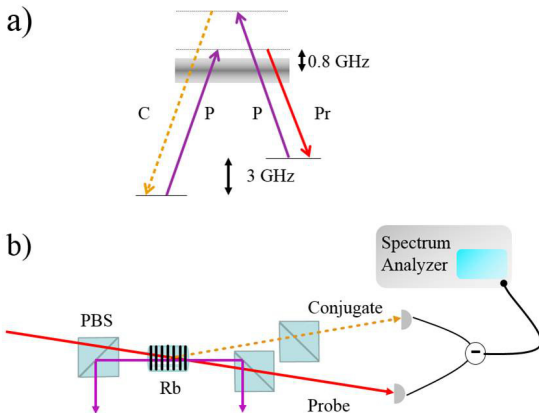
T. Horrom, et al. **PRA**, 86, 023803, (2012).

Self-squeezed magnetometry



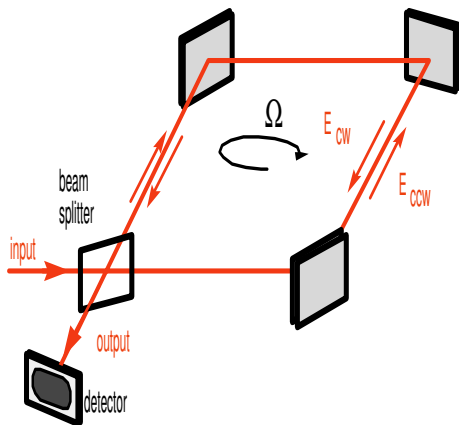
Irina Novikova, Eugeny E. Mikhailov, Yanhong Xiao, "Excess optical quantum noise in atomic sensors", arXiv:1410.3810, (2014).

20 pT/ $\sqrt{\text{Hz}}$ self-squeezed magnetometry with 4WM



N. Otterstrom, R. C. Pooser, and B. J. Lawrie, "Nonlinear optical magnetometry with accessible in situ optical squeezing", *Optics Letters*, **39**, Issue 22, pp. 6533-6536 (2014)

Sagnac effect in interferometer

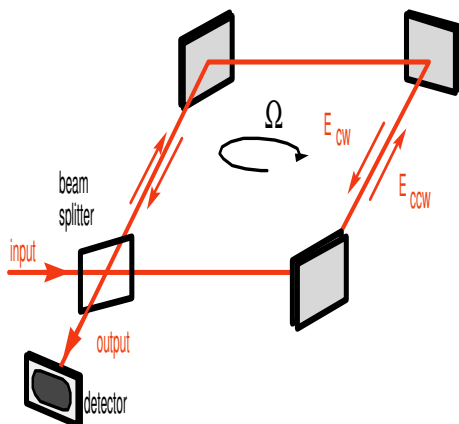


$$t_{\pm} = \frac{2\pi R}{c} \left(1 \pm \frac{R\Omega}{c} \right)$$

$$\Delta t = t_{+} - t_{-} = \frac{4\pi R^2 \Omega}{c^2}$$

$$\Delta\phi = 2\pi \frac{c\Delta t}{\lambda} = \frac{8\pi A\Omega}{c\lambda}$$

Sagnac effect and cavity response



$$\Delta p = c\Delta t = \frac{4A\Omega}{c}$$

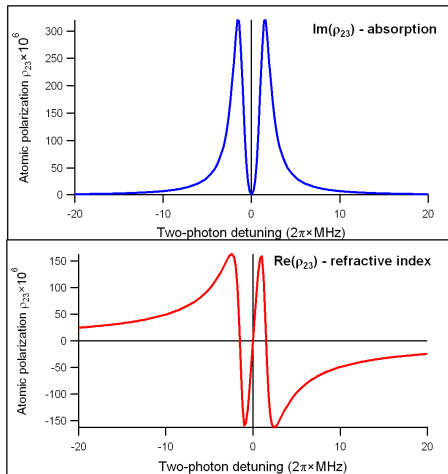
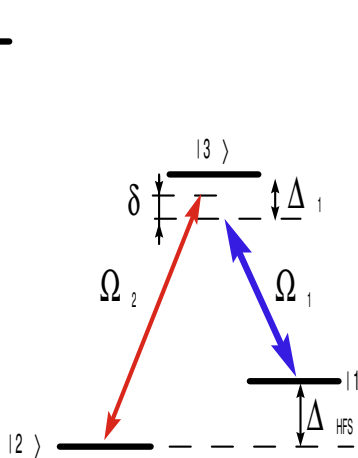
$$\Delta f = f_0 \frac{\Delta p}{p} \frac{1}{n_g} = \Delta f_{empty} \frac{1}{n_g}$$

Group index

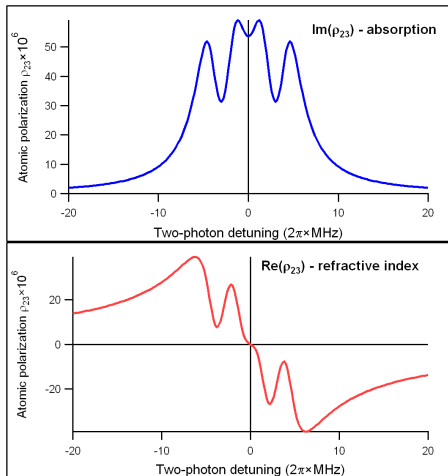
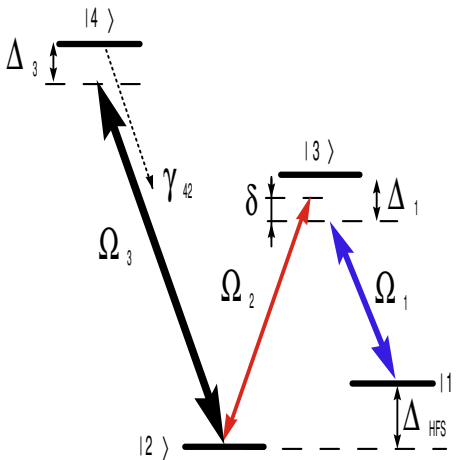
$$n_g(f) = n + f_0 \frac{\partial n}{\partial f}$$

Cavity response enhanced if $n_g < 1$ i.e. under the **fast light** condition
Shahriar et al., PRA **75**, 053807 (2007)

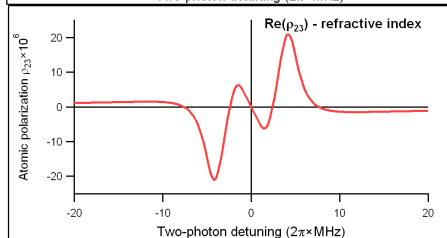
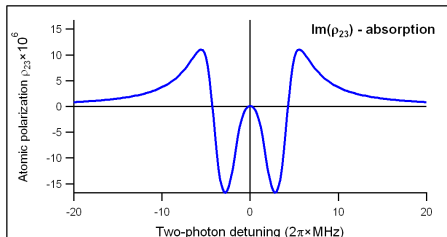
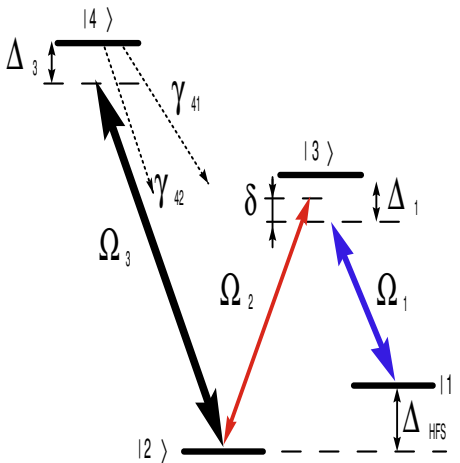
EIT - slow light



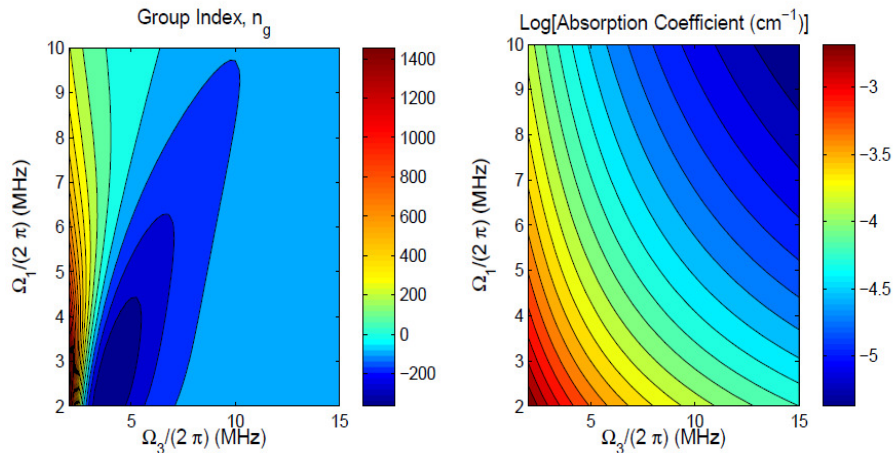
N-scheme, with forbidden transition - fast but no gain



N-bar with four-wave mixing - fast and with gain



N-bar with four-wave mixing - optimal parameters

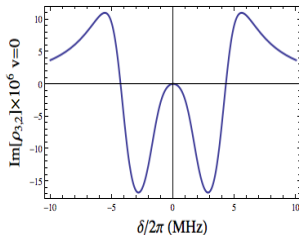
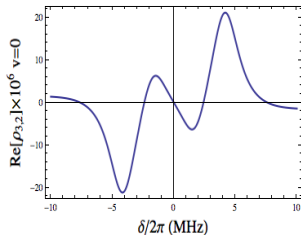


N. B. Phillips, *et al.* Journal of Modern Optics, Issues 1, 60, 64-72, (2013).

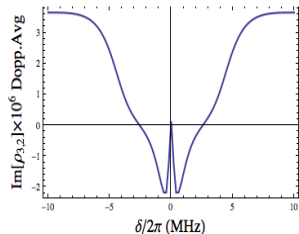
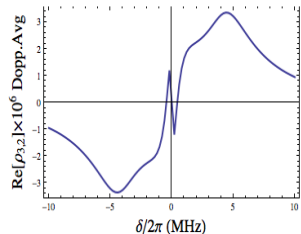
N-bar with Doppler averaging

Refractive index

Absorption



Stationary atoms



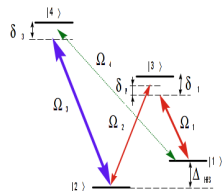
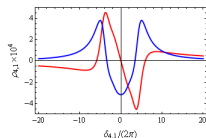
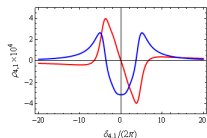
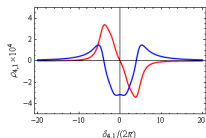
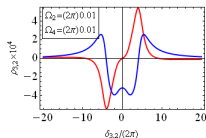
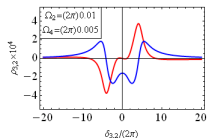
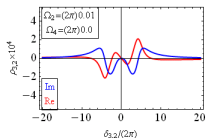
Room temperature
Doppler averaged

N-bar field competition

$$\Omega_4 = 0$$

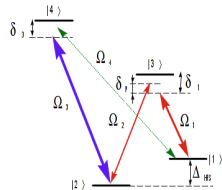
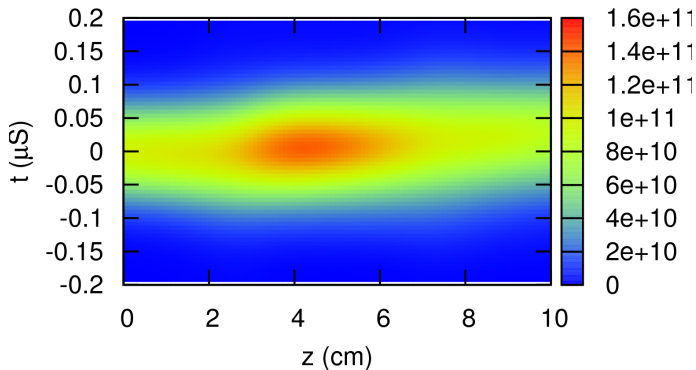
$$\Omega_4 = \Omega_2/2$$

$$\Omega_4 = \Omega_2$$



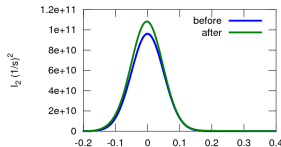
$$\Omega_1 = 2\pi 3 \text{ MHz}, \Omega_3 = 2\pi 6 \text{ MHz}, N = 10^9 \text{ cm}^{-3}$$

N-bar beam propagation



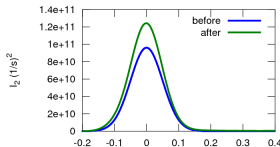
$z=1.6$ cm

I_2 before and after cell



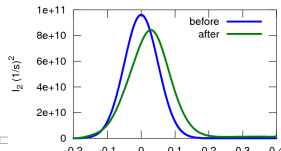
$z=2.4$ cm

I_2 before and after cell

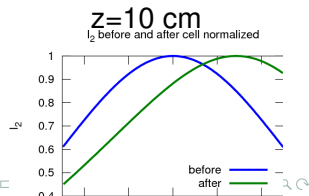
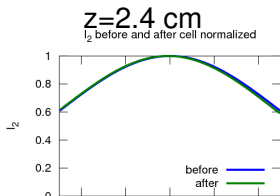
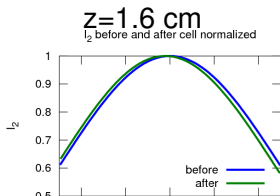
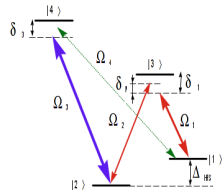
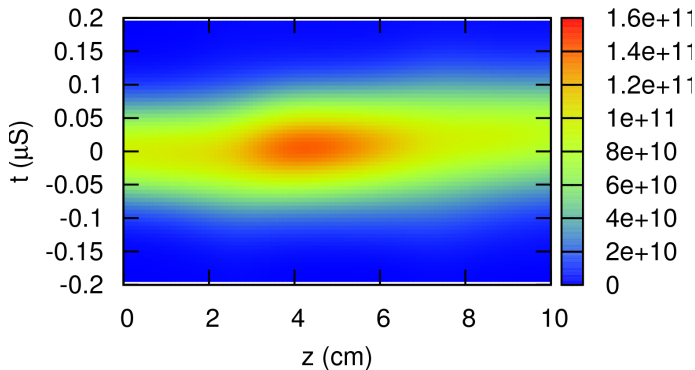


$z=10$ cm

I_2 before and after cell

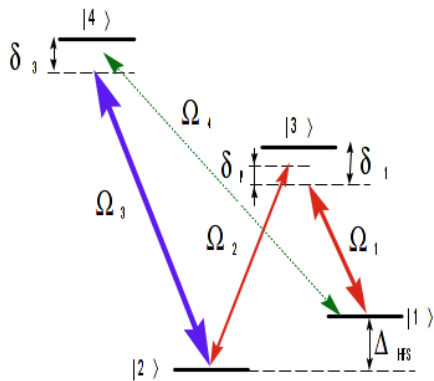


N-bar beam propagation

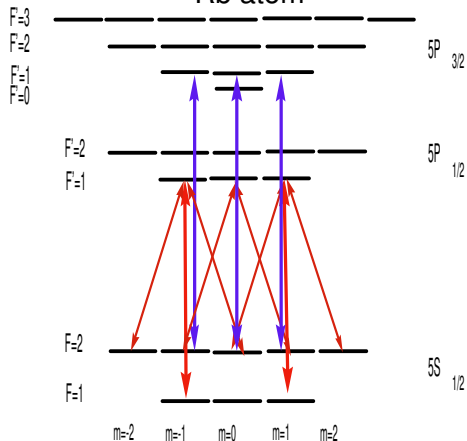


N-bar levels and fields diagram

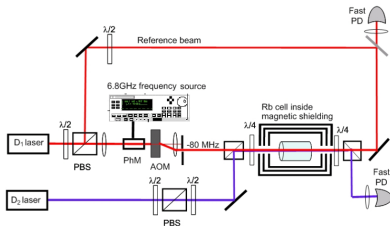
Artificial atom



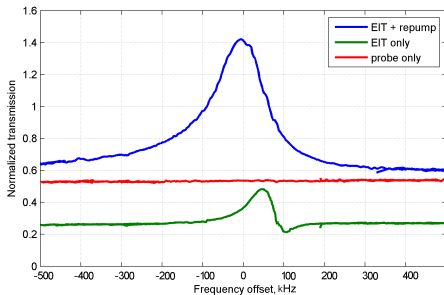
^{87}Rb atom



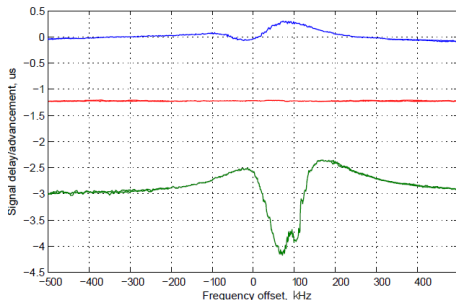
N-bar scheme linearly polarized pumps - single pass



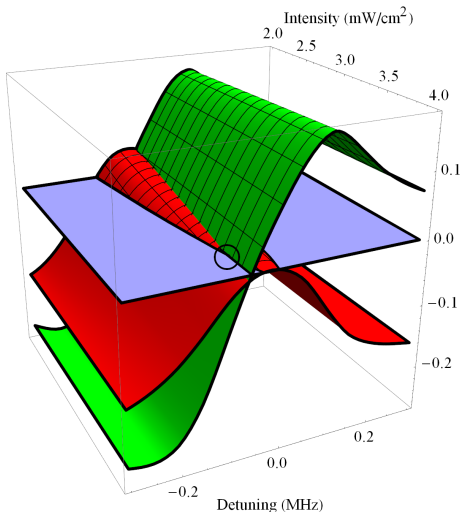
Transmission



Delay



Gyro lasing - no fast light condition



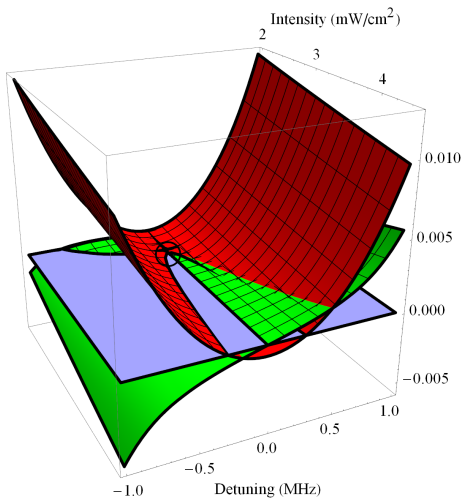
Lasing conditions

- round trip gain is 1
- round trip phase shift is 0

Simulation parameters

- $N = 9.6 \times 10^{10}$ 1/cm³
- $\delta_1 = -2\pi \times 800$ MHz
- $\delta_3 = 0$ MHz
- $\Omega_1 = 6$ mW/cm²
- $\Omega_3 = 10$ mW/cm²
- Finesse = 18

Gyro lasing - fast light condition, single frequency



Lasing conditions

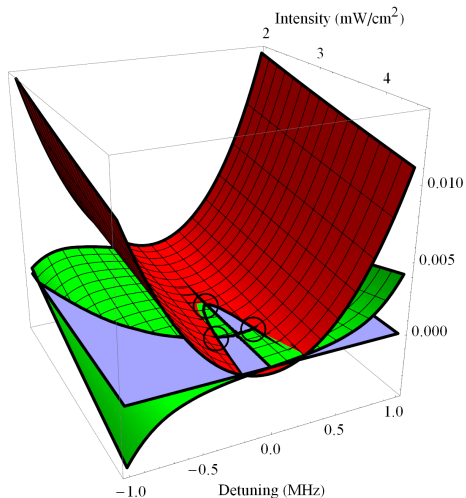
- round trip gain is 1
- round trip phase shift is 0

Simulation parameters

- $N = 4.8 \times 10^{10}$ 1/cm³
- $\delta_1 = 0$ MHz
- $\delta_3 = 0$ MHz
- $\Omega_1 = 48.2$ mW/cm²
- $\Omega_3 = 192.9$ mW/cm²
- Finesse = 60

$$\Delta f = 150 \Delta f_{empty.cavity}$$

Gyro lasing - fast light condition, multiple frequencies



Lasing conditions

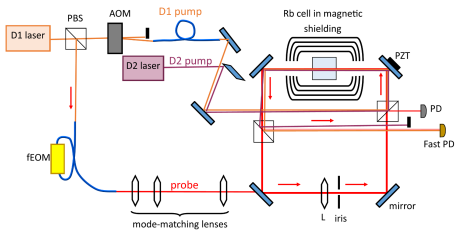
- round trip gain is 1
- round trip phase shift is 0

Simulation parameters

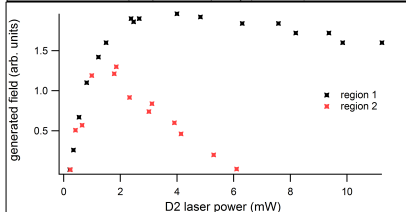
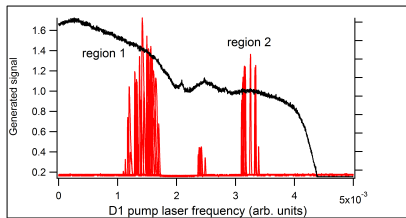
- $N = 4.8 \times 10^{10}$ 1/cm³
- $\delta_1 = 0$ MHz
- $\delta_3 = 0$ MHz
- $\Omega_1 = 50.6$ mW/cm², 5% higher
- $\Omega_3 = 192.9$ mW/cm²
- Finesse = 60

Multiple lasing points might be problematic in the experiment

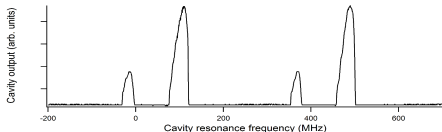
First gyro setup and its performance



Ω_2 tuned around $F = 1 \rightarrow F' = 1, 2$

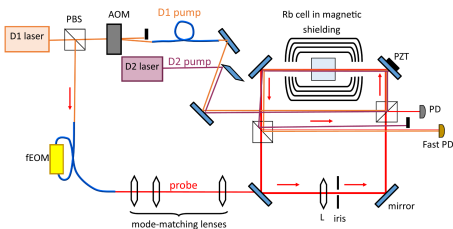


Finesse = 20

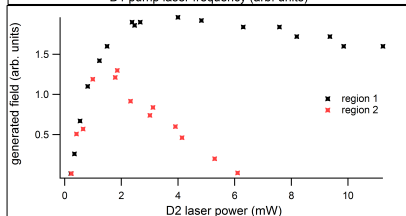
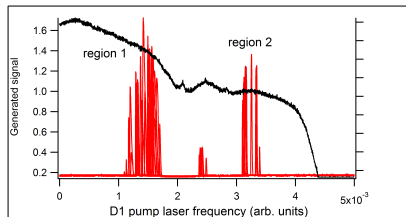


E. Mikhailov, *et al.* Optical Engineering, Issue 10, 53, 102709, (2014)

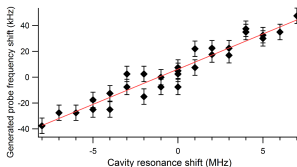
First gyro setup and its performance



Ω_2 tuned around $F = 1 \rightarrow F' = 1, 2$

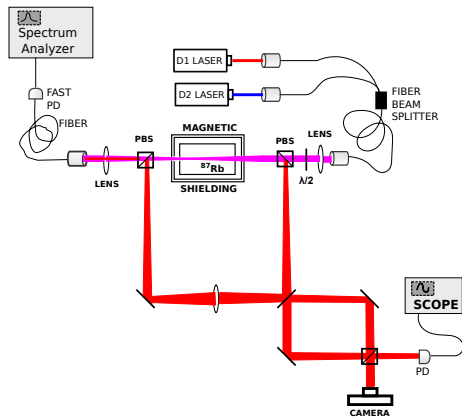
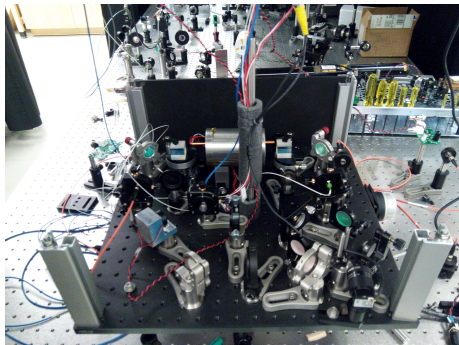


Finesse = 20 \rightarrow Pulling 1/200

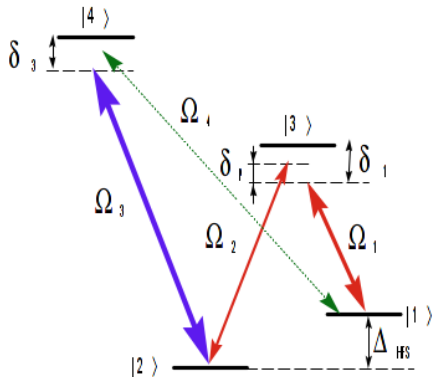
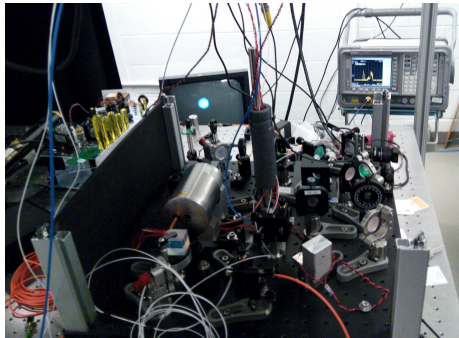


E. Mikhailov, *et al.* Optical Engineering, Issue 10, 53, 102709, (2014)

Gyro setup

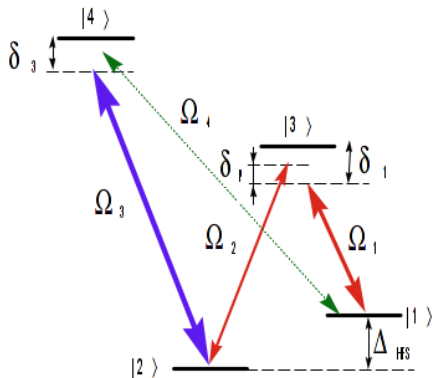
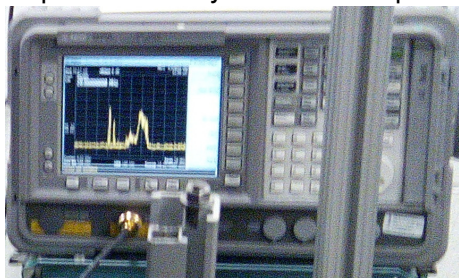


Gyro setup and lasing beat-note



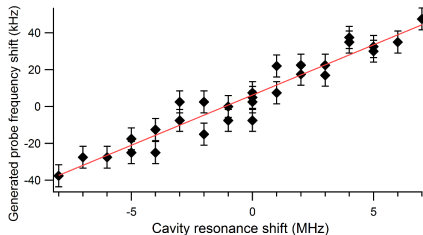
Gyro setup and lasing beat-note

Spectrum analyzer 20 MHz span

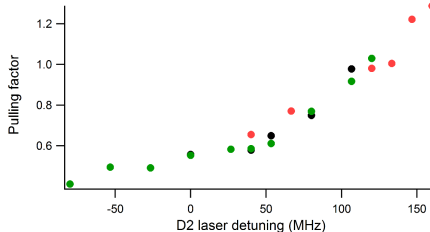


Gyro pulling - response to cavity length change

Old



New



Pulling drastically improved by mainly two factors

- Higher finesse 20 \rightarrow 70
- Higher pumping powers

Higher pulling happens at D_2 detuning where lasing tends to stop

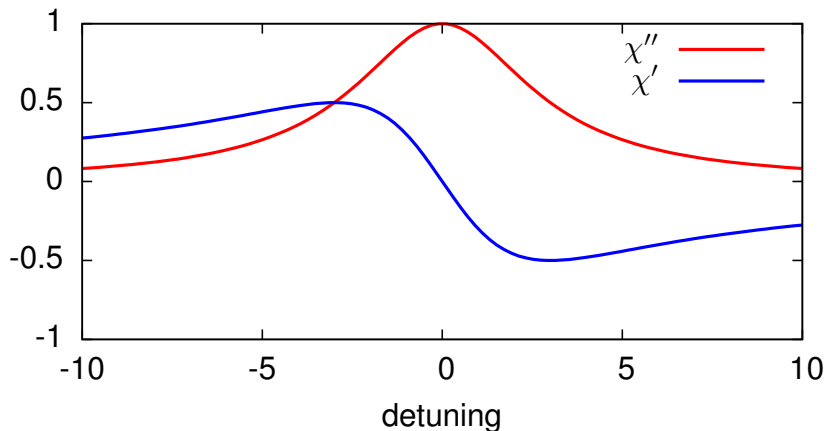
Why superluminal squeezing?

- Quantum memories
- M. S. Shahriar, et al. “Ultrahigh enhancement in absolute and relative rotation sensing using fast and slow light”, Phys. Rev. A **75**(5), 053807, 2007.
- R. W. Boyd, et al. “Noise properties of propagation through slow- and fast- light media”, Journal of Optics **12**, 104007 (2010).

Light group velocity

$$\text{Group velocity } v_g = \frac{c}{\omega \frac{\partial n}{\partial \omega}}$$

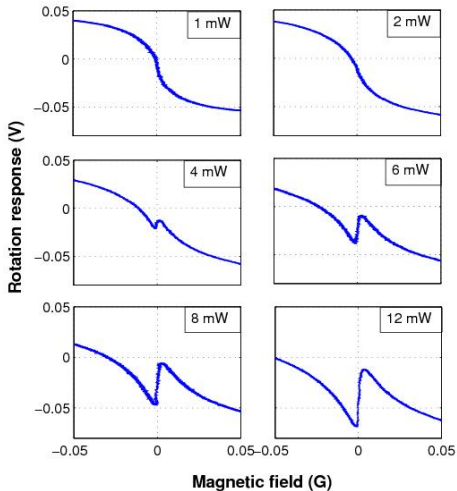
Susceptibility



Light group velocity

$$\text{Group velocity } v_g = \frac{c}{\omega \frac{\partial n}{\partial \omega}}$$

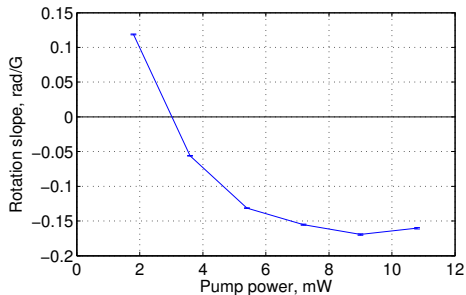
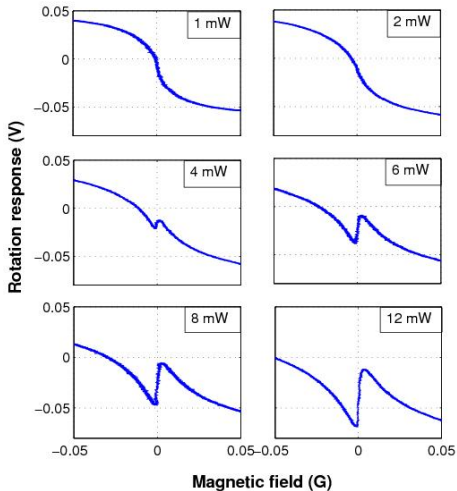
$$\text{Delay } \tau = \frac{L}{v_g} \sim \frac{\partial n}{\partial \omega} \sim \frac{\partial R}{\partial B}$$



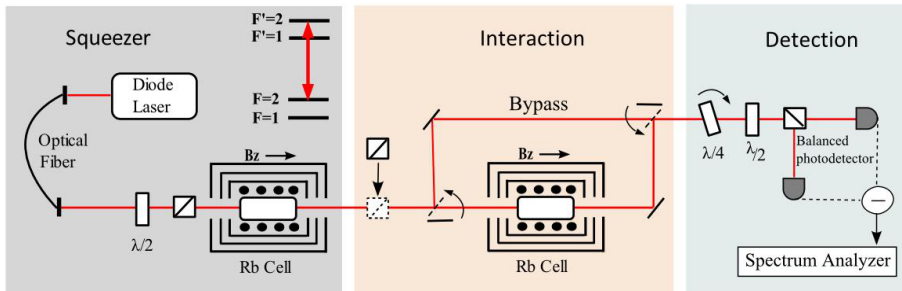
Light group velocity

$$\text{Group velocity } v_g = \frac{c}{\omega \frac{\partial n}{\partial \omega}}$$

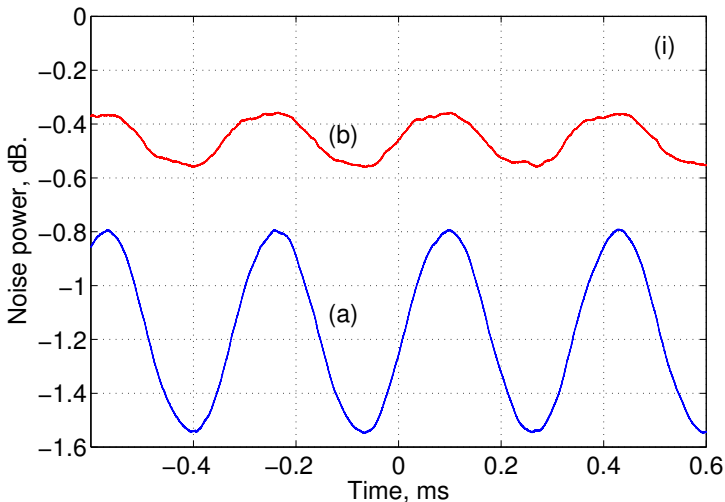
$$\text{Delay } \tau = \frac{L}{v_g} \sim \frac{\partial n}{\partial \omega} \sim \frac{\partial R}{\partial B}$$



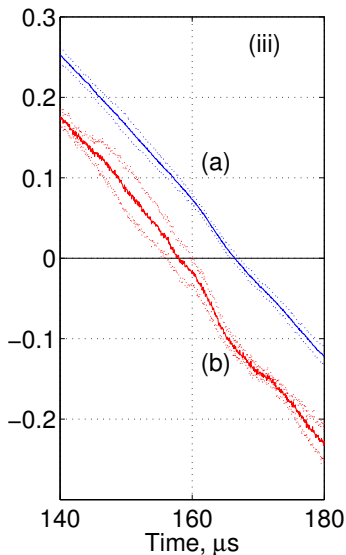
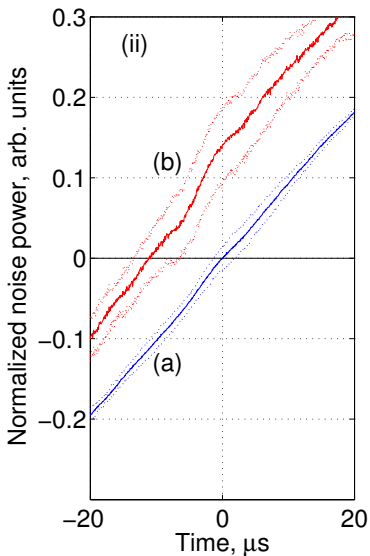
Time advancement setup



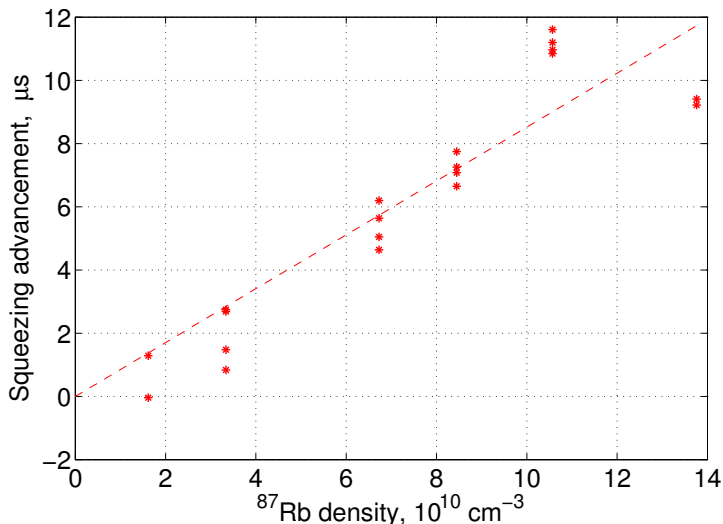
Squeezing modulation and time advancement



Squeezing modulation and time advancement



Squeezing advancement vs atomic density

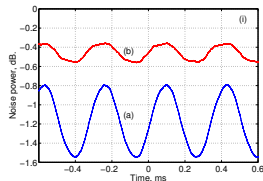
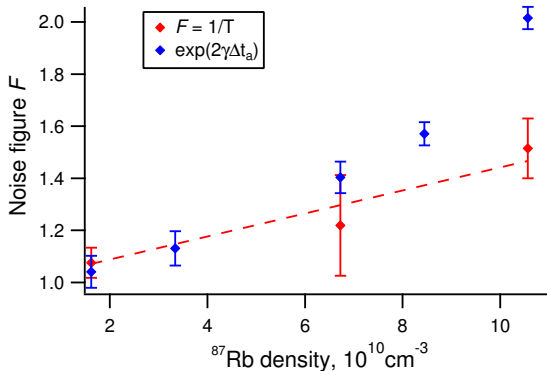


G. Romanov, et al. Optics Letters, Issue 4, 39, 1093-1096, (2014).

Noise figure and advancement

R. W. Boyd, et al. "Noise properties of propagation through slow- and fast- light media", Journal of Optics **12**, 104007 (2010).

$$F = \frac{SNR_{in}}{SNR_{out}} = 1/T = e^{2\gamma\Delta t_a}$$



Summary

Utilizing non-linear light-atom interaction, we demonstrated

- atomic squeezer
- fully atomic squeezed enhanced magnetometer with sensitivity as low as $1 \text{ pT}/\sqrt{\text{Hz}}$
- feasibility of fast laser gyroscope
- superluminal squeezing propagation with $v_g \approx -7'000 \text{ m/s} \approx -c/43'000$ or time advancement of $11 \mu\text{S}$

Support from

