Squeezed states of light with hot atoms

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\[ E(\phi) = |a| e^{-i\phi} = |a| \cos(\phi) + i |a| \sin(\phi) = X_1 + iX_2, \quad \phi = \omega t - kz \]
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Transition from classical to quantum field

Classical analog

- Field amplitude $a$
- Field real part $X_1 = (a^* + a)/2$
- Field imaginary part $X_2 = i(a^* - a)/2$

$$E(\phi) = |a|e^{-i\phi} = X_1 + iX_2$$

Quantum approach

- Field operator $\hat{a}$
- Amplitude quadrature $\hat{X}_1 = (\hat{a}^\dagger + \hat{a})/2$
- Phase quadrature $\hat{X}_2 = i(\hat{a}^\dagger - \hat{a})/2$

$$\hat{E}(\phi) = \hat{X}_1 + i\hat{X}_2$$
Light consists of photons

\[ \hat{N} = a^\dagger a \]

Commutator relationship

\[ [a, a^\dagger] = 1 \]
\[ [X_1, X_2] = i/2 \]

Detectors measure

- number of photons \( \hat{N} \)
- Quadratures \( \hat{X}_1 \) and \( \hat{X}_2 \)

Uncertainty relationship

\[ \Delta X_1 \Delta X_2 \geq 1/4 \]
Heisenberg uncertainty principle and its optics equivalent

**Heisenberg uncertainty principle**

\[ \Delta p \Delta x \geq \frac{\hbar}{2} \]

The more precisely the POSITION is determined, the less precisely the MOMENTUM is known, and vice versa.

**Optics equivalent**

\[ \Delta \phi \Delta N \geq 1 \]

The more precisely the PHASE is determined, the less precisely the AMPLITUDE is known, and vice versa.

**Optics equivalent strict definition**

\[ \Delta X_1 \Delta X_2 \geq \frac{1}{4} \]
Heisenberg uncertainty principle and its optics equivalent

Heisenberg uncertainty principle

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Squeezed quantum states zoo

Unsqueezed coherent
Squeezed quantum states zoo

Unsqueezed coherent

Amplitude squeezed

Vacuum squeezed

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Squeezed quantum states zoo

Unsqueezed coherent

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Squeezed field generation recipe

Take a vacuum state $|0\rangle$

$$H = \frac{1}{2}$$
Squeezed field generation recipe

Take a vacuum state $|0\rangle$

Apply squeezing operator $|\xi\rangle = \hat{S}(\xi)|0\rangle$

$$\hat{S}(\xi) = e^{\frac{1}{2} \xi^* a^2 - \frac{1}{2} \xi a^\dagger^2}$$

$$H = \frac{1}{2}$$
Squeezed field generation recipe

Take a vacuum state \( |0\rangle \)

Apply squeezing operator \( |\xi\rangle = \hat{S}(\xi)|0\rangle \)

Apply displacement operator \( |\alpha, \xi\rangle = \hat{D}(\alpha)|s\rangle \)

\[
\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^\dagger 2}
\]

\[
\hat{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}
\]

\[
<\alpha, \xi|X_1|\alpha, \xi> = Re(\alpha),
\]

\[
<\alpha, \xi|X_2|\alpha, \xi> = Im(\alpha)
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Squeezed field generation recipe

Take a vacuum state $|0\rangle$

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\[
H = \frac{1}{2}
\]

Notice $\Delta X_1 \Delta X_2 = \frac{1}{4}$

$\langle \alpha, \xi | X_1 | \alpha, \xi \rangle = Re(\alpha)$,

$\langle \alpha, \xi | X_2 | \alpha, \xi \rangle = Im(\alpha)$
Photon number of squeezed state $|\xi>$

Probability to detect given number of photons $C = \langle n|\xi>$ for squeezed vacuum

- even

$$C_{2m} = (-1)^m \frac{\sqrt{(2m)!} \left(e^{i\theta} \tanh r\right)^m}{2^m m! \sqrt{\cosh r}}$$

- odd

$$C_{2m+1} = 0$$

Average number of photons in general squeezed state

$$\langle \alpha, \xi| a\dagger a|\alpha, \xi> = \alpha + \sinh^2 r$$
Two photon squeezing picture

Squeezing operator

\[ \hat{S}(\xi) = e^{\frac{1}{2} \xi^* a^2 - \frac{1}{2} \xi a^\dagger 2} \]

Parametric down-conversion in crystal

\[ \hat{H} = i\hbar \chi^{(2)}(a^2 b^\dagger - a^\dagger 2 b) \]

Squeezing

maximum squeezing value detected 11.5 dB at 1064 nm
Squeezed States Generated by Four-Wave Mixing

Squeezed States Generated by Four-Wave Mixing


Analysis frequency = 422 MHz
Coupling induced squeezing

M. G. Raizen, L. A. Orozco, Min Xiao, T. L. Boyd, and H. J. Kimble

Analysis frequency = 270 MHz
Doppler-free spectroscopy of Cs with squeezing


Analysis frequency = 2.7 MHz
Squeezing in Fluorescence of Two-Level Atoms $^{174}$Yb

Four-wave-mixing induced squeezing

Vincent Boyer, Alberto M. Marino, Raphael C. Pooser and Paul D. Lett
Four-wave-mixing induced squeezing

Vincent Boyer, Alberto M. Marino, Raphael C. Pooser and Paul D. Lett

Analysis frequency = 3.5 MHz
Degenerate vacuum squeezing via four-wave-mixing

Self-rotation of elliptical polarization in atomic medium

A.B. Matsko et al., PRA 66, 043815 (2002): theoretically prediction of 4-6 dB noise suppression

\[ a_{out} = a_{in} + \frac{igL}{2}(a_{in}^\dagger - a_{in}) \]
Self-rotation of elliptical polarization in atomic medium

A.B. Matsko et al., PRA 66, 043815 (2002): theoretically prediction of 4-6 dB noise suppression

\[ a_{out} = a_{in} + \frac{igL}{2}(a_{in}^{\dagger} - a_{in}) \]
 Noise contrast vs detuning in hot $^{87}$Rb vacuum cell

$F_g = 2 \rightarrow F_e = 1, 2$

Noise vs detuning

$F_g = 1 \rightarrow F_e = 1, 2$

Noise vs detuning

Noise vs quadrature angle

Noise vs quadrature angle
Observation of reduction of quantum noise below the shot noise limit is corrupted by the excess noise due to atomic interaction with atoms.
Maximally squeezed spectrum with $^{87}$Rb

W&M team. $^{87}$Rb $F_g = 2 \rightarrow F_e = 2$, laser power 7 mW, $T=65° C$

Lezama et.al report 3 dB squeezing in similar setup
Optical magnetometer based on Faraday effect

$^{87}$Rb D$_1$ line

- $F'=2$
- $F'=1$
- $F=2$
- $F=1$
- $m=0$
- $m'=0$
- $m=1$
- $m=-1$

Susceptibility vs B

- $\chi''$
- $\chi'$

Polarization rotation vs B

- $\Delta \chi$
Optical magnetometer based on Faraday effect

$^{87}$Rb D$_1$ line

Susceptibility vs B

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Optical magnetometer based on Faraday effect

$^{87}\text{Rb D}_1 \text{ line}$

Susceptibility vs B

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$^{87}\text{Rb D}_1$ line

$F'=2$

$F'=1$

$F=2$

$m=0$

$m=1$

$F=1$

$m'=0$

$m=0$

$m=1$

Susceptibility vs B

$\chi''(+\Delta)$

$\chi''(-\Delta)$

$\chi'(+\Delta)$

$\chi'(-\Delta)$
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Susceptibility vs B

\[ \chi''(\pm \Delta) \]

Polarization rotation vs B

\[ \Delta \chi' \]

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Optical magnetometer based on Faraday effect

$^{87}\text{Rb D}_1$ line

Susceptibility vs B

Polarization rotation vs B
Optical magnetometer and non linear Faraday effect

Naive model of rotation

Experiment

\[ \Delta \chi' \]

Experiment setup:
- Laser
- SMPM fiber
- Lens
- \( \lambda/2 \)
- GP
- Magneto-optical trapping
- \( ^{87}\text{Rb} \)
- \( \lambda/2 \)
- PBS
- BPD
- Scope
- Magnetic shielding

\[ B \text{ field} \]

Graph shows the variation of \( \Delta \chi' \) with respect to the B field.
Optical magnetometer and non-linear Faraday effect

Naive model of rotation

Experiment

![Graph showing B field vs. Δχ′](image1)

![Graphs showing rotation response at different power levels](image2)
Magnetometer response vs atomic density

- **Slope of rotation signal (V/µT)** vs **Atomic density (atoms/cm³)**

Normalized transmission

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Shot noise limit of the magnetometer

\[
S = \left| E_p + E_v \right|^2 - \left| E_p - E_v \right|^2
\]

\[
S = 4E_pE_v
\]

\[
< \Delta S > \sim E_p < \Delta E_v >
\]
Note: Squeezed enhanced magnetometer was first demonstrated by Wolfgramm et. al Phys. Rev. Lett, 105, 053601, 2010.
Magnetometer noise floor improvements

![Graph showing noise spectral density vs. noise frequency.(a) and (b)](image)
Magnetometer with squeezing enhancement

Sensitivity $\text{pT}/\sqrt{\text{Hz}}$ vs. Atomic density (atoms/cm$^3$)

(a) Coherent probe
(b) Squeezed probe

Self-squeezed magnetometry

20 pT/√Hz self-squeezed magnetometry with 4WM

Why superluminal squeezing?

- Quantum memories
Light group velocity

Group velocity \( v_g = \frac{c}{\omega} \frac{\partial n}{\partial \omega} \)

Susceptibility

![Graph showing susceptibility vs. detuning](image_url)
**Light group velocity**

Group velocity \( v_g = \frac{c}{\omega \frac{\partial n}{\partial \omega}} \)

Delay \( \tau = \frac{L}{v_g} \sim \frac{\partial n}{\partial \omega} \sim \frac{\partial R}{\partial B} \)
Light group velocity

Group velocity \( v_g = \frac{c}{\omega \frac{\partial n}{\partial \omega}} \)

Delay \( \tau = \frac{L}{v_g} \sim \frac{\partial n}{\partial \omega} \sim \frac{\partial R}{\partial B} \)
Time advancement setup

Squeezing with hot atoms
Squeezing modulation and time advancement

<table>
<thead>
<tr>
<th>Time, ms</th>
<th>Noise power, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>-0.4</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.4</td>
</tr>
<tr>
<td>0.0</td>
<td>-0.4</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.4</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.4</td>
</tr>
<tr>
<td>0.6</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

(i)
Squeezing advancement vs atomic density

Noise figure and advancement


\[ F = \frac{SNR_{in}}{SNR_{out}} = \frac{1}{T} = e^{2\gamma \Delta t_a} \]
Squeezing and self-focusing

Noise with knife edge mask

Same side cut (= before BS)

Opposite sides cut

![Diagram showing optical setup for noise with knife edge mask.]

- Noise with knife edge mask
- Same side cut (= before BS)
- Opposite sides cut
- Squeezing with hot atoms
  - LAOP, 2014 49 / 56
Simple round mask

![Graph 1](image1.png)

![Graph 2](image2.png)
Transmissive ring mask

8% blocking disk

25% blocking disk
Constant transmission map

![Diagram of a setup with labeled components: L, Mask, Collimated Beam, Magnetic Shield, Rb Cell, PBS.]

**Graph:**
- **Y-axis:** Min Noise Level (dB)
- **X-axis:** Position (cm)
- **Legend:**
  - Circle: 55%
  - Square: 70%
  - Cross: 75%
  - Plus: 95%

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Constant iris size map

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Squeezing with hot atoms

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Irina Novikova, Mi Zhang, Gleb Romanov and Travis Horrom (NIST), LSU group of Jonathan P. Dowling, Yanhong Xiao (Fudan, China) and Arturo Lezama (Instituto de Física, Uruguay)