Quantum enhanced magnetometer and squeezed state of light tunable filter

Eugeniy E. Mikhailov

The College of William & Mary

October 5, 2012
Transition from classical to quantum field

Classical analog
- Field amplitude $a$
- Field real part
  \[ X_1 = (a^* + a)/2 \]
- Field imaginary part
  \[ X_2 = i(a^* - a)/2 \]

\[ E(\phi) = |a|e^{-i\phi} = X_1 + iX_2 \]

Quantum approach
- Field operator $\hat{a}$
- Amplitude quadrature
  \[ \hat{X}_1 = (\hat{a}^\dagger + \hat{a})/2 \]
- Phase quadrature
  \[ \hat{X}_2 = i(\hat{a}^\dagger - \hat{a})/2 \]

\[ \hat{E}(\phi) = \hat{X}_1 + i\hat{X}_2 \]
Notice $\Delta X_1 \Delta X_2 \geq \frac{1}{4}$
Notice $\Delta X_1 \Delta X_2 \geq \frac{1}{4}$
Squeezed quantum states zoo

Notice $\Delta X_1 \Delta X_2 \geq \frac{1}{4}$
Squeezed quantum states zoo

Notice $\Delta X_1 \Delta X_2 \geq \frac{1}{4}$
Self-rotation of elliptical polarization in atomic medium

A.B. Matsko et al., PRA 66, 043815 (2002): theoretically prediction of 4-6 dB noise suppression

\[ a_{out} = a_{in} + \frac{igL}{2}(a^\dagger_{in} - a_{in}) \]
Simplified setup

PBS
PBS
LO
V. Sq

$^{87}\text{RB}$
Noise contrast vs detuning in hot $^{87}$Rb vacuum cell

$F_g = 2 \rightarrow F_e = 1, 2$

Noise vs detuning

Transmission  |  PSR noise

Noise vs quadrature angle

$F_g = 1 \rightarrow F_e = 1, 2$

Noise vs detuning

Transmission  |  PSR noise

Noise vs quadrature angle

Eugeni E. Mikhailov (W&M)  Squeezed light  October 5, 2012
Atomic low frequency squeezing source
Optical magnetometer based on Faraday effect

$^{87}$Rb D$_1$ line

F' = 2
F' = 1
F = 2
F = 1

Susceptibility vs B

Susceptibility vs detuning

$\chi''$ (red) and $\chi'$ (blue)

Polarization rotation vs B

Polarization rotation vs B field

$\Delta \chi'$
Optical magnetometer based on Faraday effect

$^{87}$Rb D$_1$ line

Susceptibility vs B

Polarization rotation vs B
Optical magnetometer based on Faraday effect

$^{87}$Rb D$_1$ line

$F'=2$

$F'=1$

$F=2$

$F=1$

Susceptibility vs B

$\chi''(+\Delta)$

$\chi''(-\Delta)$

$\chi'(+\Delta)$

$\chi'(-\Delta)$
Optical magnetometer based on Faraday effect

$^{87}$Rb D$_1$ line

Susceptibility vs B

Squeezed light

Eugeniy E. Mikhailov (W&M)
Optical magnetometer based on Faraday effect

$^{87}$Rb D$_1$ line

Susceptibility vs B

\[
\chi''(\pm \Delta) = \chi'(\pm \Delta)
\]
Optical magnetometer based on Faraday effect

$^{87}$Rb D$_1$ line

Susceptibility vs B

Polarization rotation vs B
Optical magnetometer and non linear Faraday effect

Naive model of rotation

Experiment

\[ \Delta \chi' \]

\[ -10 -5 0 5 10 \]

B field

\[ \lambda/2 \]

SMPM

FIBER

GPLENS

MAGNETIC

SHIELDING

PBS

BPD

SCOPE

LASER

Rb Cell

\[ ^{87}\text{Rb} \]

\[ \lambda/2 \]
Optical magnetometer and non-linear Faraday effect

Naive model of rotation

Experiment

-10 -5 0 5 10

\( \Delta \chi' \)

B field

1 mW
2 mW
4 mW
6 mW
8 mW
12 mW

Rotation response (V)

Magnetic field (G)

Squeezed light

Eugeniy E. Mikhailov (W&M)

October 5, 2012
Shot noise limit of the magnetometer

\[ S = \left| E_p + E_v \right|^2 - \left| E_p - E_v \right|^2 \]

\[ S = 4E_pE_v \]

\[ < \Delta S > \sim E_p < \Delta E_v > \]
Squeezed enhanced magnetometer setup

Note: Squeezed enhanced magnetometer was first demonstrated by Wolfgramm et. al/ Phys. Rev. Lett, 105, 053601, 2010.
Magnetometer noise floor improvements

![Graph showing noise spectral density vs. noise frequency.](image)

(a) Squeezed light

Eugeniy E. Mikhailov (W&M)

October 5, 2012 16 / 42
Magnetometer noise spectra

![Graph showing noise spectra for different probe types: coherent, squeezed, and SQL.](image)

- Coherent probe
- Squeezed probe
- SQL

Noise spectral density (dBm/Hz)

Frequency (kHz)
Noise suppression and response vs atomic density

Noise suppression

Response
Magnetometer with squeezing enhancement

---

- **SQUEEZER**
  - Laser
  - SMPM Fiber
  - Lens
  - GP
  - PBS
- **MAGNETOMETER**
  - Laser
  - SMPM Fiber
  - Lens
  - PBS
  - Scope
  - Squeezed Probe

---

**Sensitivity (pT/√Hz)** vs **Atomic Density (atoms/cm³)**

- (a) Coherent probe
- (b) Squeezed probe

---

Eugeniya E. Mikhailov (W&M)

Squeezed light

October 5, 2012
Group velocity \( v_g = \frac{c}{\omega \frac{\partial n}{\partial \omega}} \)

### Susceptibility

Rotation vs B field
Light group velocity expression

Group velocity \( v_g = \frac{c}{\omega \frac{\partial n}{\partial \omega}} \)

Susceptibility

Rotation vs B field
Light group velocity estimate

Group velocity \( v_g = \frac{c}{\omega \frac{\partial n}{\partial \omega}} \)

Delay \( \tau = \frac{L}{v_g} \sim \frac{\partial n}{\partial \omega} \sim \frac{\partial R}{\partial B} \)
Light group velocity estimate

Group velocity $v_g = \frac{c}{\omega \frac{\partial n}{\partial \omega}}$

Delay $\tau = \frac{L}{v_g} \sim \frac{\partial n}{\partial \omega} \sim \frac{\partial R}{\partial B}$
Squeezing vs magnetic field

Spectrum analyzer settings: Central frequency = 1 MHz, VBW = 3 MHz, RBW = 100 kHz

![Diagram of the experimental setup](image)

---

Eugeniy E. Mikhailov (W&M)
Squeezing vs magnetic field

Spectrum analyzer settings: Central frequency = 1 MHz, VBW = 3 MHz, RBW = 100 kHz

(a) antisqueezed
(b) squeezed
(c) squeezed
(d) antisqueezed
(e) squeezed
(f) squeezed

SNL

Noise power (dB)

Magnetic field (G)

0 0.2 0.4 0.6 0.8 1

-2.5 -2 -1.5 -1 -0.5 0

-0.5 0 0.5 1

(a) antisqueezed
(b) squeezed

Eugeniy E. Mikhailov (W&M)
Time advancement setup

![Diagram of time advancement setup](image)
Squeezing modulation and time advancement
Squeezing after advancement cell

![Graph showing squeezing level (dB) vs. probe power (mW). The graph compares input and output signals, with the shot noise level indicated by a horizontal line.]
Advancement vs power

Arrival times

- Coherent probe
- Noise trial 1
- Noise trial 2

Pulse delay (μs) vs Probe power (mW)

- Advanced
- Delayed
- Squeezed
Advancement vs power

Arrival times

- Coherent probe
- Noise trial 1
- Noise trial 2

Pulse delay (μs) vs Probe power (mW)

Response slope (V/G) vs Probe power (mW)
Quantum limited interferometers revisited

Vacuum input

Squeezed input

Eugeniy E. Mikhailov (W&M)

Squeezed light

October 5, 2012
Next generation of LIGO will be **quantum optical noise limited** at almost all detection frequencies.
Next generation of LIGO will be quantum optical noise limited at almost all detection frequencies.

**shot noise**

Uncertainty in number of photons

\[ h \sim \sqrt{\frac{1}{P}} \quad (2) \]
Next generation of LIGO will be quantum optical noise limited at almost all detection frequencies.

### shot noise
Uncertainty in number of photons

\[ h \sim \sqrt{\frac{1}{P}} \]  \hspace{1cm} (2)

### radiation pressure noise
Photons impart momentum to mirrors

\[ h \sim \sqrt{\frac{P}{M^2 f^4}} \]  \hspace{1cm} (3)
Next generation of LIGO will be quantum optical noise limited at almost all detection frequencies.

shot noise

Uncertainty in number of photons

\[ h \sim \sqrt{\frac{1}{P}} \]  

(2)

radiation pressure noise

Photons impart momentum to mirrors

\[ h \sim \sqrt{\frac{P}{M^2 f^4}} \]  

(3)

There is no optimal light power to suit all detection frequency. Optimal power depends on desired detection frequency.
Interferometer sensitivity improvement with squeezing

Projected advanced LIGO sensitivity

EIT filter

Probe transparency dependence on its detuning.

\[ |a\rangle \]

\[ |b\rangle \]

\[ |c\rangle \]

\[ \omega_p \]

\[ \omega_{bc} \]
EIT filter

Probe transparency dependence on its detuning.

\[ \begin{align*}
|a\rangle \\
|b\rangle \\
|c\rangle
\end{align*} \]

\[ \omega_p, \omega_d, \omega_{bc} \]

\[ \begin{align*}
\text{Transparency [Arb. Unit]} \\
\text{Probe detuning [Arb. Unit]}
\end{align*} \]
\[
\begin{pmatrix}
V_{1}^{\text{out}} \\
V_{2}^{\text{out}}
\end{pmatrix}
= \begin{pmatrix}
A_{+}^{2} & A_{-}^{2} \\
A_{-}^{2} & A_{+}^{2}
\end{pmatrix}
\begin{pmatrix}
V_{1}^{\text{in}} \\
V_{2}^{\text{in}}
\end{pmatrix}
+ \left[ 1 - \left( A_{+}^{2} + A_{-}^{2} \right) \right]
\begin{pmatrix}
1 \\
1
\end{pmatrix}
\]

\[\varphi_{\pm} = \frac{1}{2} (\Theta_{+} \pm \Theta_{-})\]

\[A_{\pm} = \frac{1}{2} \left( T_{+} \pm T_{-} \right)\]
Squeezing and EIT filter

\[
\begin{pmatrix}
V_{1\text{out}} \\
V_{2\text{out}}
\end{pmatrix}
= 
\begin{pmatrix}
A_+^2 & A_-^2 \\
A_-^2 & A_+^2
\end{pmatrix}
\begin{pmatrix}
V_{1\text{in}} \\
V_{2\text{in}}
\end{pmatrix}
+ [1 - (A_+^2 + A_-^2)]
\begin{pmatrix}
1 \\
1
\end{pmatrix}
\]

\[\phi_{\pm} = \frac{1}{2} (\Theta_+ \pm \Theta_-)\]

\[A_{\pm} = \frac{1}{2} (T_+ \pm T_-)\]
\[
\begin{pmatrix}
V_{1}^{\text{out}} \\
V_{2}^{\text{out}}
\end{pmatrix}
= \begin{pmatrix}
A_{2}^{+} & A_{2}^{-} \\
A_{-}^{2} & A_{+}^{2}
\end{pmatrix}
\begin{pmatrix}
V_{1}^{\text{in}} \\
V_{2}^{\text{in}}
\end{pmatrix}
+ \left[1 - \left(A_{+}^{2} + A_{-}^{2}\right)\right]
\begin{pmatrix}
1 \\
1
\end{pmatrix}
\]

\begin{align*}
\varphi_{\pm} &= \frac{1}{2} (\Theta_{+} \pm \Theta_{-}) \\
A_{\pm} &= \frac{1}{2} (T_{+} \pm T_{-})
\end{align*}
\[
\begin{pmatrix}
V_{1\text{out}} \\
V_{2\text{out}}
\end{pmatrix} =
\begin{pmatrix}
A_+^2 & A_-^2 \\
A_-^2 & A_+^2
\end{pmatrix}
\begin{pmatrix}
V_{1\text{in}} \\
V_{2\text{in}}
\end{pmatrix} + \left[1 - (A_+^2 + A_-^2)\right]
\begin{pmatrix}
1 \\
1
\end{pmatrix}
\]

\[\varphi_{\pm} = \frac{1}{2} (\Theta_+ \pm \Theta_-)\]

\[A_{\pm} = \frac{1}{2} (T_+ \pm T_-)\]
Squeezing and EIT filter setup

Laser → Squeezing → EIT
EIT filter and measurements without light

Signal in the noise quadratures

Coherent signal

Eugeniy E. Mikhailov (W&M)  Squeezed light  October 5, 2012  33 / 42
Squeezing angle rotation

\[
\begin{pmatrix}
V_{1}^{\text{out}} \\
V_{2}^{\text{out}}
\end{pmatrix} =
\begin{pmatrix}
\cos^2 \varphi_+ & \sin^2 \varphi_+ \\
\sin^2 \varphi_+ & \cos^2 \varphi_+
\end{pmatrix}
\begin{pmatrix}
A_+^2 \\
A_-^2
\end{pmatrix}
\begin{pmatrix}
V_{1}^{\text{in}} \\
V_{2}^{\text{in}}
\end{pmatrix} + \left[ 1 - \left( A_+^2 + A_-^2 \right) \right]
\begin{pmatrix}
1 \\
1
\end{pmatrix}
\]

Locked at 300kHz

Locked at 1200kHz
Narrower filter

T=35° C, no control
transmission 42%

T=40° C, no control
transmission 17%
Narrower filter

T=35°C, no control transmission 42%

T=40°C, no control transmission 17%

Noise vs frequency through EIT (35 degrees)

Noise vs frequency through EIT (40 degrees)
Excess noise and leakage

**Effect of leakage photons**

- **Shot noise**
- **Output noise, 1 mV leakage**
- **Output noise, 20 mV leakage**

**dB**

**MHz**
Theoretical prediction for MOT squeezing with \(^{87}\text{Rb}\)

\[ F_g = 2 \rightarrow F_e = 1, 2 \] high optical density is very important

\[ \gamma = 10^{-1} \Gamma \]

\[ \gamma = 10^{-2} \Gamma \]

\[ \gamma = 10^{-3} \Gamma \]

\[ \gamma = 10^{-4} \Gamma \]
MOT squeezer

Cloud size = 1 mm, $T = 200 \, \mu K$, $N = 7 \times 10^9 \, 1/cm^3$, $OD = 2$, beam size = 0.1 mm, $10^5$ interacting atoms
Noise contrast in MOT with $^{87}\text{Rb} \ F_g = 2 \rightarrow \ F_e = 1$
Squeezing in MOT with $^{87}\text{Rb}$ $F_g = 2 \rightarrow F_e = 1$
Summary

- We demonstrate fully atomic squeezed enhanced magnetometer
- Magnetometer noise floor lowered in the range from several kHz to several MHz
- Demonstrated sensitivity as low as 1 pT/√Hz in our particular setup
- First demonstration of superluminal squeezing propagation with $v_g = c/2000$ or time advancement of 0.5 μS

For more details:


Support from Eugeniy E. Mikhailov (W&M)