Quantum enhanced measurements

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People



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From ray optics to semiclassical optics

Classical/Geometrical optics

- light is a ray
- which propagates straight
- cannot explain diffraction and interference



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From ray optics to semiclassical optics

Classical/Geometrical optics

- light is a ray
- which propagates straight
- cannot explain diffraction and interference
- Semiclassical optics
 - light is a wave
 - color (wavelength/frequency) is important
 - amplitude (a) and phase are important, E(t) = ae^{i(kz-ωt)}
 - cannot explain residual measurements noise





Detector quantum noise

Simple photodetector



Detector quantum noise

Simple photodetector



Balanced photodetector



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Squeezed light

 $E(\phi) = |a|e^{-i\phi} = |a|\cos(\phi) + i|a|\sin(\phi) = X_1 + iX_2, \ \phi = \omega t - kz$ Detectors sense the real part of the field (X₁) but there is a way to see X_2 as well



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Classical quadratures vs time in a rotating frame

$$E(\phi) = |a|e^{-i\phi} = |a|\cos(\phi) + i|a|\sin(\phi) = X_1 + iX_2, \ \phi = \omega t - kz$$



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$$E(\phi) = |a|e^{-i\phi} = |a|\cos(\phi) + i|a|\sin(\phi) = X_1 + iX_2, \quad \phi = \omega t - kz$$



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Transition from classical to quantum field

Classical analog

- Field amplitude a
- Field real part $X_1 = (a^* + a)/2$
- Field imaginary part $X_2 = i(a^* a)/2$



Quantum approach

- Field operator â
- Amplitude quadrature $\hat{X_1} = (\hat{a}^\dagger + \hat{a})/2$
- Phase quadrature $\hat{\chi_2} = i(\hat{a}^{\dagger} \hat{a})/2$



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Light consist of photons • $\hat{N} = a^{\dagger} a$ Commutator relationship • $[a, a^{\dagger}] = 1$ • $[X_1, X_2] = i/2$ Detectors measure number of photons N • Quadratures \hat{X}_1 and \hat{X}_2 Uncertainty relationship • $\Delta X_1 \Delta X_2 > 1/4$

Heisenberg uncertainty principle and its optics equivalent



Heisenberg uncertainty principle

 $\Delta p \Delta x \geq \hbar/2$

The more precisely the POSITION is determined, the less precisely the MOMENTUM is known, and vice versa

Heisenberg uncertainty principle and its optics equivalent



Heisenberg uncertainty principle

 $\Delta p \Delta x \geq \hbar/2$

The more precisely the POSITION is determined, the less precisely the MOMENTUM is known, and vice versa

Optics equivalent

 $\Delta \phi \Delta N \geq 1$

The more precisely the PHASE is determined, the less precisely the AMPLITUDE is known, and vice versa

Heisenberg uncertainty principle and its optics equivalent



Heisenberg uncertainty principle

 $\Delta p \Delta x \geq \hbar/2$

The more precisely the POSITION is determined, the less precisely the MOMENTUM is known, and vice versa

Optics equivalent

 $\Delta \phi \Delta N \geq 1$

The more precisely the PHASE is determined, the less precisely the AMPLITUDE is known, and vice versa

Optics equivalent strict definition












$\Delta X_1 \Delta X_2 = 1/4$



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 $\Delta X_1 \Delta X_2 = 1/4$



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 $\Delta X_1 \Delta X_2 = 1/4$



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Take a vacuum state |0>



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Take a vacuum state |0>

Apply squeezing operator $|\xi > = \hat{S}(\xi)|0 >$





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Squeezed state $|\xi>=\hat{S}(\xi)|0>$ properties



$$\begin{split} \hat{S}(\xi) &= e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}, \xi = r e^{i\theta} \\ \text{If } \theta &= 0 \\ &< \xi |(\Delta X_1)^2|\xi > = \frac{1}{4} e^{-2r} \\ &< \xi |(\Delta X_2)^2|\xi > = \frac{1}{4} e^{2r} \end{split}$$

$$<\xi|(\Delta X_1)^2|\xi> = \frac{1}{4}(\cosh^2 r + \sinh^2 r - 2\sinh r \cosh r \cos \theta)$$

$$<\xi|(\Delta X_2)^2|\xi> = \frac{1}{4}(\cosh^2 r + \sinh^2 r + 2\sinh r \cosh r \cos \theta)$$

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Photon number of squeezed state $|\xi>$



Probability to detect given number of photons $C = < n | \xi >$ for squeezed vacuum

even

$$C_{2m} = (-1) \frac{\sqrt{(2m)!}}{2^m m!} \frac{(e^{i\theta} \tanh r)^m}{\sqrt{\cosh r}}$$

odd

$$C_{2m+1} = 0$$

Average number of photons in general squeezed state

$$< \alpha, \xi | \boldsymbol{a}^{\dagger} \boldsymbol{a} | \alpha, \xi > = \alpha + \sinh^2 r$$

Tools for squeezing

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Squeezed light

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Tools for squeezing



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Tools for squeezing







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Two photon squeezing picture



Squeezing operator

$$\hat{S}(\xi) = e^{rac{1}{2}\xi^*a^2 - rac{1}{2}\xi a^{\dagger 2}}$$

Parametric down-conversion in crystal

$$\hat{H} = i\hbar\chi^{(2)}(a^2b^\dagger - a^{\dagger 2}b)$$



Squeezing

result of correlation of upper and lower sidebands

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Squeezer appearance



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Squeezer appearance



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- improvements any shot noise limited optical sensors
- noiseless signal amplification
- photon pair generation, entanglement, true single photon sources
- interferometers sensitivity boost (for example gravitational wave antennas)
- light free measurements
- quantum memory probe and information carrier

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Vacuum input



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Squeezing and interferometer

Vacuum input



Squeezed input



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Laser Interferometer Gravitational-wave Observatory





- L = 4 km
- $h \sim 2 \times 10^{-23}$
- $\Delta L \sim 10^{-20} \text{ m}$



Self-rotation of elliptical polarization in atomic medium



A.B. Matsko et al., PRA 66, 043815 (2002): theoretically prediction of 4-6 dB noise suppression

$$a_{out} = a_{in} + rac{igL}{2}(a^{\dagger}_{in} - a_{in})$$

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Self-rotation of elliptical polarization in atomic medium



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Setup



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Noise contrast vs detuning in hot ⁸⁷Rb vacuum cell



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Squeezed light

⁸⁷Rb D₁ line







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 87 Rb D₁ line







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 87 Rb D₁ line





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⁸⁷Rb D₁ line







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 87 Rb D₁ line







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Optical magnetometer based on Faraday effect

 87 Rb D₁ line



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Optical magnetometer and non linear Faraday effect

Naive model of rotation

Experiment





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Squeezed light

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Optical magnetometer and non linear Faraday effect

Naive model of rotation

Experiment





Shot noise limit of the magnetometer



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Squeezed enhanced magnetometer setup



Note: Squeezed enhanced magnetometer was first demonstrated by Wolfgramm *et. al* Phys. Rev. Lett, **105**, 053601, 2010.

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Magnetometer noise floor improvements



Noise suppression and response vs atomic density

Noise suppression (dB) 5 kHz 100 kHz 500 kHz 1 MHz -6^{-10} 10¹² 10¹¹ Atomic density (atoms/cm³) 0.25 Slope of rotation signal (V/µT) 0.2 formalized transmission 0.15 0.1 0.05 Þ -<mark>12</mark>75 1010 10¹¹ Atomic density (atoms/cm3)

Noise suppression

Response

Magnetometer with squeezing enhancement



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- Squeezing is exciting
- many applications benefit from squeezing
- there is still a lot of interesting physics to do

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