Quantum enhanced measurements

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From ray optics to semiclassical optics

Classical/Geometrical optics

- light is a ray
- which propagates straight
- cannot explain diffraction and interference

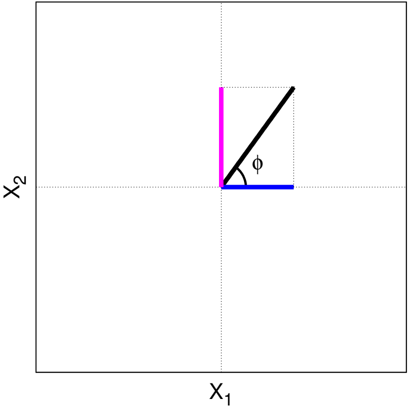
Semiclassical optics

- light is a wave
- color (wavelength/frequency) is important
- amplitude (a) and phase are important, $E(t) = ae^{i(kz-\omega t)}$
- cannot explain residual measurements noise

Classical field

$$E(\phi) \;=\; |a|e^{-i\phi} = |a|\cos(\phi) + i|a|\sin(\phi) = X_1 + iX_2, \quad \phi = \omega t - kz$$

Detectors sense the real part of the field (X_1) but there is a way to see X_2 as well



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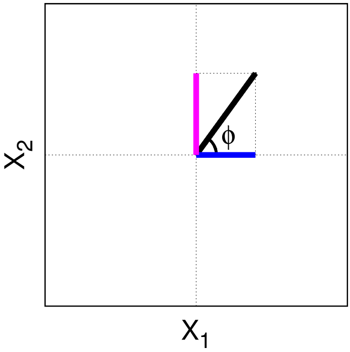

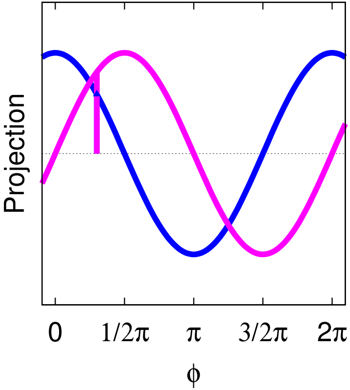

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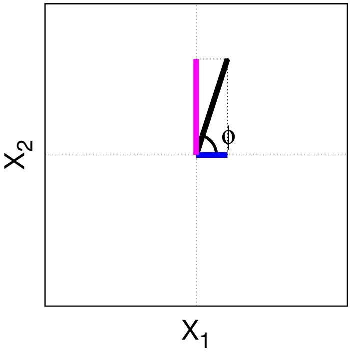

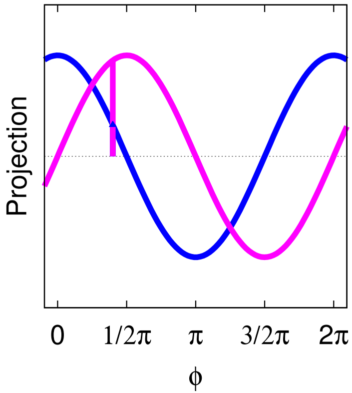

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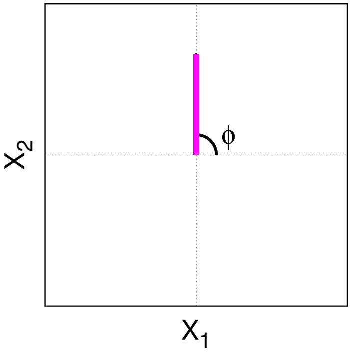

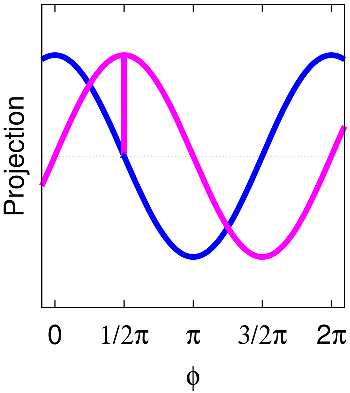

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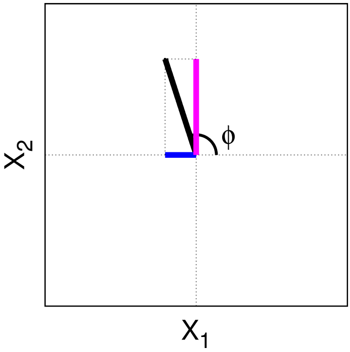

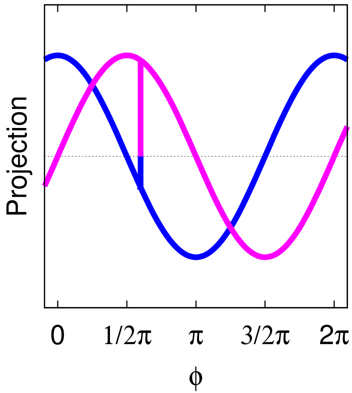

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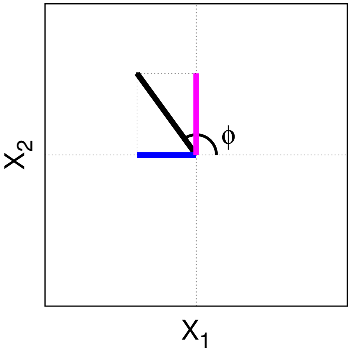

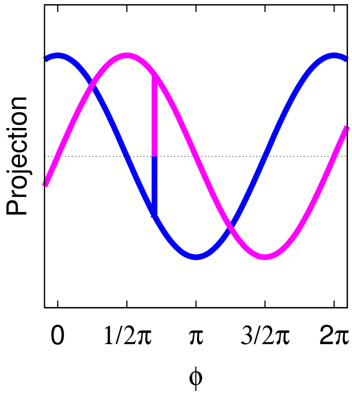

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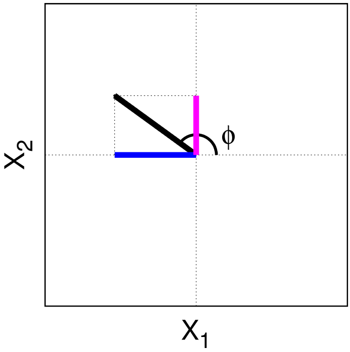

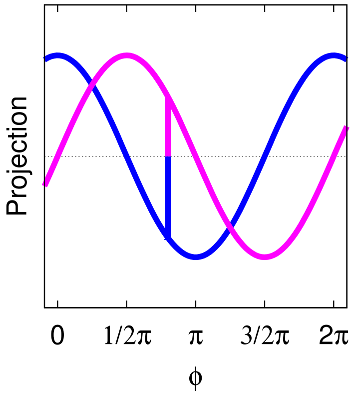

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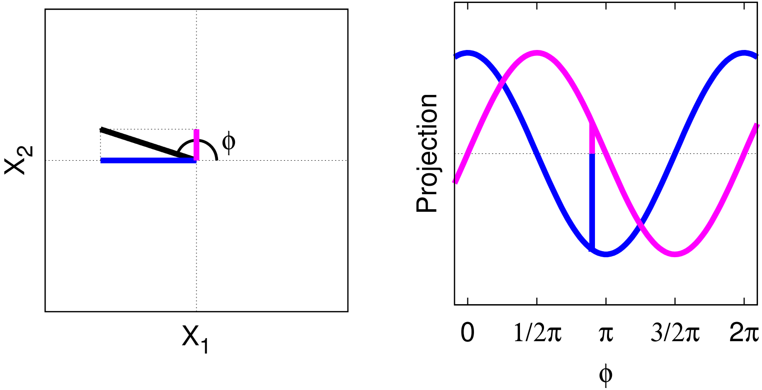

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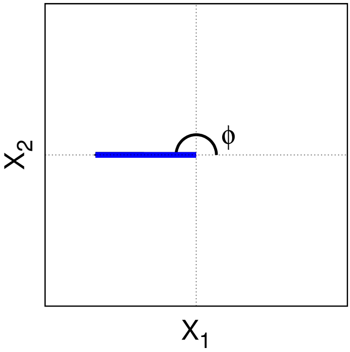

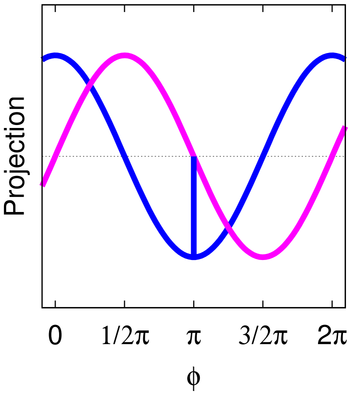

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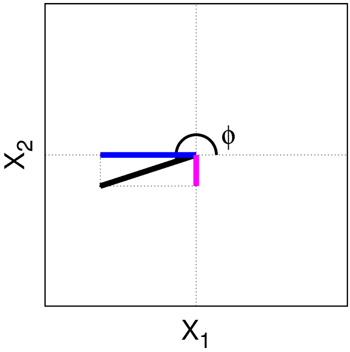

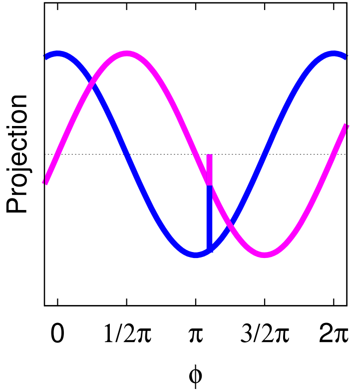

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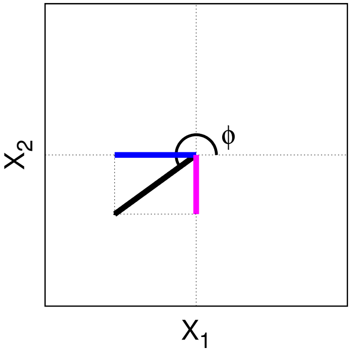

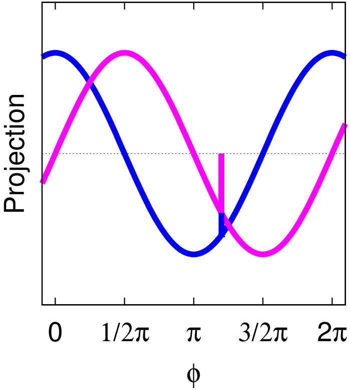

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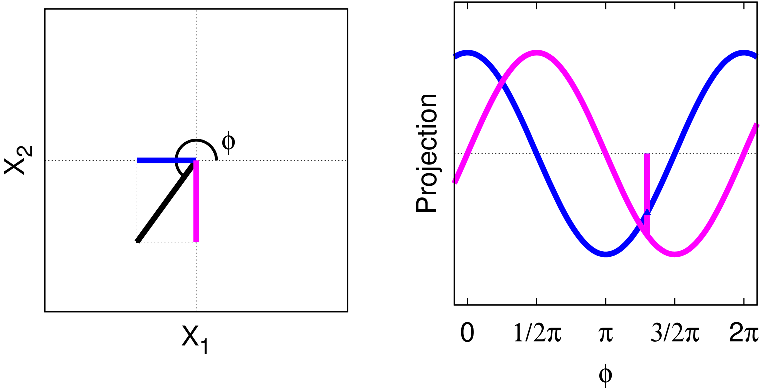

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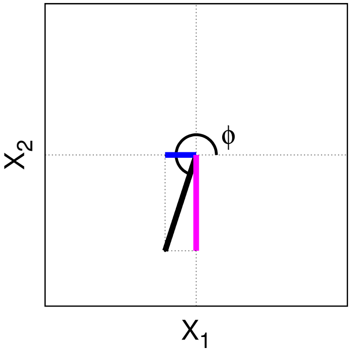

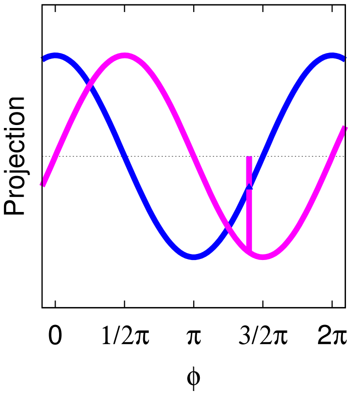

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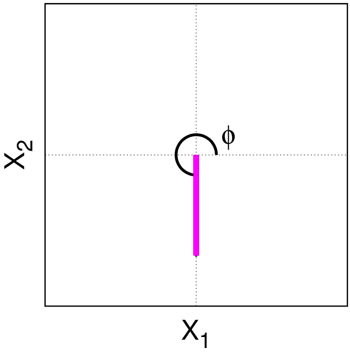

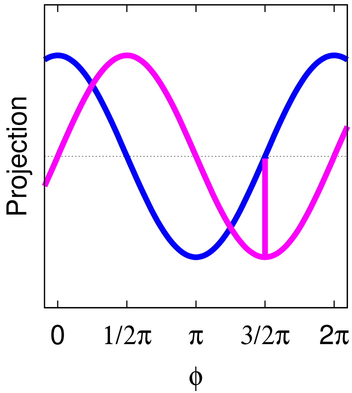

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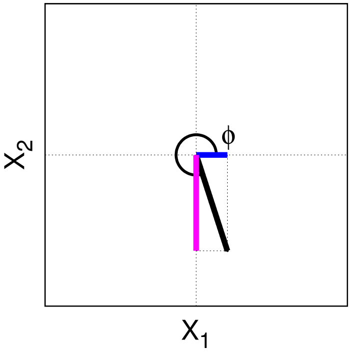

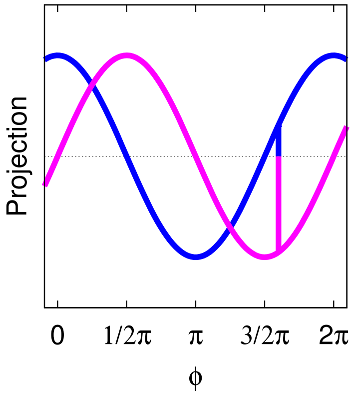

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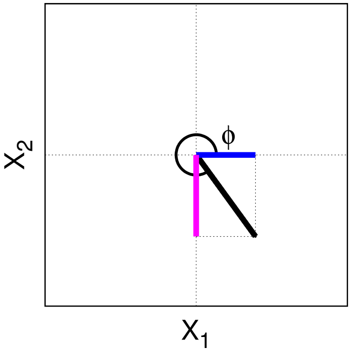

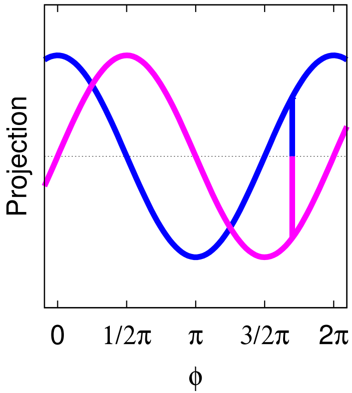

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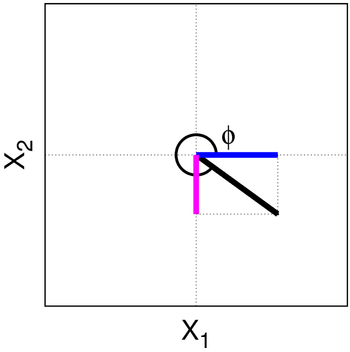

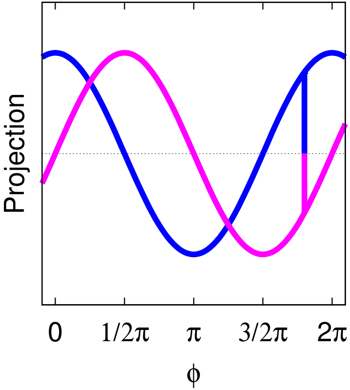

Classical quadratures vs time in a rotating frame

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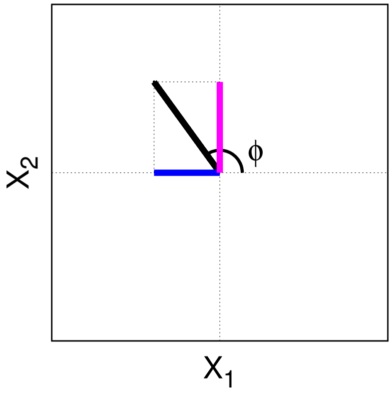

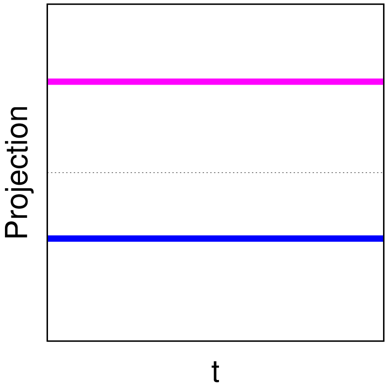

Reality check quadratures vs time

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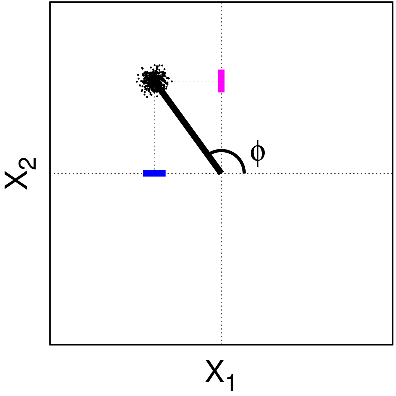
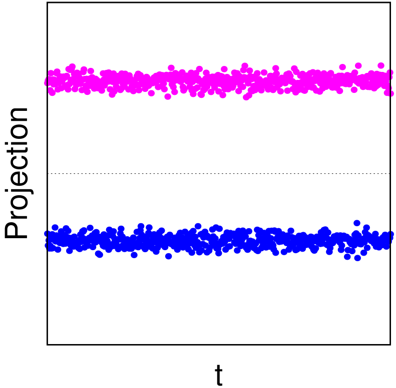
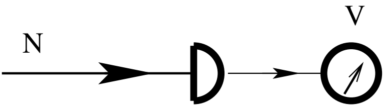




Detector quantum noise

Simple photodetector



$$V \sim N$$

$$\Delta V \sim \sqrt{N}$$

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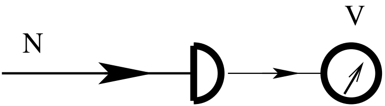

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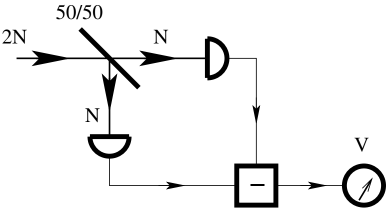
Balanced photodetector



$V = 0$

$\Delta V \sim \sqrt{N}$

Transition from classical to quantum field

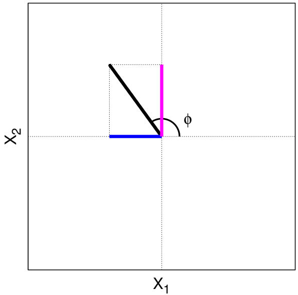
Classical analog

- Field amplitude a
- Field real part
  $X_1 = (a^* + a)/2$
- Field imaginary part
  $X_2 = i(a^* - a)/2$

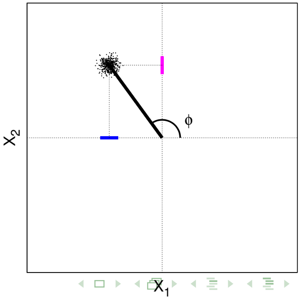
Quantum approach

- Field operator $\hat{a}$
- Amplitude quadrature
  $\hat{X}_1 = (\hat{a}^\dagger + \hat{a})/2$
- Phase quadrature
  $\hat{X}_2 = i(\hat{a}^\dagger - \hat{a})/2$

$$E(\phi) \quad = \quad |a|e^{-i\phi} = X_1 + iX_2$$



$$\hat{E}(\phi) \quad = \quad \hat{X}_1 + i\hat{X}_2$$



Heisenberg uncertainty principle and its optics equivalent

Heisenberg uncertainty principle

$\Delta p \Delta x \geq \hbar/2$
The more precisely the POSITION is determined, the less precisely the MOMENTUM is known, and vice versa

Heisenberg uncertainty principle and its optics equivalent

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Optics equivalent

$\Delta \phi \Delta N \geq 1$

The more precisely the PHASE is determined, the less precisely the AMPLITUDE is known, and vice versa

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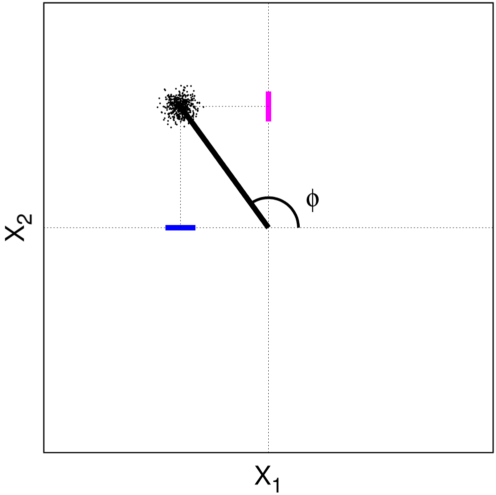
Optics equivalent strict definition

$\Delta X_1 \Delta X_2 \geq 1/4$

Quantum optics summary



Light consist of photons

- $\hat{N} = a^\dagger a$

Commutator relationship

- $[a, a^\dagger] = 1$
- $[X_1, X_2] = i/2$

Detectors measure

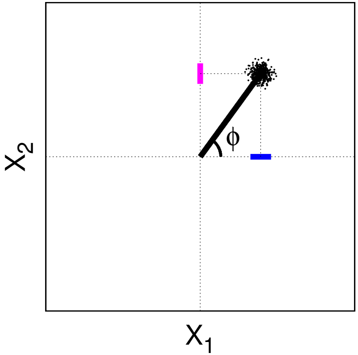
- number of photons $\hat{N}$
- Quadratures $\hat{X}_1$ and $\hat{X}_2$

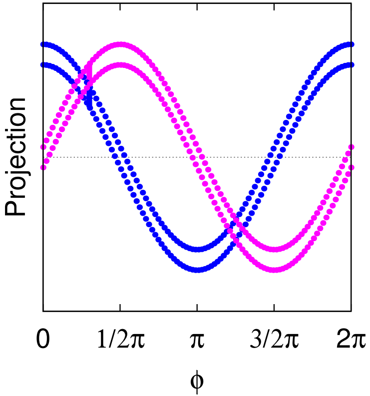
Uncertainty relationship

- $\Delta X_1 \Delta X_2 \geq 1/4$

Coherent state is minimum uncertainty state

$\Delta X_1 \Delta X_2 = 1/4$





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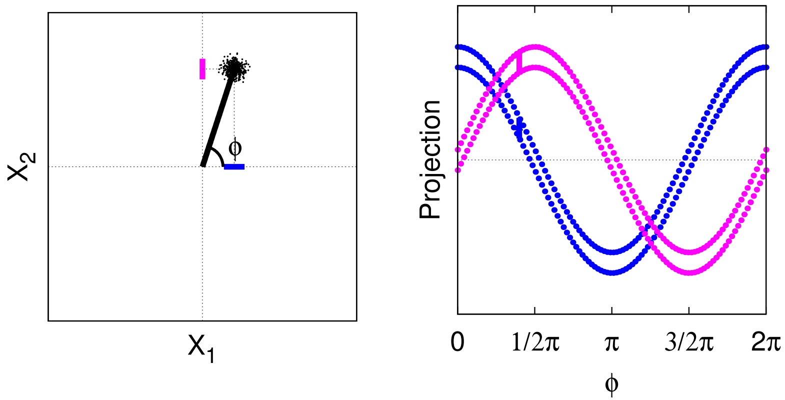

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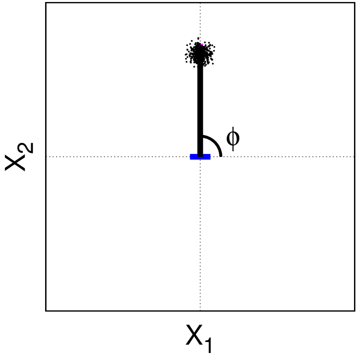

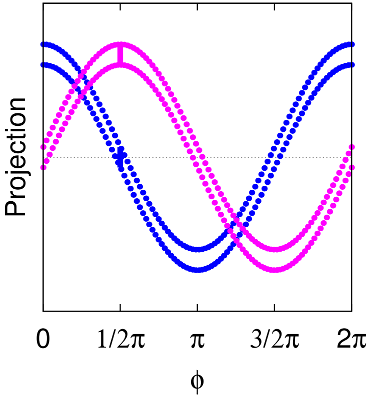

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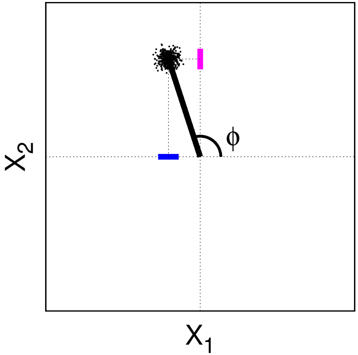

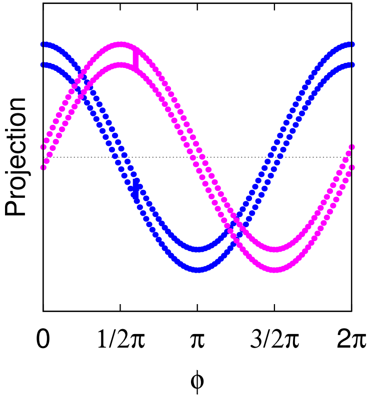

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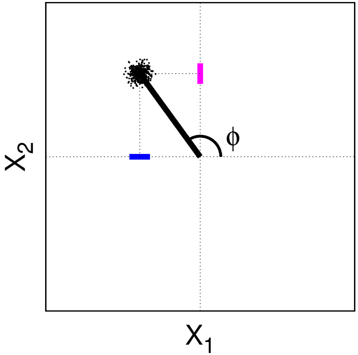

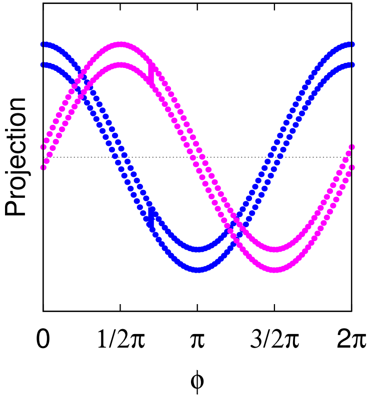

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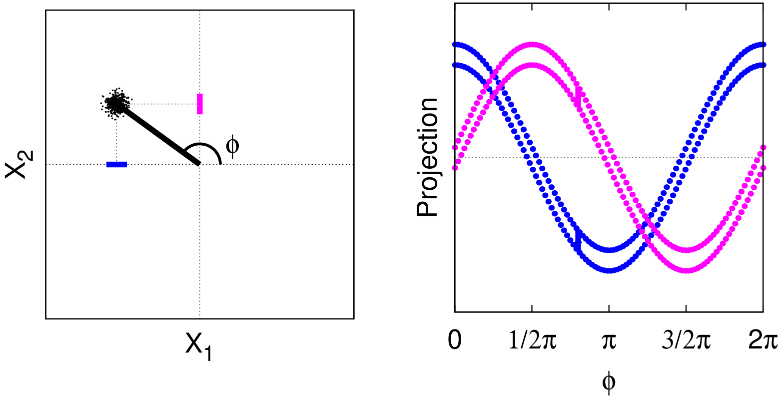

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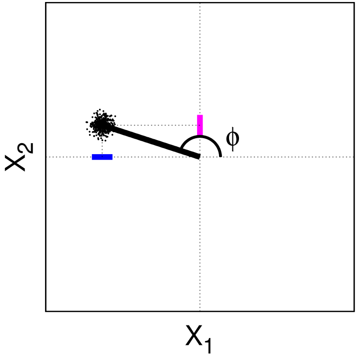

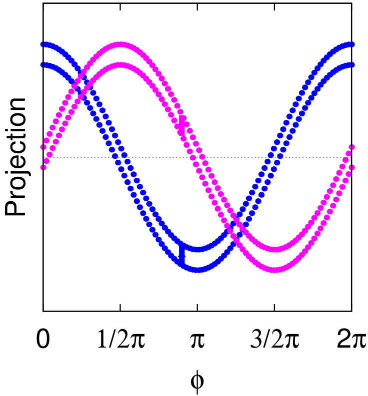

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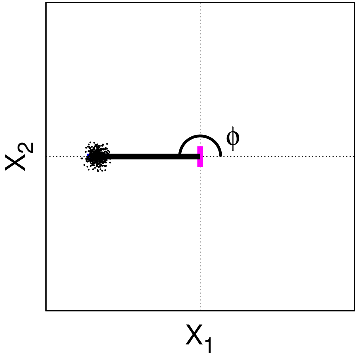

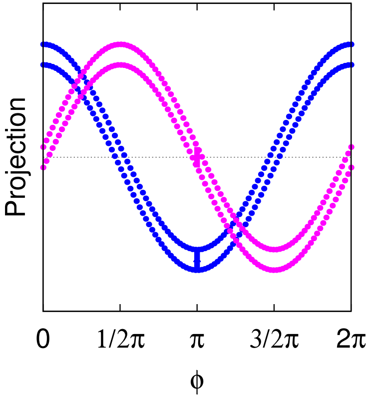

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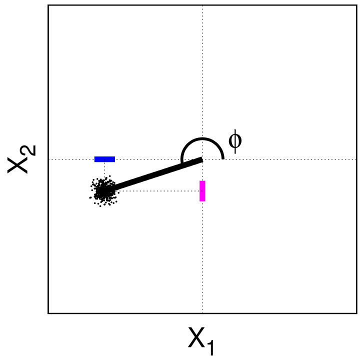
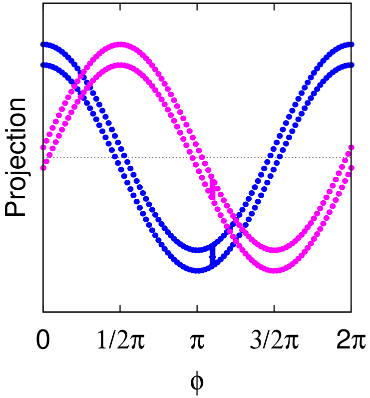
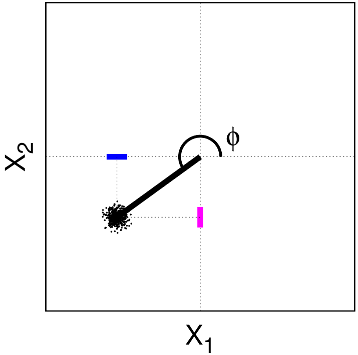
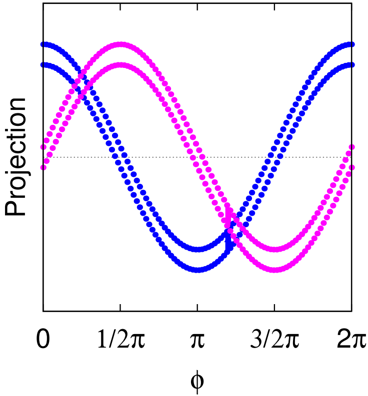




Coherent state is minimum uncertainty state

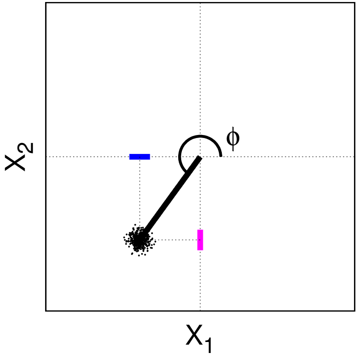
$\Delta X_1 \Delta X_2 = 1/4$

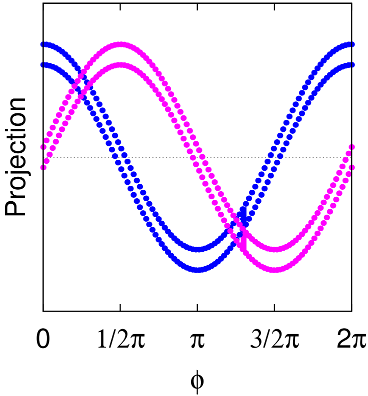




Coherent state is minimum uncertainty state

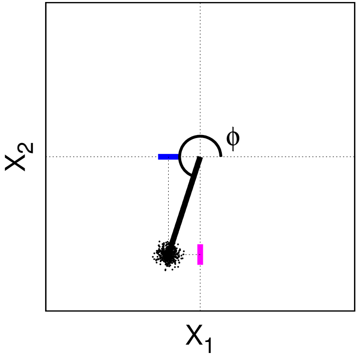
$\Delta X_1 \Delta X_2 = 1/4$

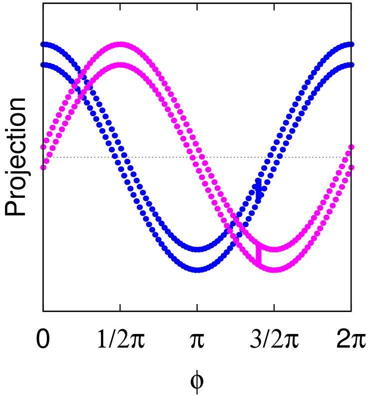




Coherent state is minimum uncertainty state

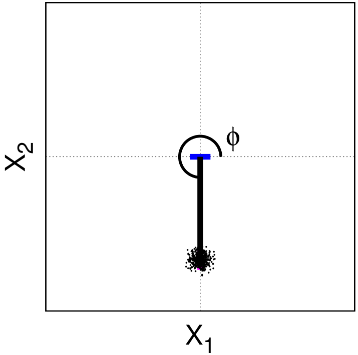
$\Delta X_1 \Delta X_2 = 1/4$

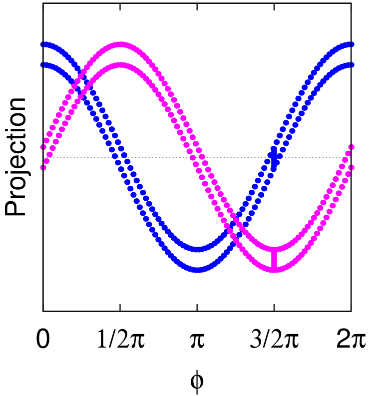




Coherent state is minimum uncertainty state

$\Delta X_1 \Delta X_2 = 1/4$





Coherent state is minimum uncertainty state

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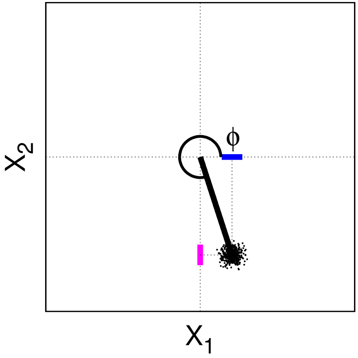
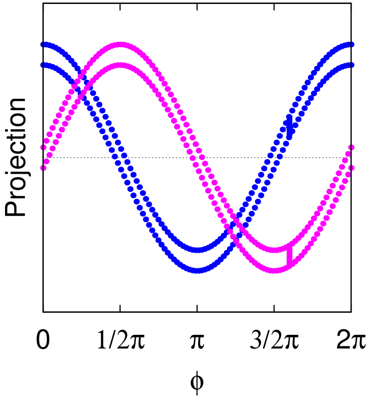
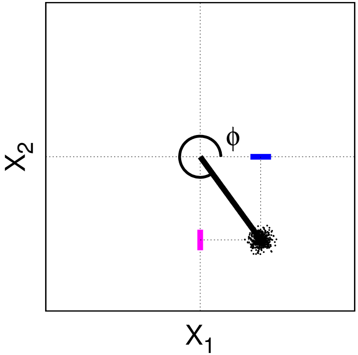
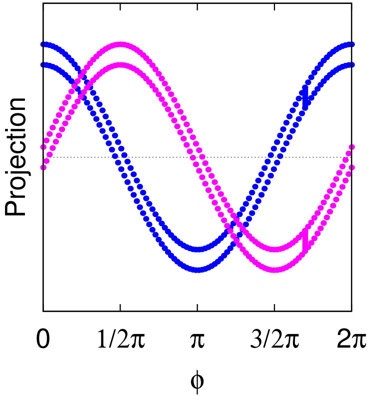




Coherent state is minimum uncertainty state

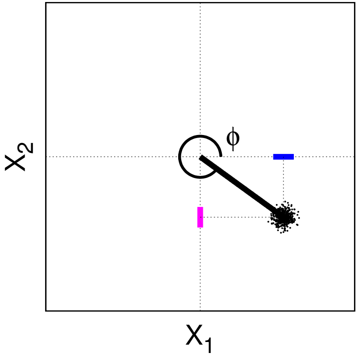
$\Delta X_1 \Delta X_2 = 1/4$

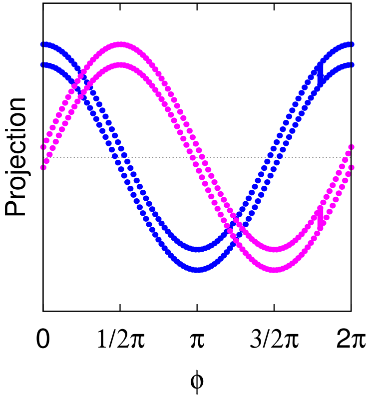




Coherent state is minimum uncertainty state

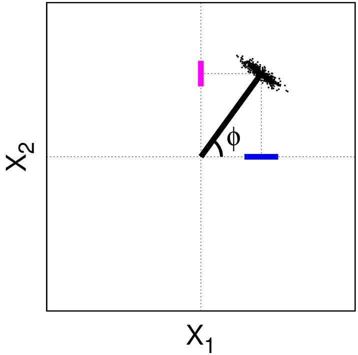
$\Delta X_1 \Delta X_2 = 1/4$

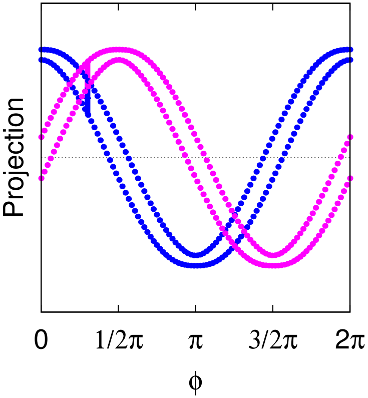




Amplitude squeezed states

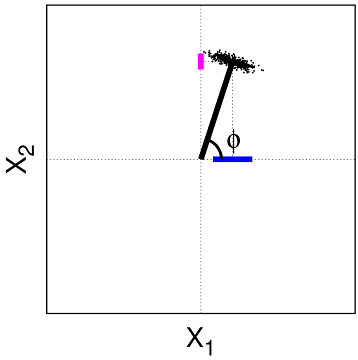
$\Delta X_1 \Delta X_2 = 1/4$

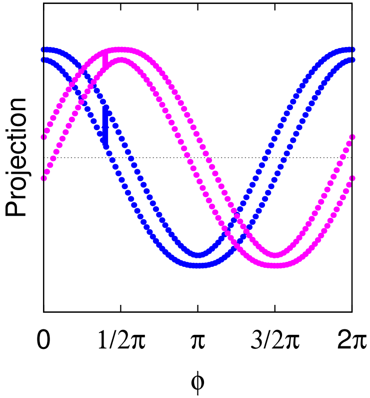




Amplitude squeezed states

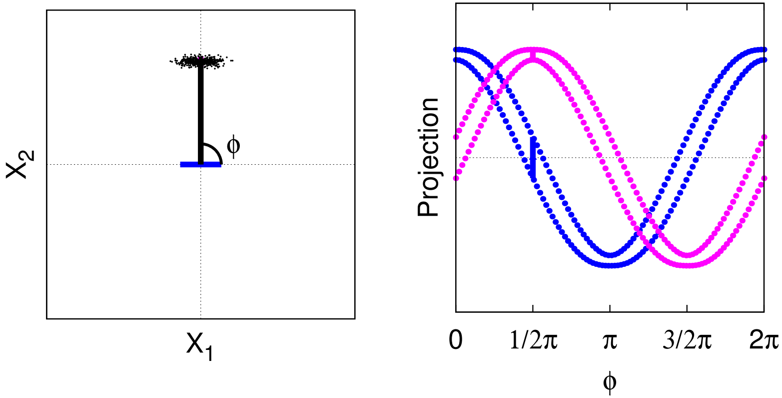
$\Delta X_1 \Delta X_2 = 1/4$





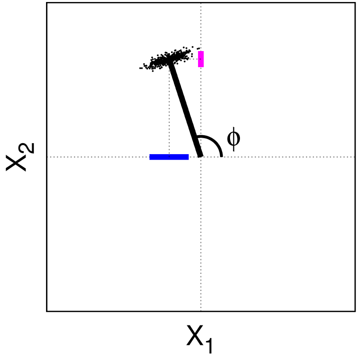
Amplitude squeezed states

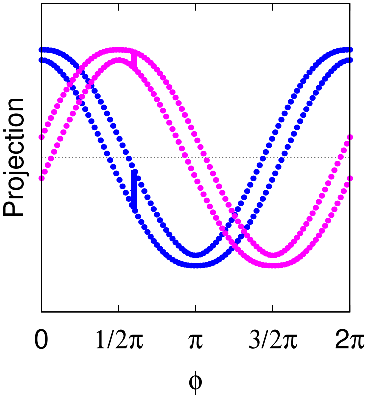
$\Delta X_1 \Delta X_2 = 1/4$



Amplitude squeezed states

$\Delta X_1 \Delta X_2 = 1/4$





Amplitude squeezed states

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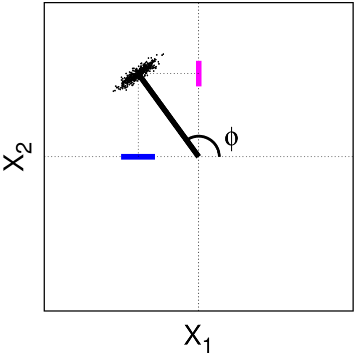

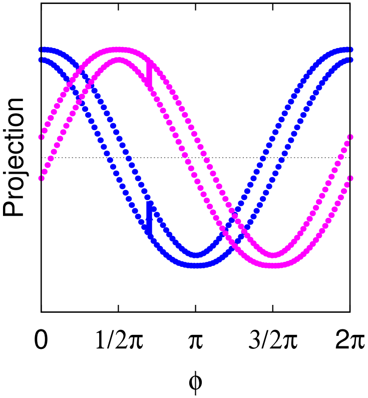

Amplitude squeezed states

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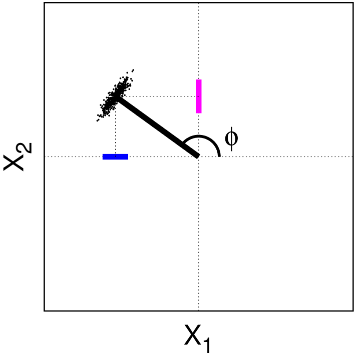
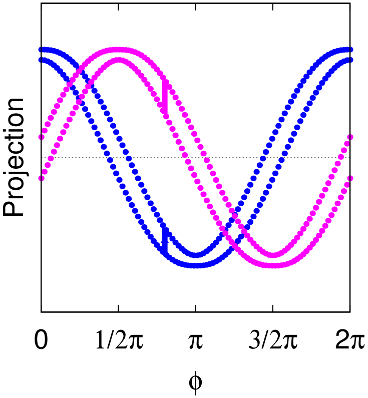
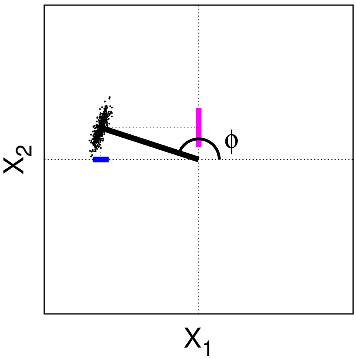
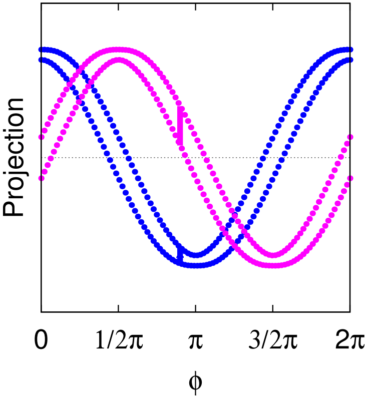




Amplitude squeezed states

$\Delta X_1 \Delta X_2 = 1/4$





Amplitude squeezed states

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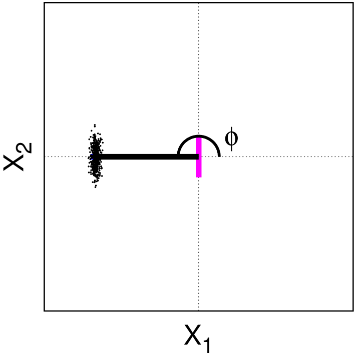

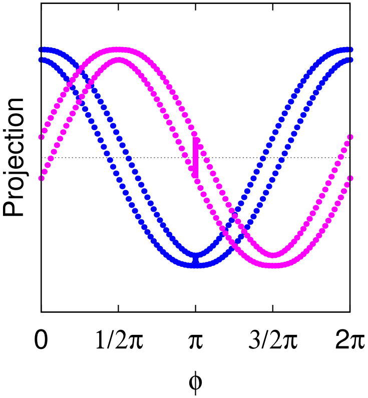

Amplitude squeezed states

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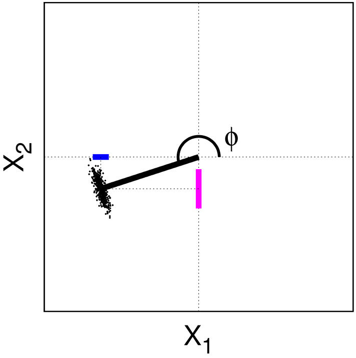

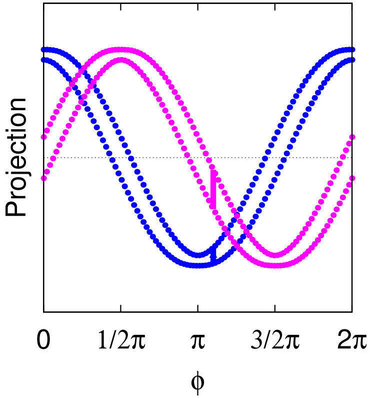

Amplitude squeezed states

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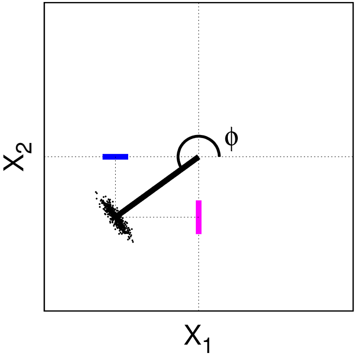

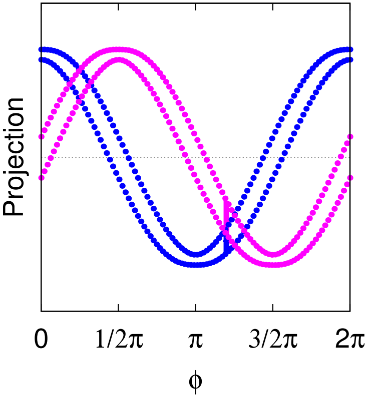

Amplitude squeezed states

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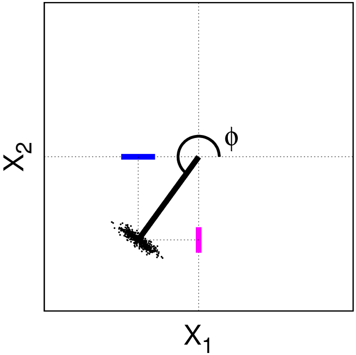
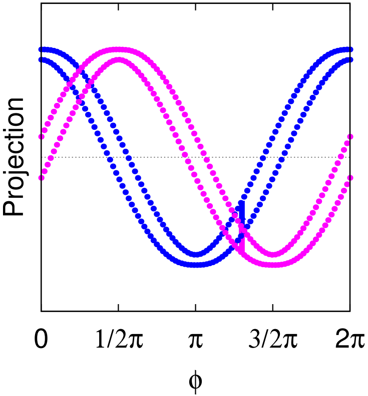
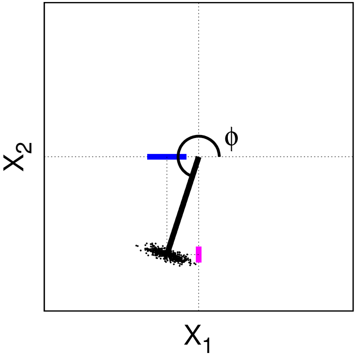
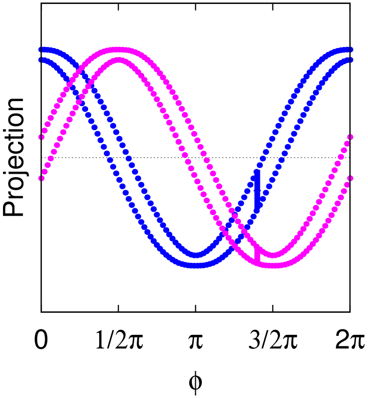




Amplitude squeezed states

$\Delta X_1 \Delta X_2 = 1/4$





Amplitude squeezed states

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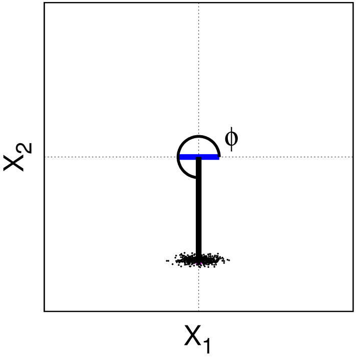

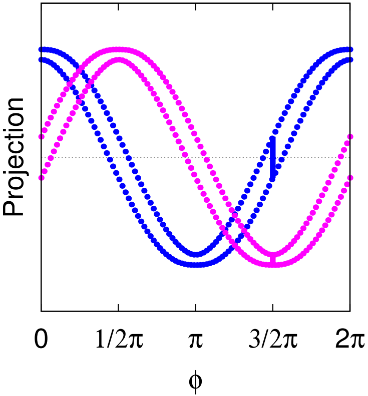

Amplitude squeezed states

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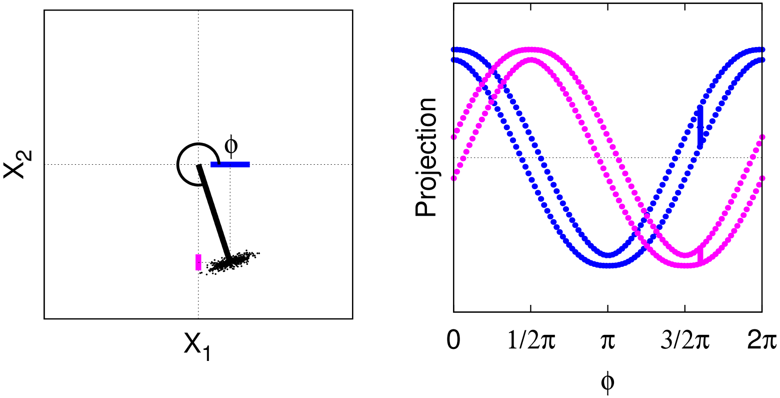

Amplitude squeezed states

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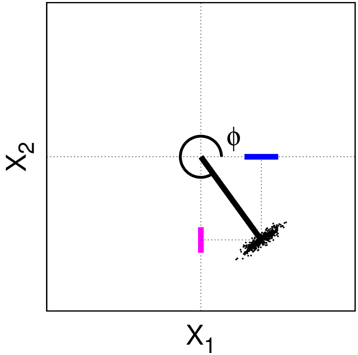

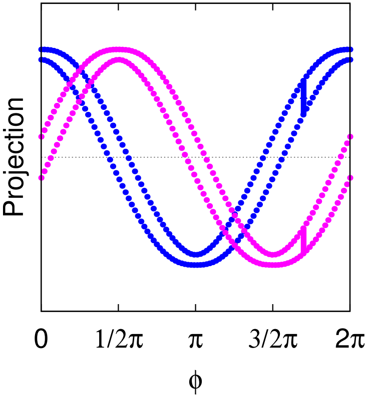

Amplitude squeezed states

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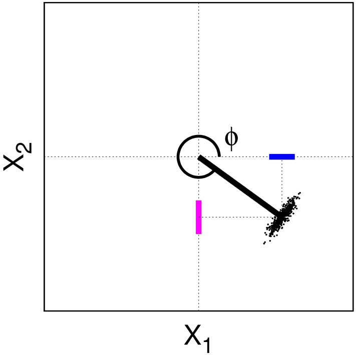
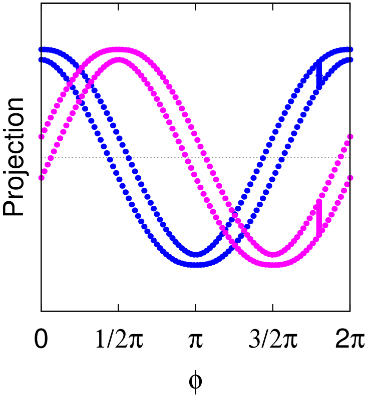
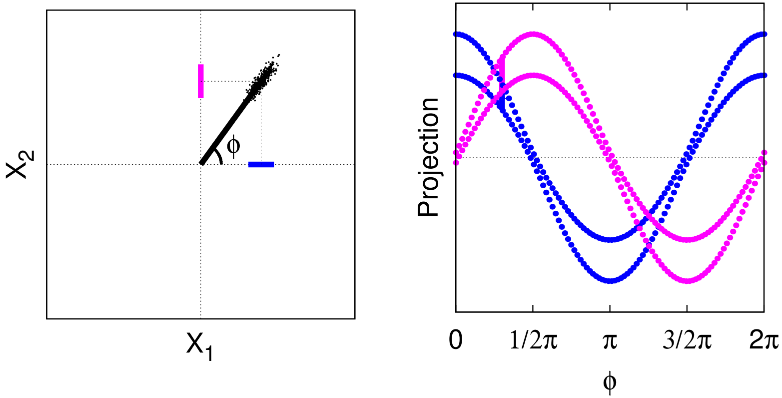




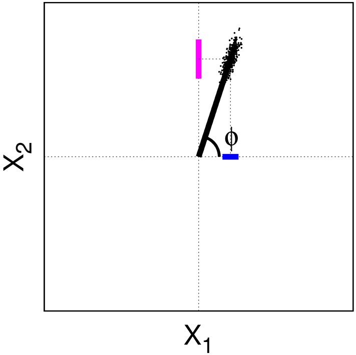
Phase squeezed states

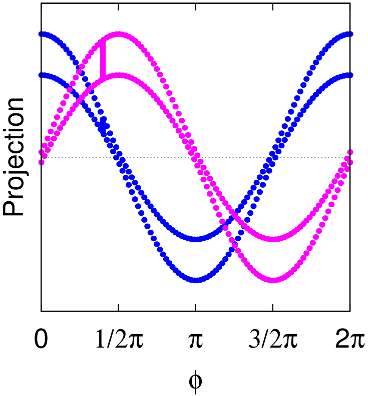
$\Delta X_1 \Delta X_2 = 1/4$



Phase squeezed states

$\Delta X_1 \Delta X_2 = 1/4$





Phase squeezed states

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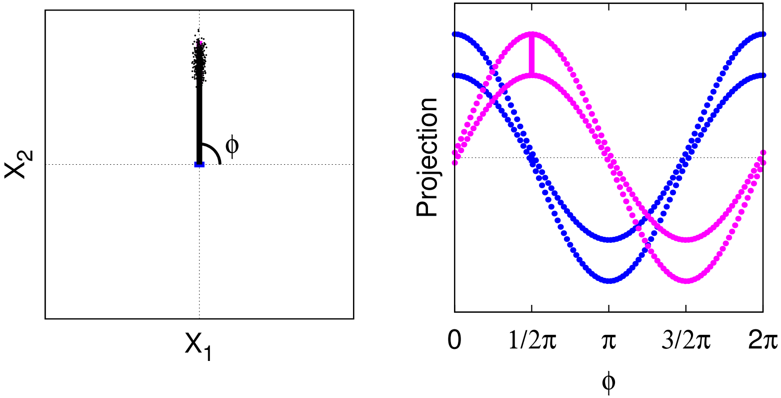

Phase squeezed states

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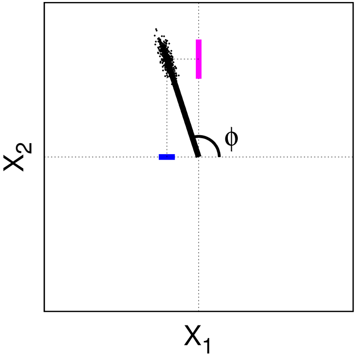

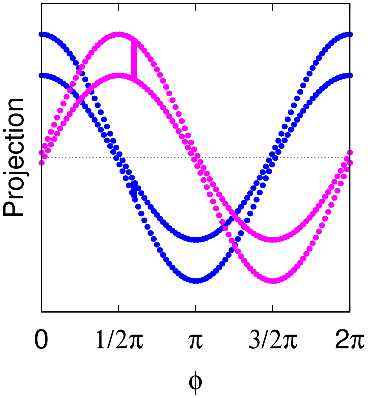

Phase squeezed states

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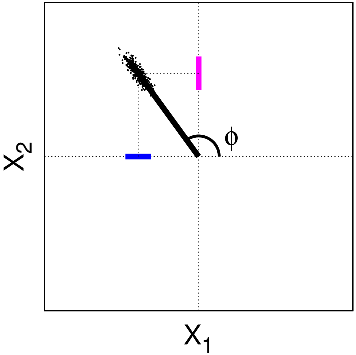

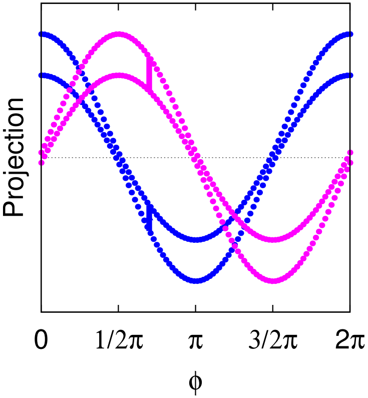

Phase squeezed states

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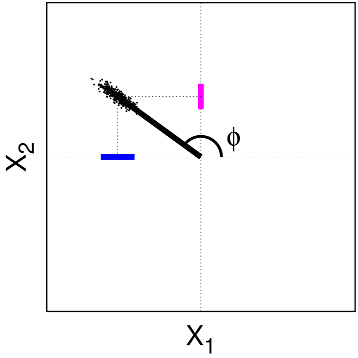

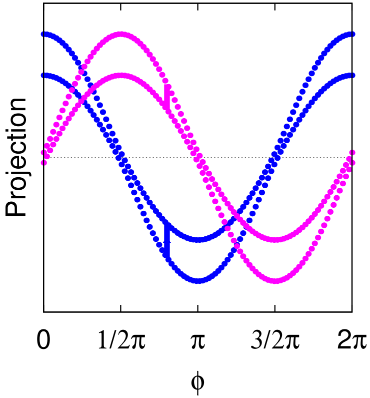

Phase squeezed states

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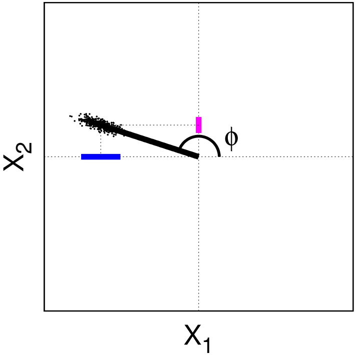
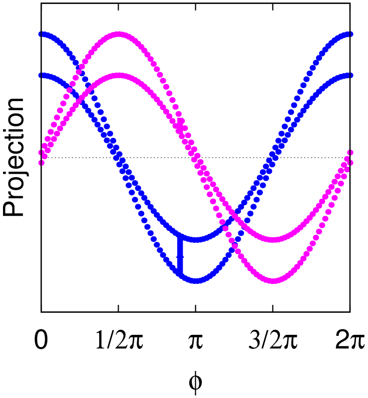
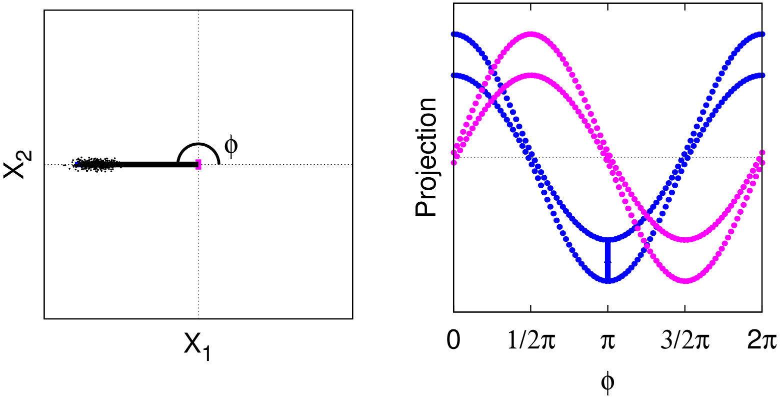




Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



Phase squeezed states

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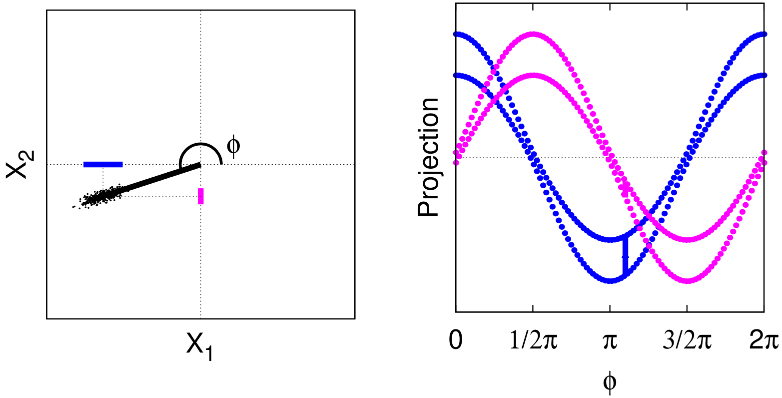

Phase squeezed states

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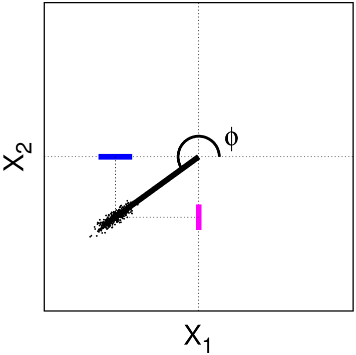

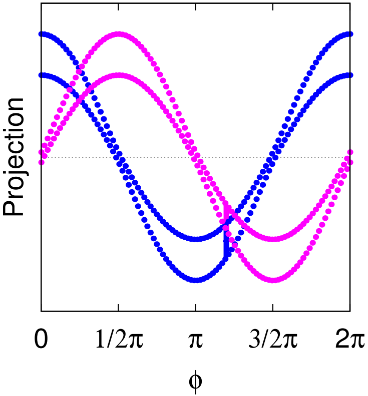

Phase squeezed states

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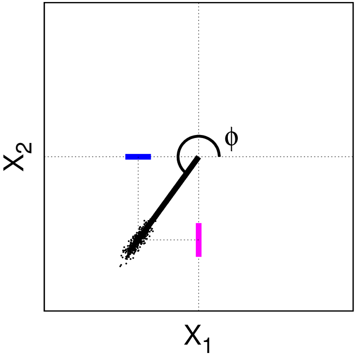

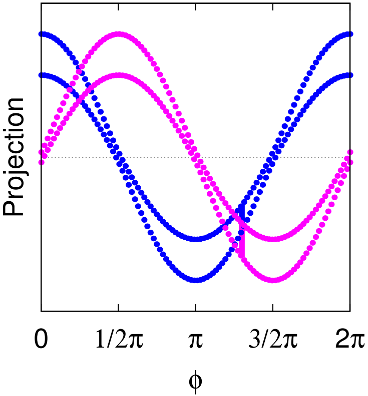

Phase squeezed states

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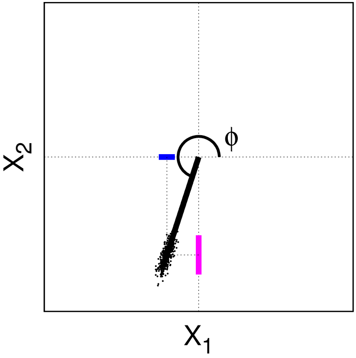

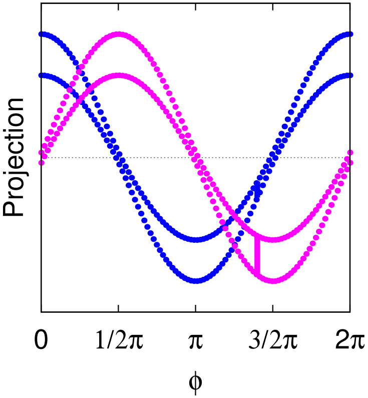

Phase squeezed states

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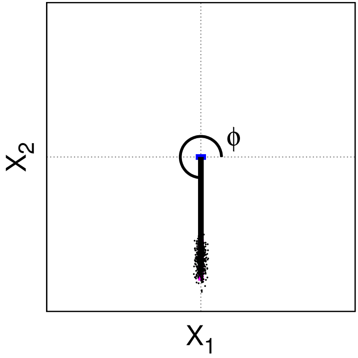

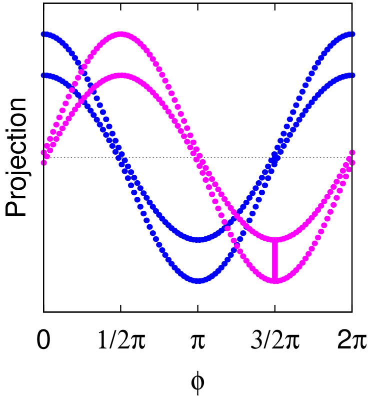

Phase squeezed states

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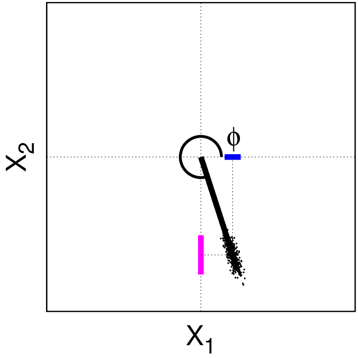
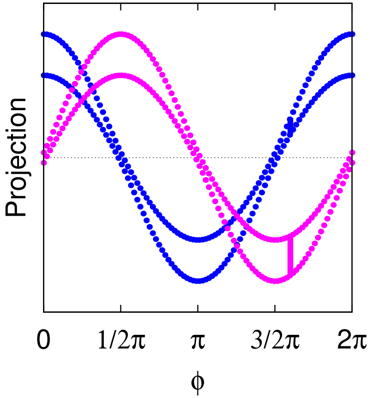
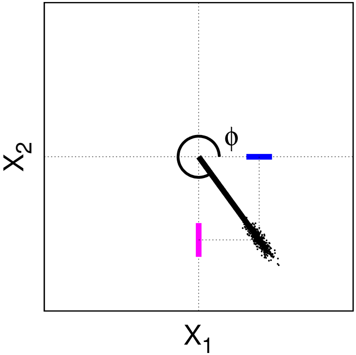
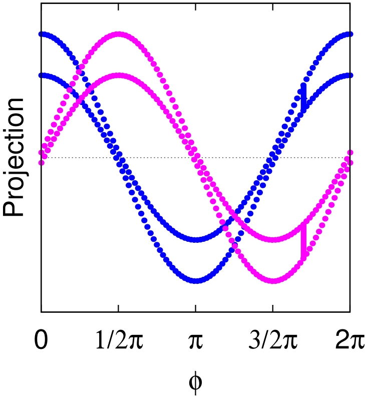




Phase squeezed states

$\Delta X_1 \Delta X_2 = 1/4$





Phase squeezed states

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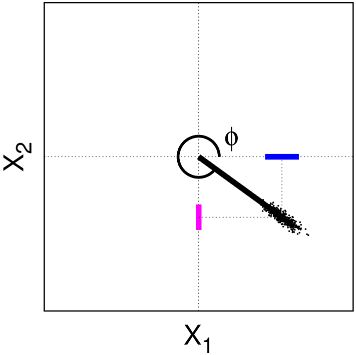
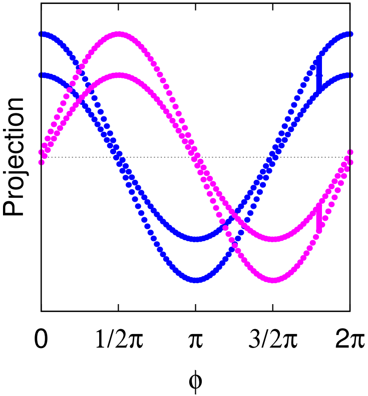
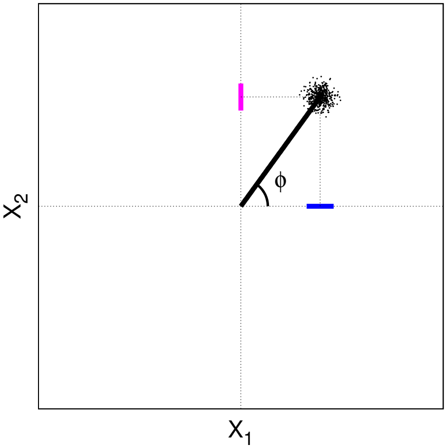
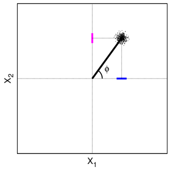




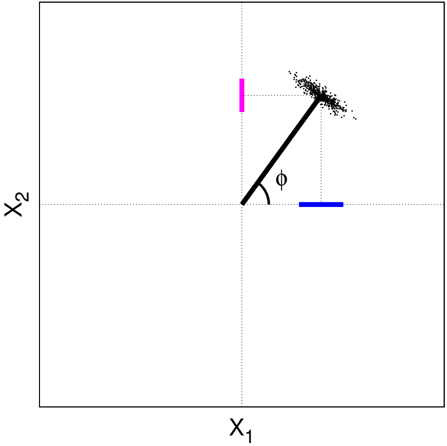
Squeezed quantum states zoo



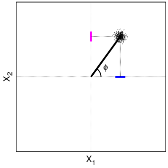
Unsqueezed coherent



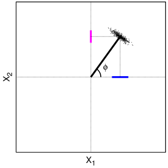
Squeezed quantum states zoo



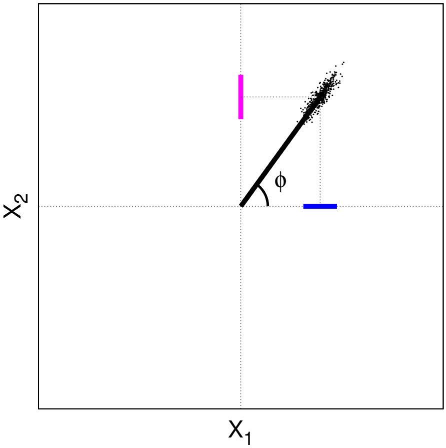
Unsqueezed coherent



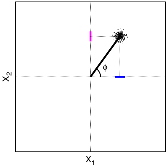
Amplitude squeezed



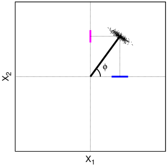
Squeezed quantum states zoo



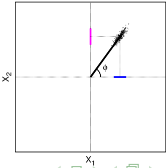
Unsqueezed coherent



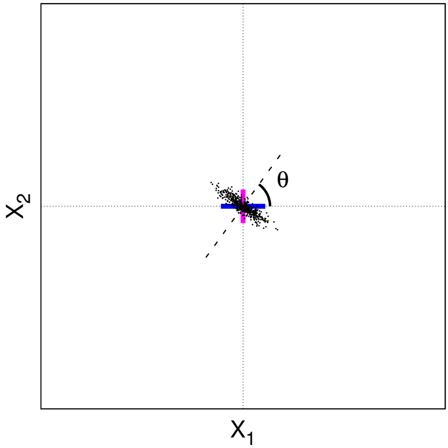
Amplitude squeezed



Phase squeezed



Squeezed quantum states zoo



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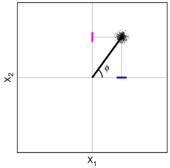
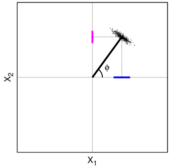
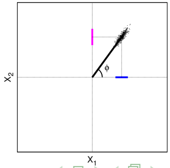


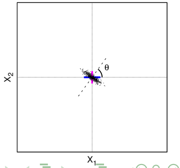
Amplitude squeezed



Phase squeezed

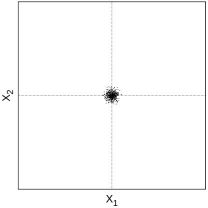


Vacuum squeezed



Squeezed field generation recipe

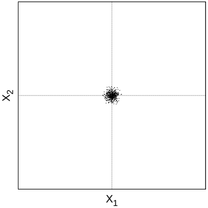
Take a vacuum state $|0>$



$$H = \frac{1}{2}$$

Squeezed field generation recipe

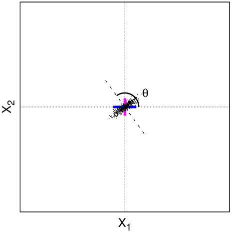
Take a vacuum state $|0>$



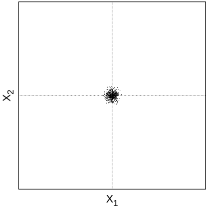
$$H = \frac{1}{2}$$

Apply squeezing operator $|\xi>= \hat{S}(\xi)|0>$

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$



Take a vacuum state $|0>$



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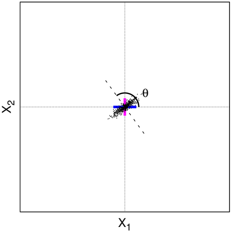
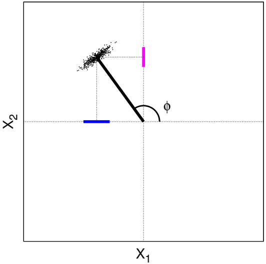


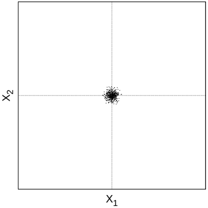
Apply displacement operator $|\alpha, \xi >= \hat{D}(\alpha)|s>$

$$\hat{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$



$$\begin{aligned} <\alpha, \xi|X_1|\alpha, \xi> &= Re(\alpha), \\ <\alpha, \xi|X_2|\alpha, \xi> &= Im(\alpha) \end{aligned}$$

Take a vacuum state $|0>$

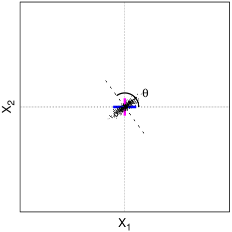


$$H = \frac{1}{2}$$

Notice $\Delta X_1 \Delta X_2 = \frac{1}{4}$

Apply squeezing operator $|\xi >= \hat{S}(\xi)|0>$

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$



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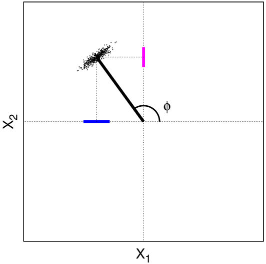
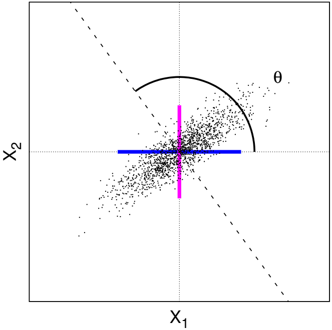


$$< \alpha, \xi | X_1 | \alpha, \xi > = Re(\alpha),$$
$$< \alpha, \xi | X_2 | \alpha, \xi > = Im(\alpha)$$



$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}, \xi = re^{i\theta}$$

If $\theta = 0$

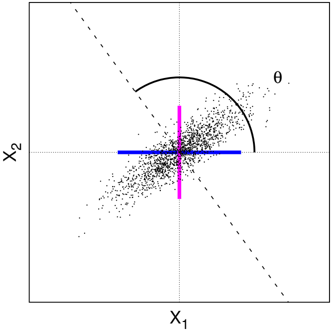
$$< \xi|(\Delta X_1)^2|\xi > = \frac{1}{4}e^{-2r}$$

$$< \xi|(\Delta X_2)^2|\xi > = \frac{1}{4}e^{2r}$$

$$< \xi|(\Delta X_1)^2|\xi > = \frac{1}{4}(\cosh^2 r + \sinh^2 r - 2\sinh r \cosh r \cos\theta)$$

$$< \xi|(\Delta X_2)^2|\xi > = \frac{1}{4}(\cosh^2 r + \sinh^2 r + 2\sinh r \cosh r \cos\theta)$$

Photon number of squeezed state $|\xi>$



Probability to detect given number of photons $C =< n|\xi >$ for squeezed vacuum

- even

$$C_{2m} = (-1)\frac{\sqrt{(2m)!}}{2^m m!}\frac{(e^{i\theta}\tanh r)^m}{\sqrt{\cosh r}}$$

- odd

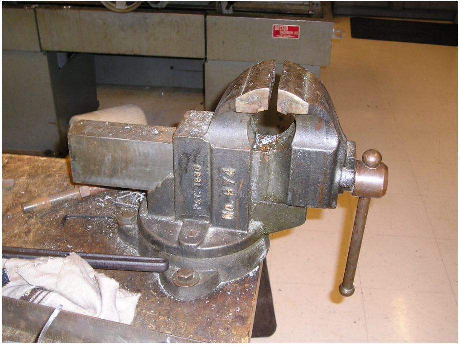
$$C_{2m+1} = 0$$

Average number of photons in general squeezed state

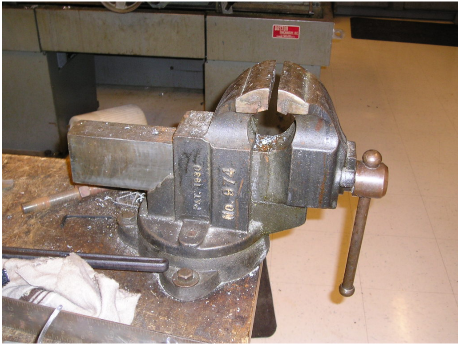
$$< \alpha, \xi | a^\dagger a | \alpha, \xi >= \alpha + \sinh^2 r$$

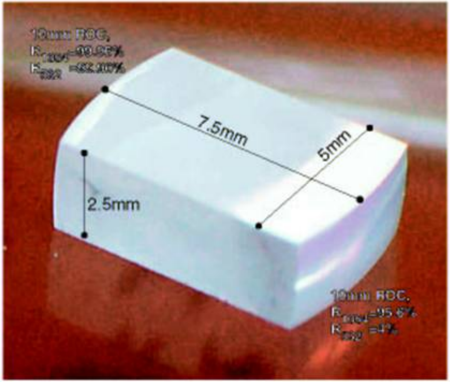
Tools for squeezing

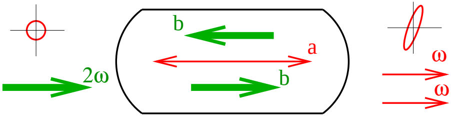
Tools for squeezing



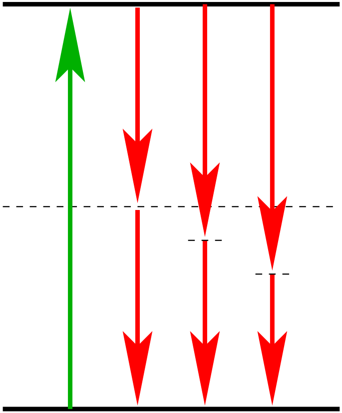
Tools for squeezing







Two photon squeezing picture

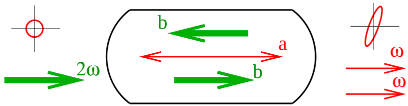


Squeezing operator

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$

Parametric down-conversion in crystal

$$\hat{H} = i\hbar\chi^{(2)}(a^2 b^\dagger - a^{\dagger 2} b)$$

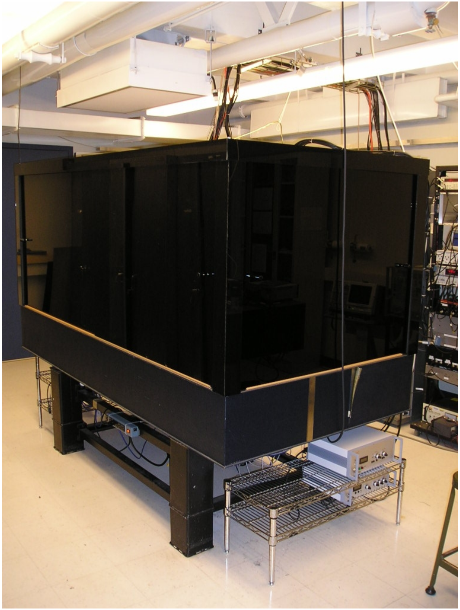


Squeezing

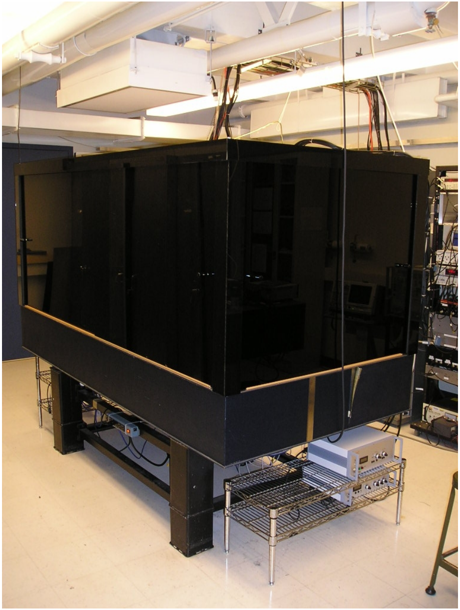
result of correlation of upper and lower sidebands

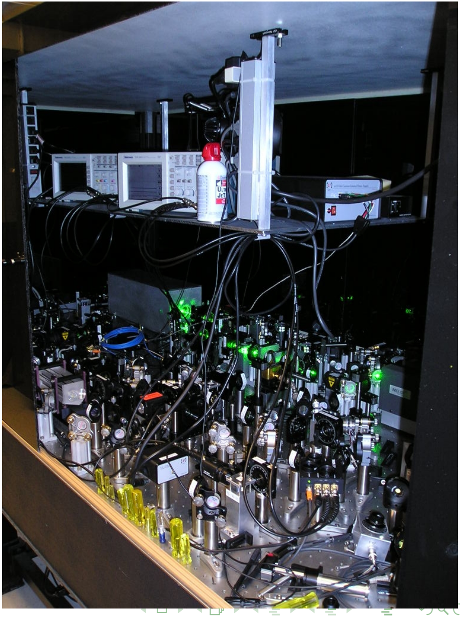
Squeezer appearance

Squeezer appearance

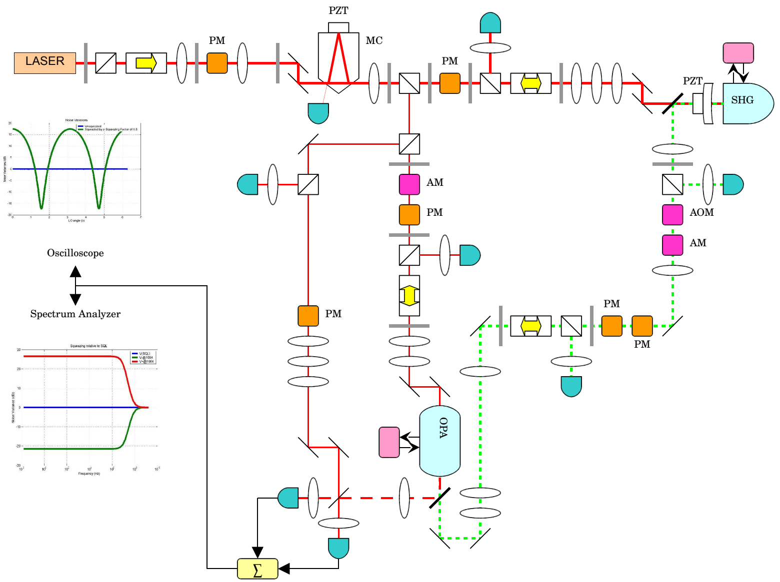


Squeezer appearance





Crystal squeezing setup scheme

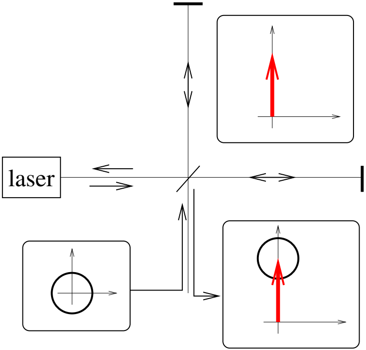


Possible squeezing applications

- improvements any shot noise limited optical sensors
- noiseless signal amplification
- secure communications (you would notice eavesdropper)
- photon pair generation, entanglement, true single photon sources
- interferometers sensitivity boost (for example gravitational wave antennas)
- light free measurements
- quantum memory probe and information carrier

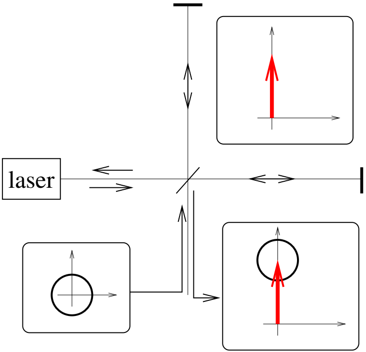
Squeezing and interferometer

Vacuum input

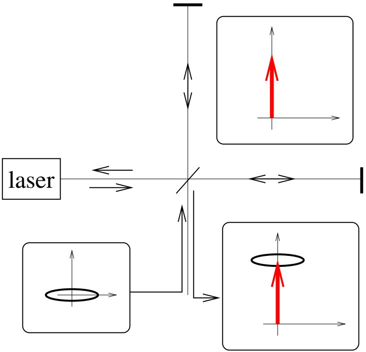


Squeezing and interferometer

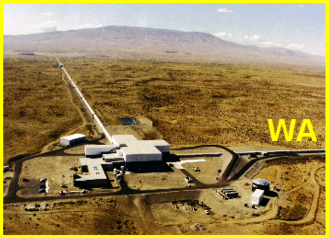
Vacuum input



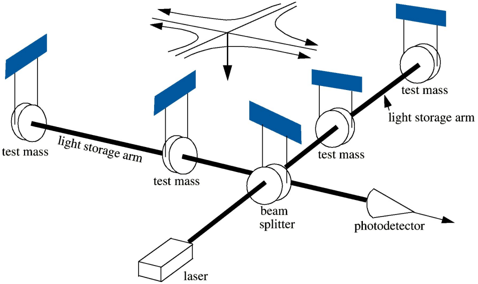
Squeezed input



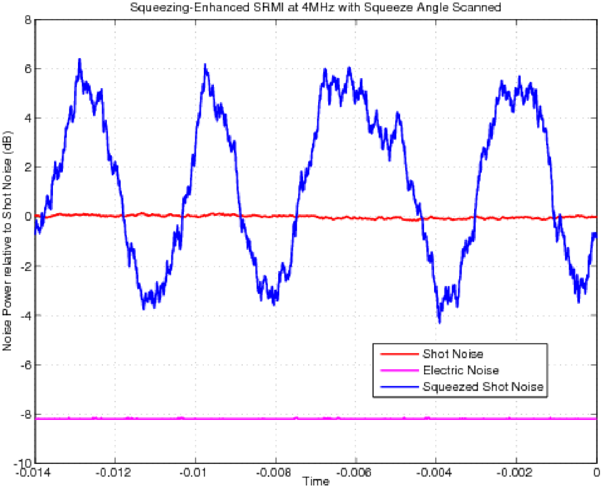
Laser Interferometer Gravitational-wave Observatory



- $L = 4$ km
- $h \sim 10^{-21}$
- $\Delta L \sim 10^{-18}$ m
- $\Delta\phi \sim 10^{-10}$ rad

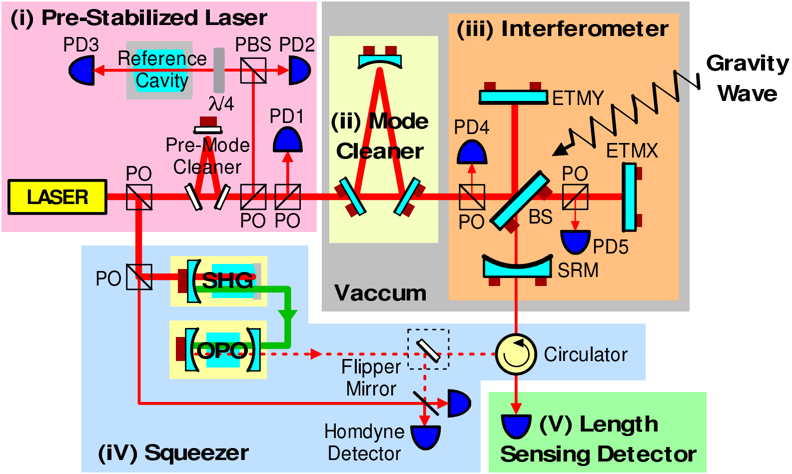


Squeezing level vs time (unlocked)

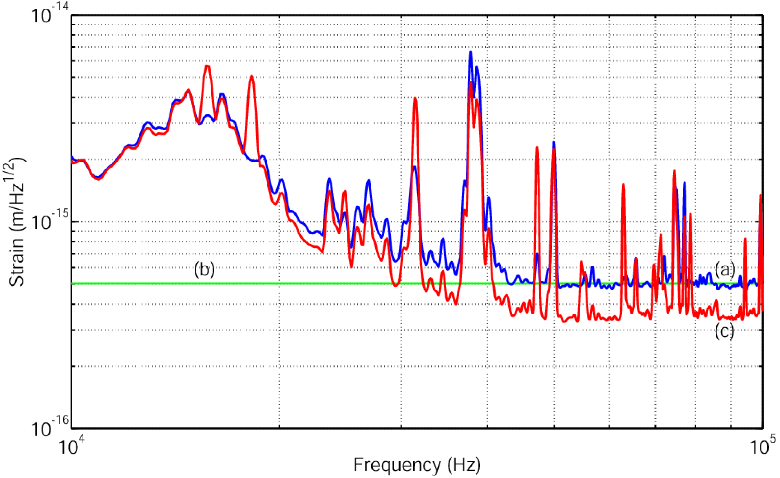


"A quantum-enhanced prototype gravitational-wave detector", Nature Physics, **4**, 472-476, (2008).

GW 40m detector and squeezer

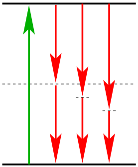


GW 40m detector with 4dB of squeezed vacuum



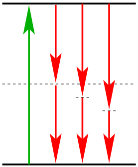
Signal to noise improvement by factor of 1.43

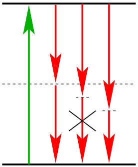
Cavity parameters with squeezing

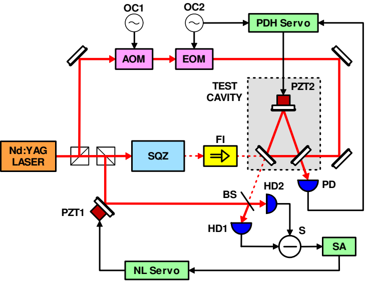


Cavity parameters with squeezing

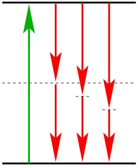
Cavity parameters with squeezing

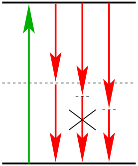


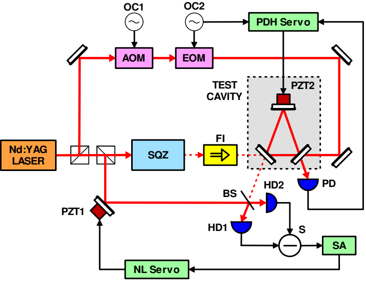


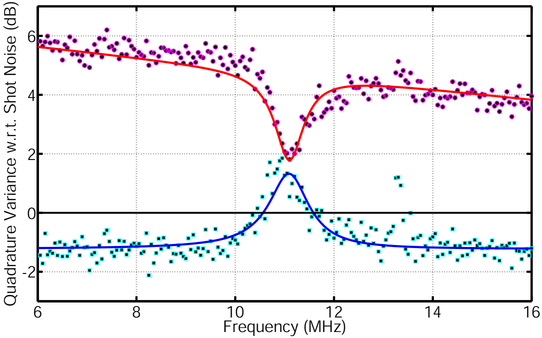


Cavity parameters with squeezing









"Noninvasive measurements of cavity parameters by use of squeezed vacuum", Physical Review A, **74**, 033817, (2006).

Summary for crystal squeezing

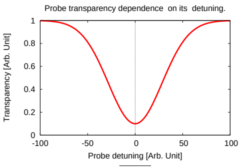
Pros

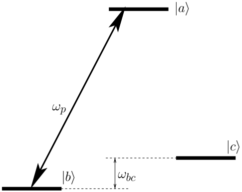
- mainstream: many different nonlinear crystals available
- so far the best squeezers
  - maximum squeezing value detected 11.5 dB at 1064 nm
  - Moritz Mehmet, Henning Vahlbruch, Nico Lastzka, Karsten Danzmann, and Roman Schnabel, "Observation of squeezed states with strong photon-number oscillations", Phys. Rev. A **81**, 013814 (2010)
- well understood

Cons

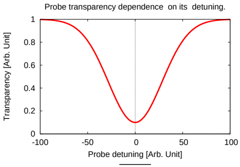
- crystals have limited transparency window
- thus squeezing is hard to generate at visible wavelength
  - at 795 nm only 4-6 dB squeezing is reported
- this limits applications of such squeezers for spectroscopy

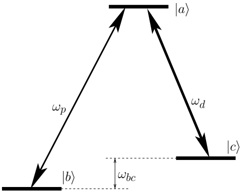
Quantum memory with atomic ensembles

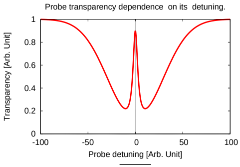


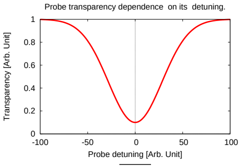


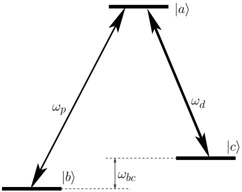
Quantum memory with atomic ensembles

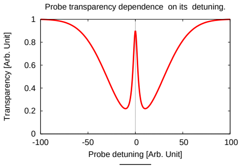






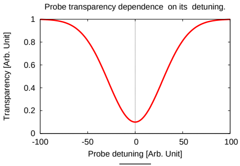


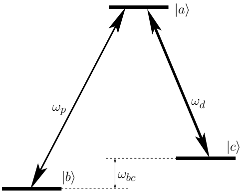


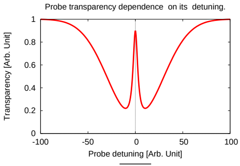


Storage and retrieval

Quantum memory with atomic ensembles



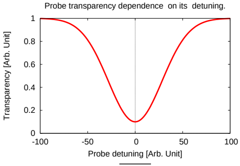


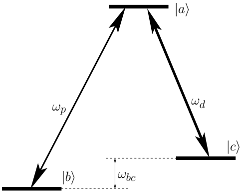


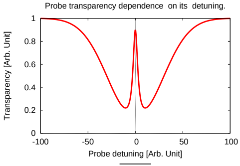
Storage and retrieval

- single photon

Quantum memory with atomic ensembles



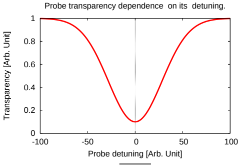


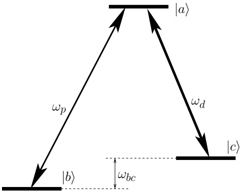


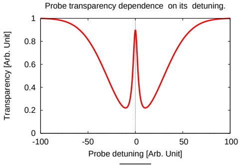
Storage and retrieval

- single photon
- squeezed state (Furusawa and Lvovsky PRL **100** 2008)

Quantum memory with atomic ensembles







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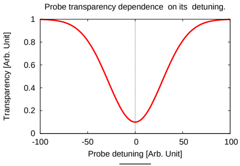
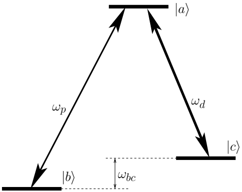
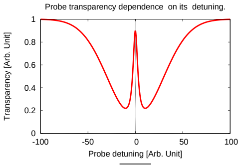
Squeezed state requirements for a quantum memory probe

Quantum memory with atomic ensembles







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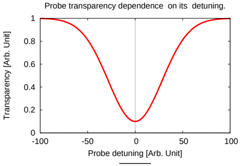
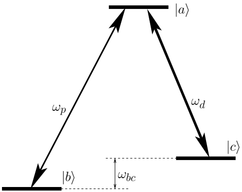
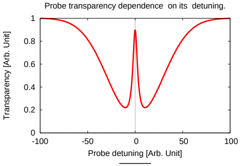
Squeezed state requirements for a quantum memory probe

- squeezing carrier at atomic wavelength (780nm, 795nm)
- squeezing within narrow resonance window at frequencies($<$100kHz)

Quantum memory with atomic ensembles







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Squeezed state requirements for a quantum memory probe

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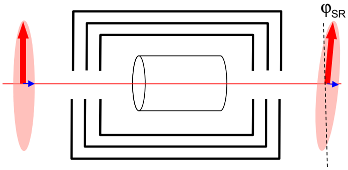
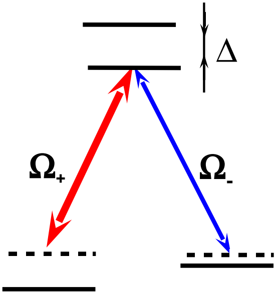
Traditional nonlinear crystal based squeezers are capable of it, but they are extremely technically challenging especially at short wave length.

Self-rotation of elliptical polarization in atomic medium





A.B. Matsko et al., PRA 66, 043815 (2002): theoretically prediction of 4-6 dB noise suppression

$$a_{out} = a_{in} + \frac{igL}{2}(a_{in}^{\dagger} - a_{in}) \quad (2)$$

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- Yes! J. Ries, B. Brezger, and A. I. Lvovsky, Experimental vacuum squeezing in rubidium vapor via self-rotation, PRA **68**, 025801 (2003).
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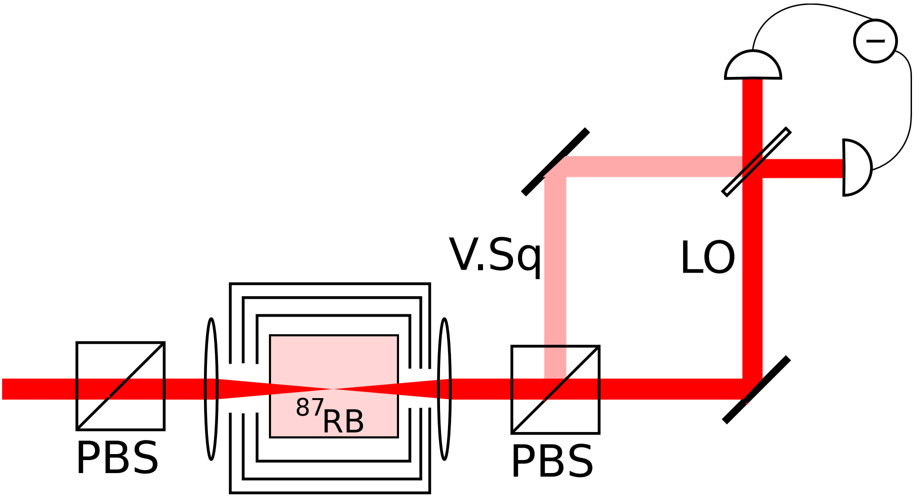
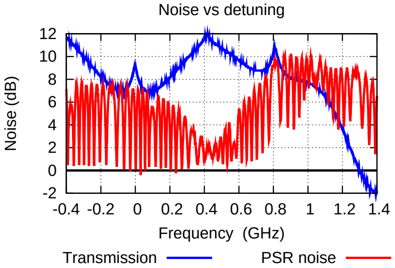
- Definitely Eugeniy E. Mikhailov et al. Optics Letters, Issue 11, **33**, 1213-1215, (2008).
- Definitely Eugeniy E. Mikhailov et al. JMO , Issues 18&19, **56**, 1985-1992, (2009).
- Definitely Philippe Grangier et al. Optics Express, **18**, Issue 5, pp. 4198-4205 (2010)
  - 1.4 dB of squeezing

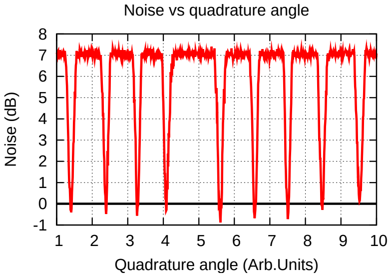
Setup



Noise contrast vs detuning in hot ^{87}Rb vacuum cell

$F_g = 2 \rightarrow F_e = 1, 2$





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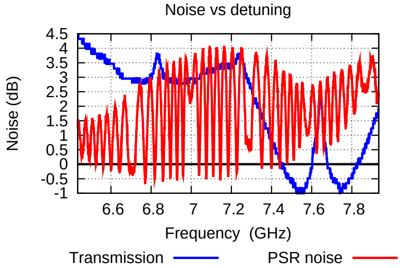
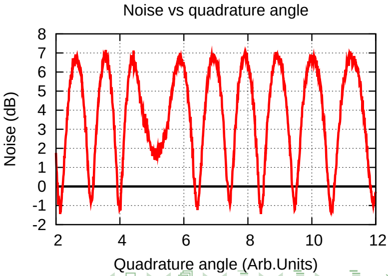
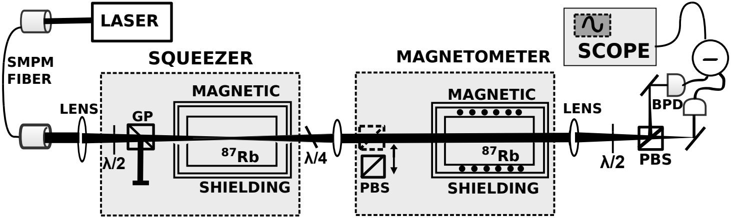
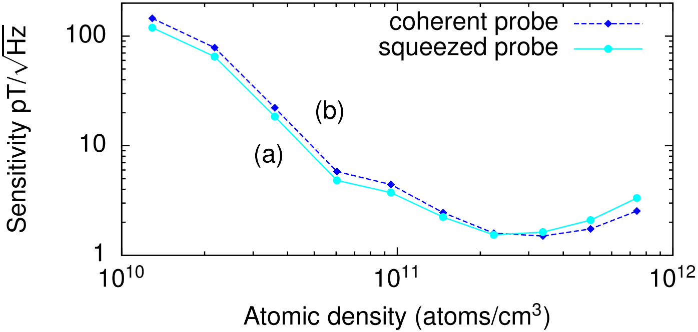


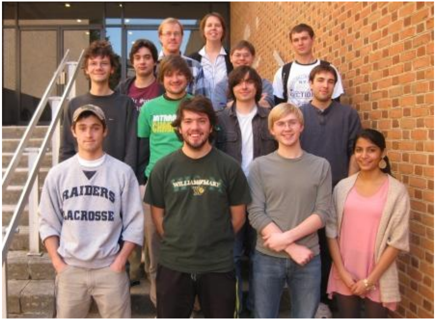


Magnetometer with squeezing enhancement





People



Support from NSF

Summary

- Squeezing is exiting
- many applications benefit from squeezing
- there is still a lot of interesting physics to do