

Squeezed light, generation and applications.

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From ray optics to semiclassical optics

Classical/Geometrical optics

- light is a ray
- which propagates straight
- cannot explain diffraction and interference

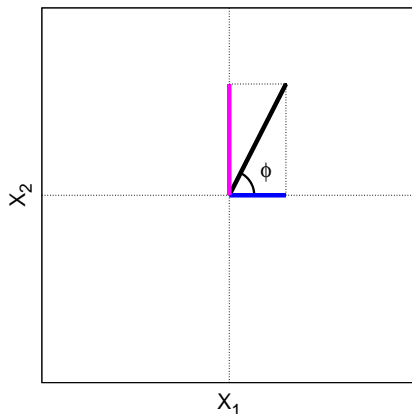
Semiclassical optics

- light is a wave
- color (wavelength/frequency) is important
- amplitude (a) and phase are important, $E(t) = ae^{i(kz-\omega t)}$
- cannot explain residual measurements noise

Classical field

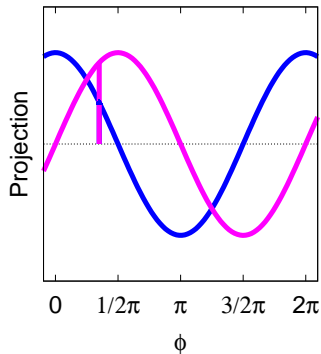
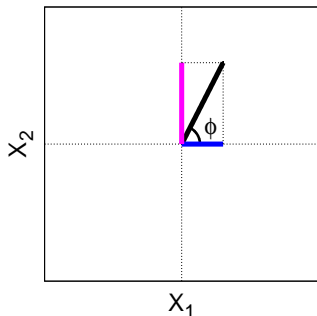
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Detectors sense the **real** part of the field (X_1) but there is a way to see X_2 as well



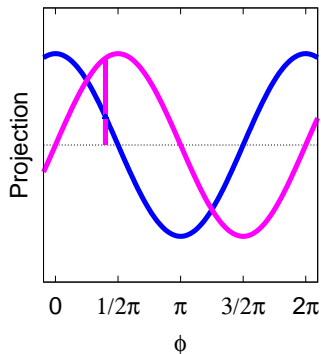
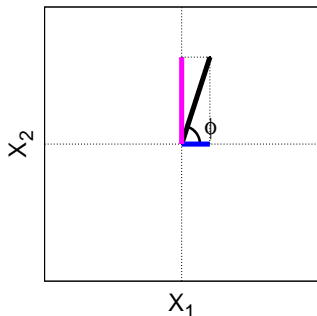
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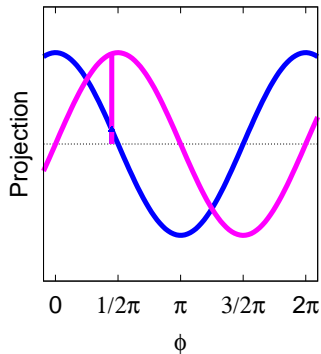
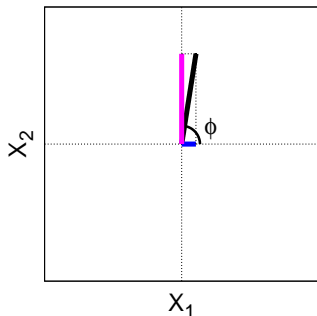
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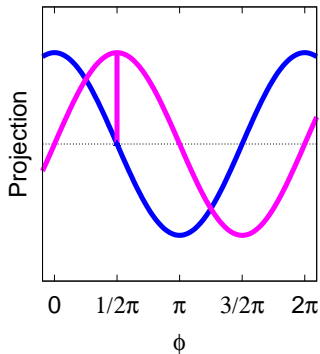
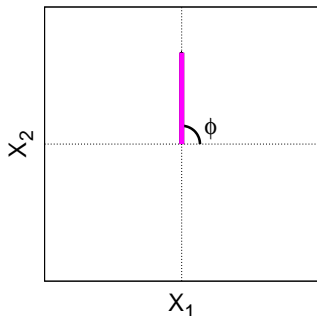
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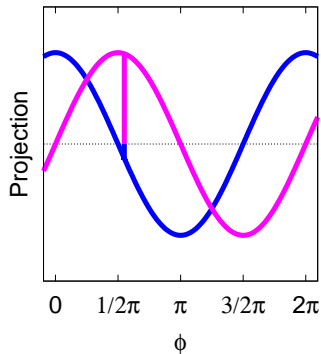
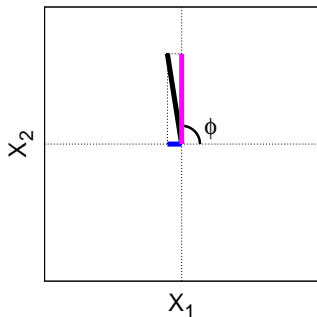
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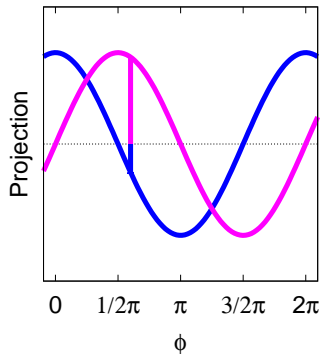
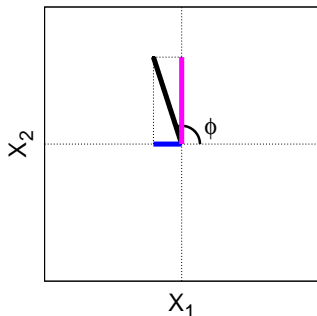
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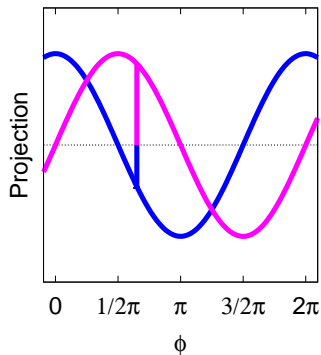
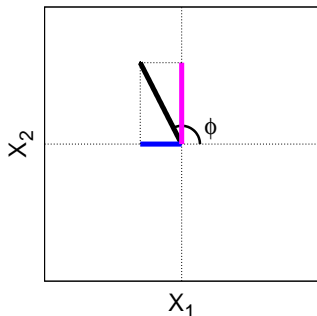
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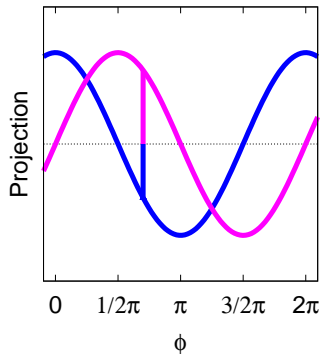
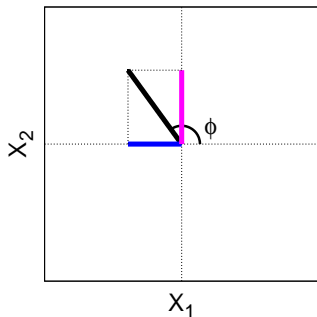
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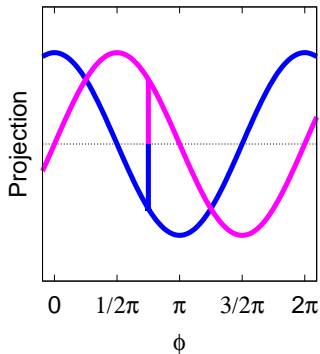
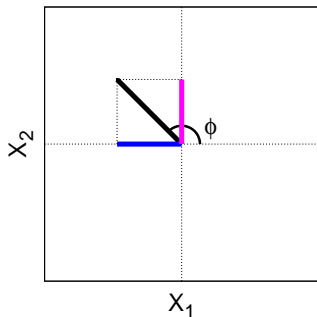
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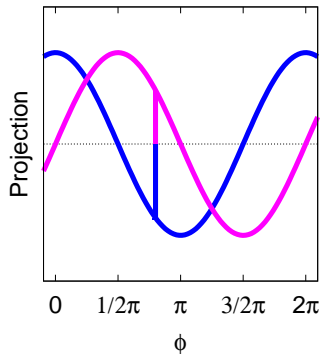
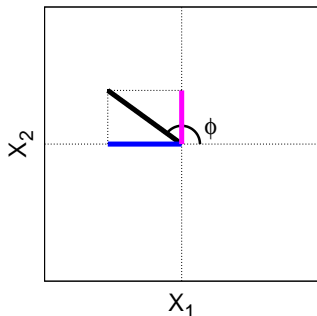
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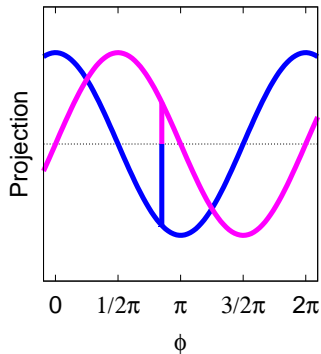
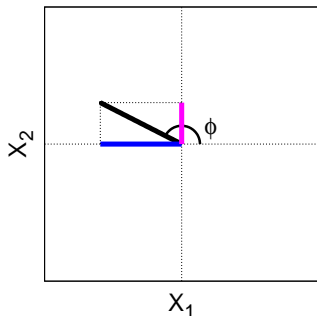
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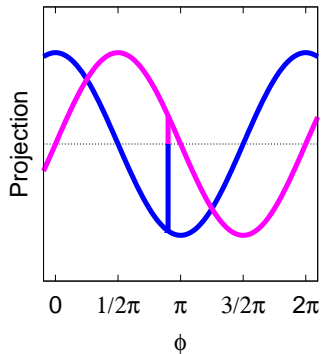
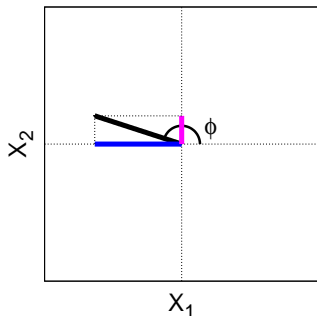
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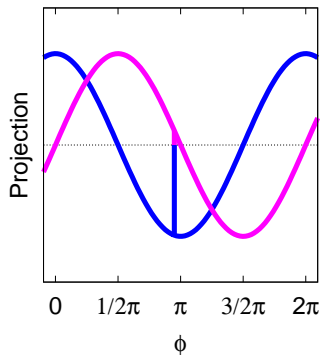
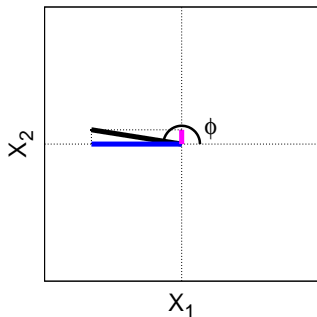
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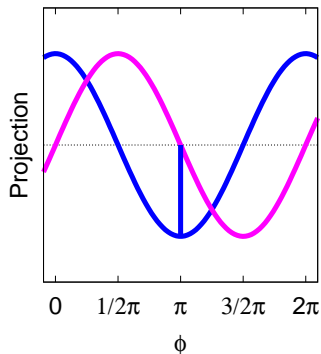
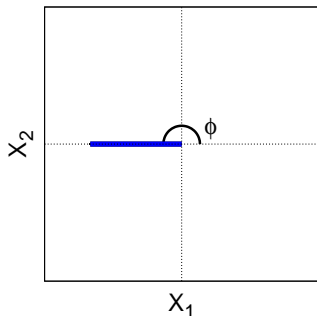
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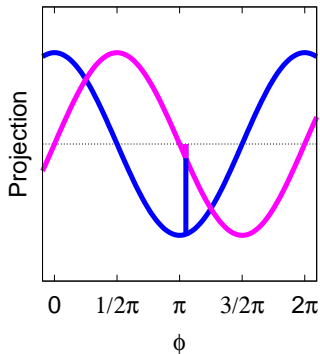
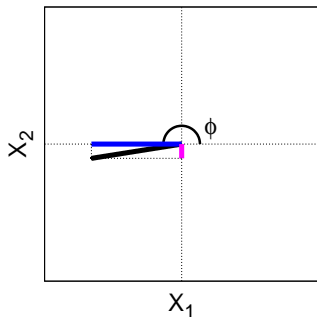
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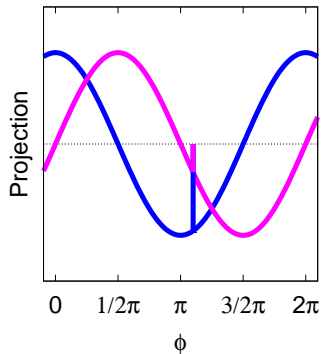
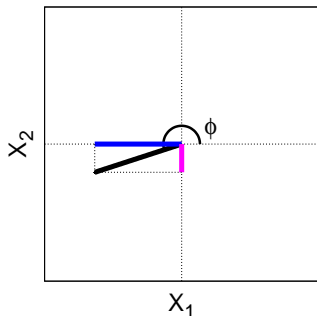
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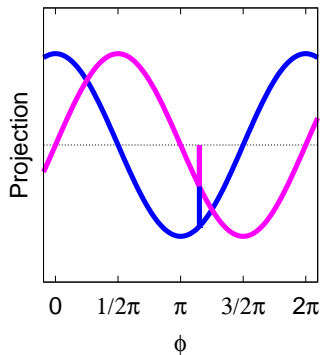
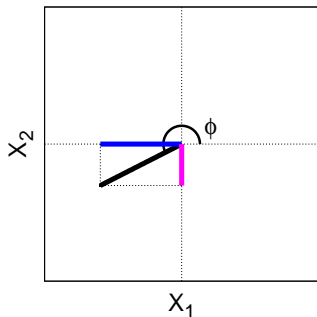
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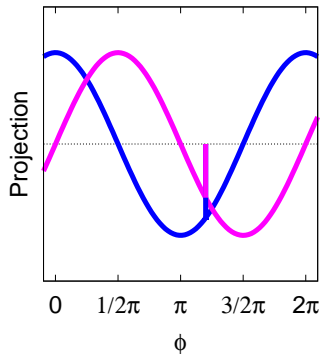
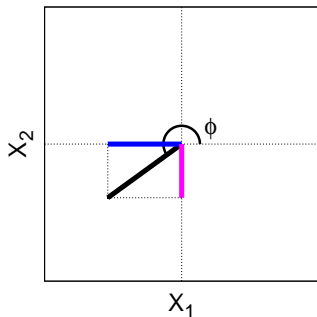
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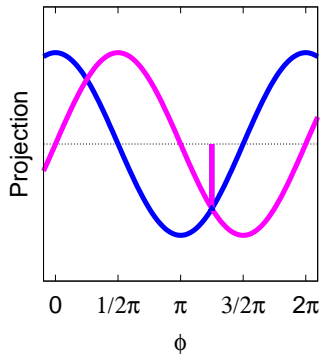
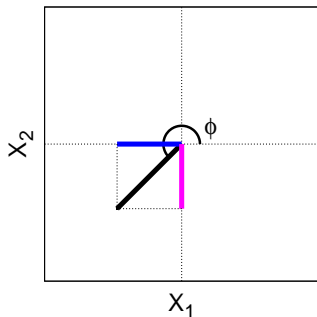
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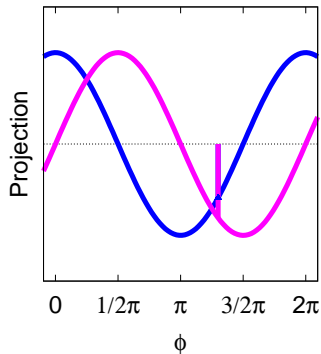
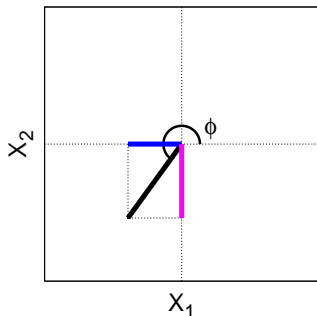
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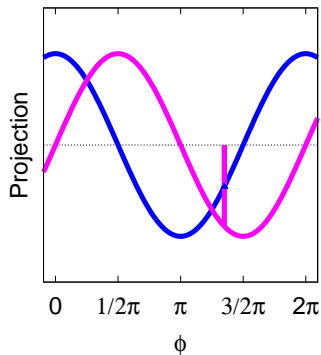
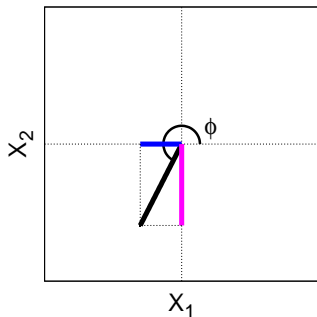
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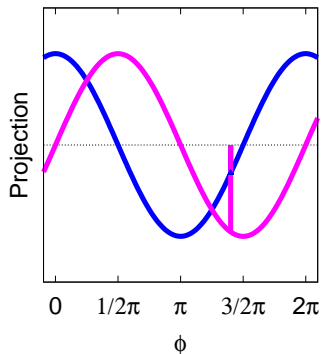
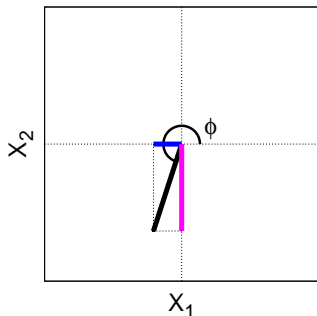
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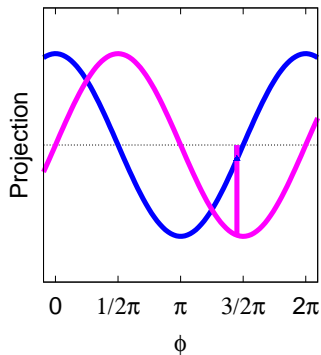
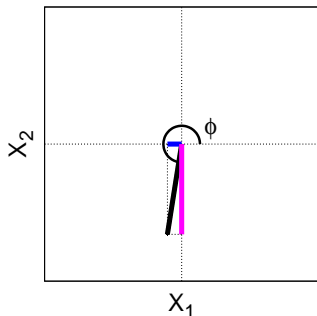
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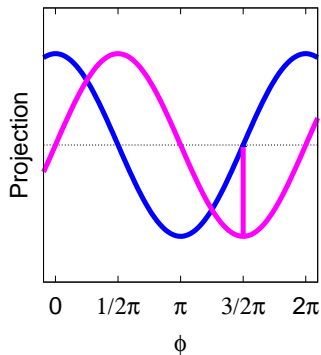
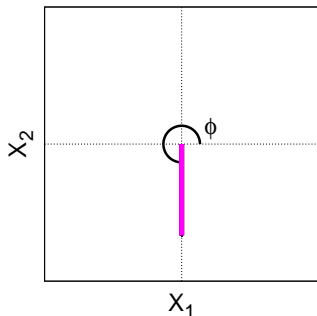
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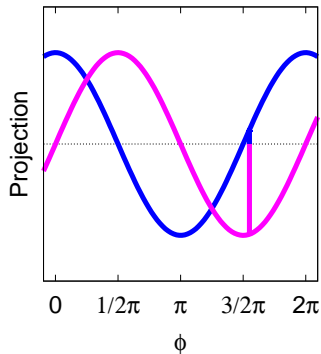
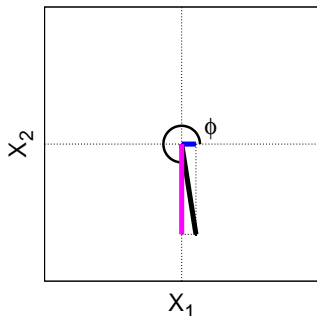
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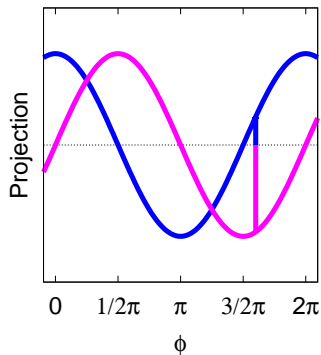
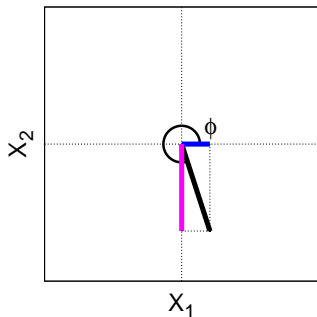
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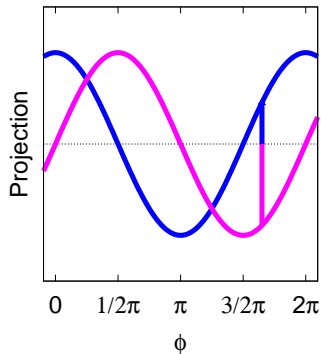
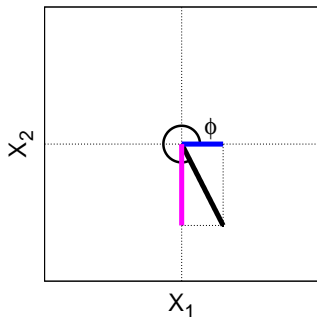
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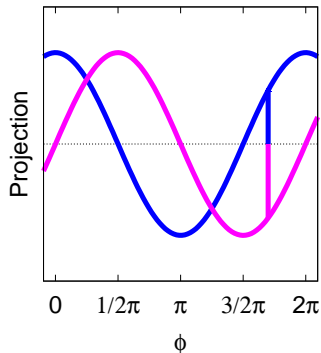
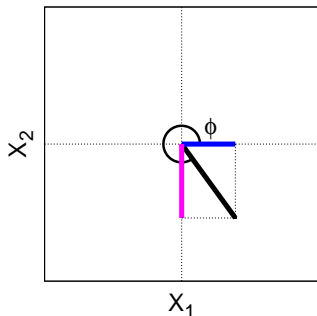
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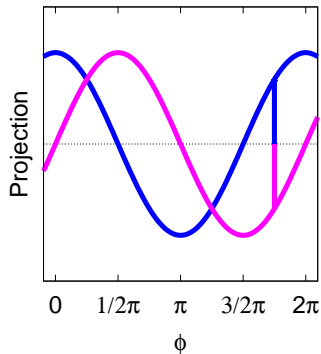
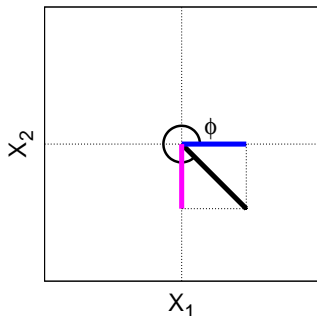
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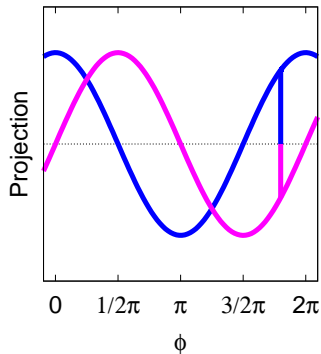
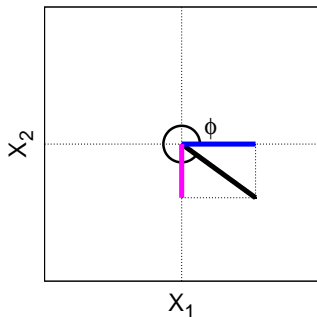
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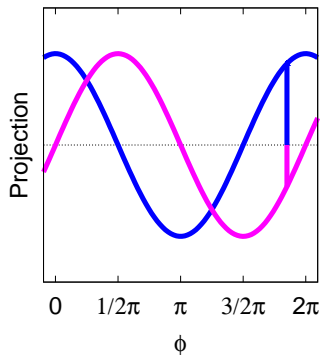
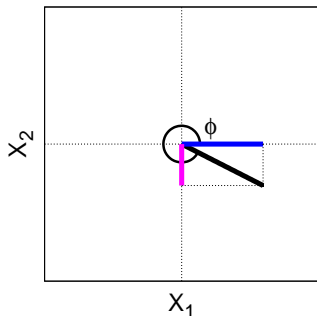
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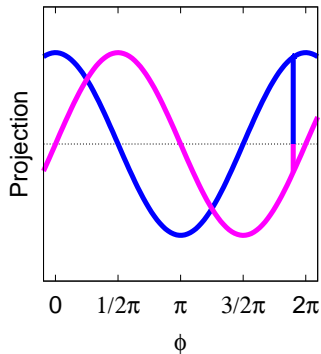
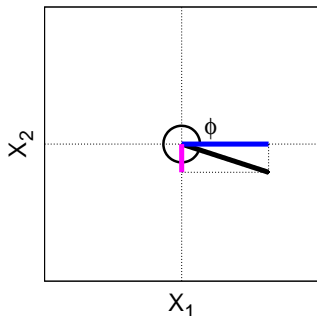
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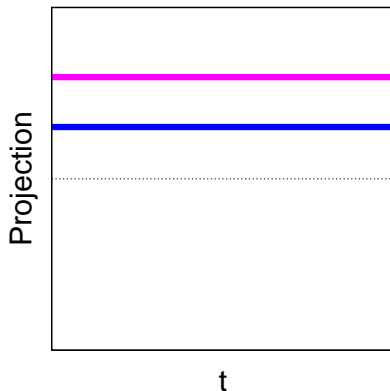
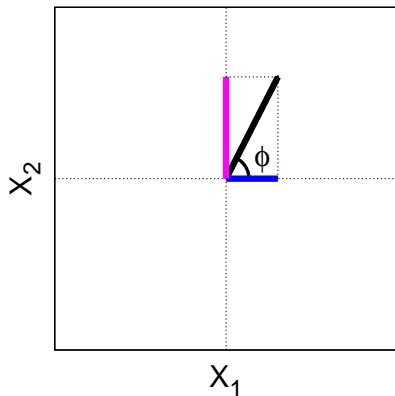
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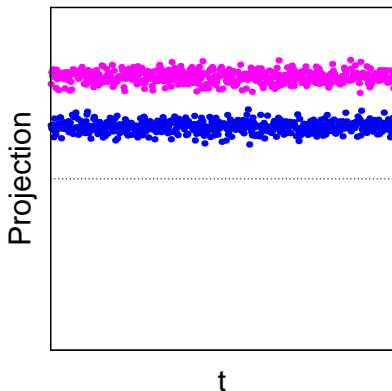
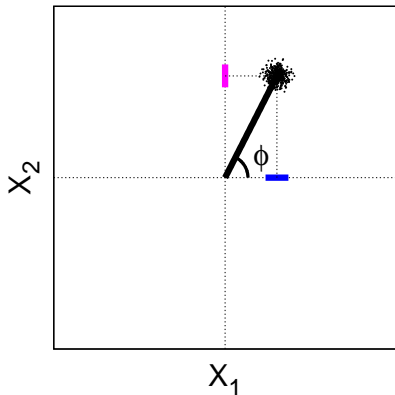
Classical quadratures vs time in a rotating frame

$$E(\phi) = |a|e^{-i\phi} = |a|\cos(\phi) + i|a|\sin(\phi) = X_1 + iX_2, \quad \phi = \omega t - kz$$



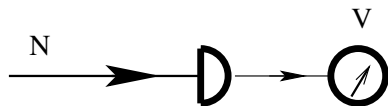
Reality check quadratures vs time

$$E(\phi) = |a|e^{-i\phi} = |a|\cos(\phi) + i|a|\sin(\phi) = X_1 + iX_2, \quad \phi = \omega t - kz$$



Detector quantum noise

Simple photodetector

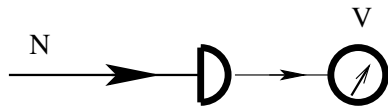


$$V \sim N$$

$$\Delta V \sim \sqrt{N}$$

Detector quantum noise

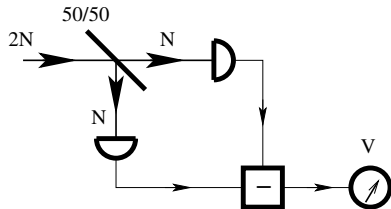
Simple photodetector



$$V \sim N$$

$$\Delta V \sim \sqrt{N}$$

Balanced photodetector



$$V = 0$$

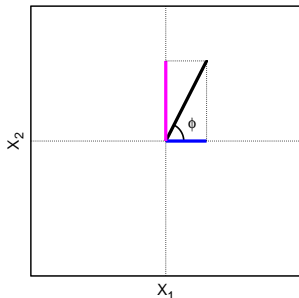
$$\Delta V \sim \sqrt{N}$$

Transition from classical to quantum field

Classical analog

- Field amplitude a
- Field real part
 $X_1 = (a^* + a)/2$
- Field imaginary part
 $X_2 = i(\hat{a}^* - a)/2$

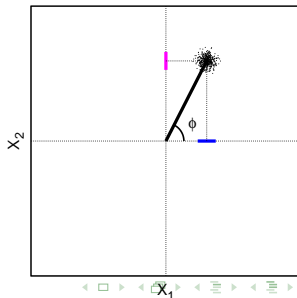
$$E(\phi) = |a|e^{-i\phi} = X_1 + iX_2$$



Quantum approach

- Field operator \hat{a}
- Amplitude quadrature
 $\hat{X}_1 = (\hat{a}^\dagger + \hat{a})/2$
- Phase quadrature
 $\hat{X}_2 = i(\hat{a}^\dagger - \hat{a})/2$

$$\hat{E}(\phi) = \hat{X}_1 + i\hat{X}_2$$



Heisenberg uncertainty principle and its optics equivalent

Heisenberg uncertainty principle

$$\Delta p \Delta x \geq \hbar/2$$

The more precisely the POSITION is determined, the less precisely the MOMENTUM is known, and vice versa



Heisenberg uncertainty principle and its optics equivalent



Heisenberg uncertainty principle

$$\Delta p \Delta x \geq \hbar/2$$

The more precisely the POSITION is determined, the less precisely the MOMENTUM is known, and vice versa

Optics equivalent

$$\Delta \phi \Delta N \geq 1$$

The more precisely the PHASE is determined, the less precisely the AMPLITUDE is known, and vice versa

Heisenberg uncertainty principle and its optics equivalent



Heisenberg uncertainty principle

$$\Delta p \Delta x \geq \hbar/2$$

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Optics equivalent

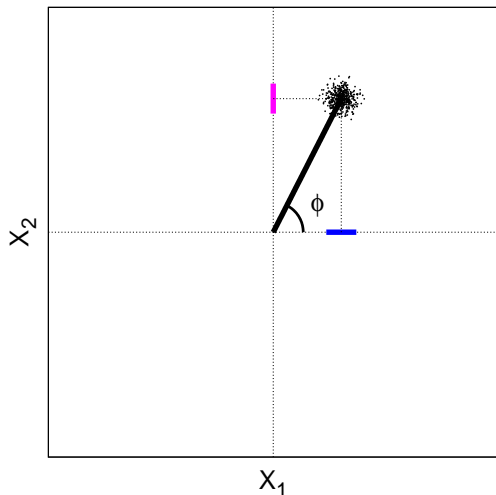
$$\Delta \phi \Delta N \geq 1$$

The more precisely the PHASE is determined, the less precisely the AMPLITUDE is known, and vice versa

Optics equivalent strict definition

$$\Delta X_1 \Delta X_2 \geq 1/4$$

Quantum optics summary



Light consist of photons

- $\hat{N} = a^\dagger a$

Commutator relationship

- $[a, a^\dagger] = 1$

- $[X_1, X_2] = i/2$

Detectors measure

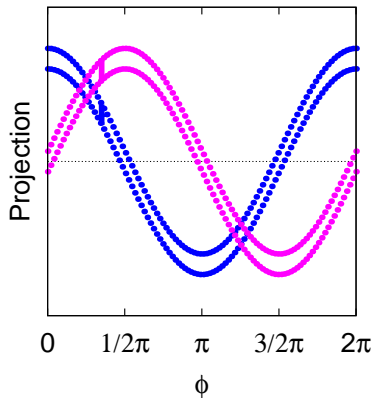
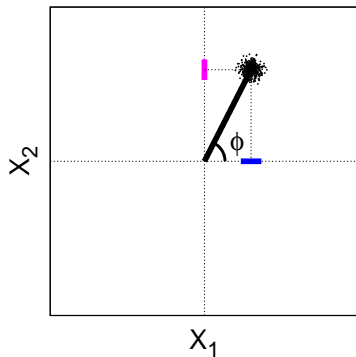
- number of photons N
- Quadratures \hat{X}_1 and \hat{X}_2

Uncertainty relationship

- $\Delta X_1 \Delta X_2 \geq 1/4$

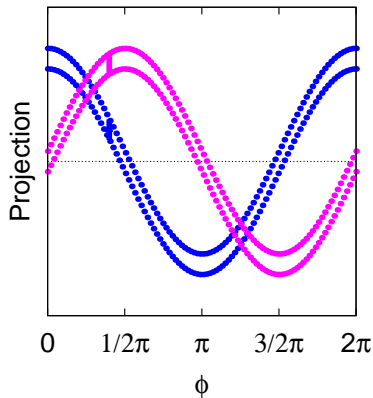
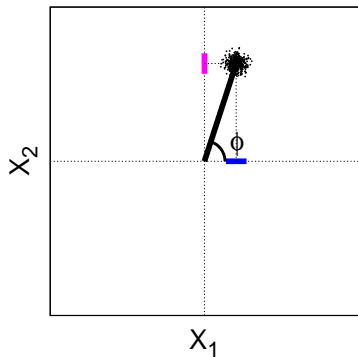
Coherent state is minimum uncertainty state

$$\Delta X_1 \Delta X_2 = 1/4$$



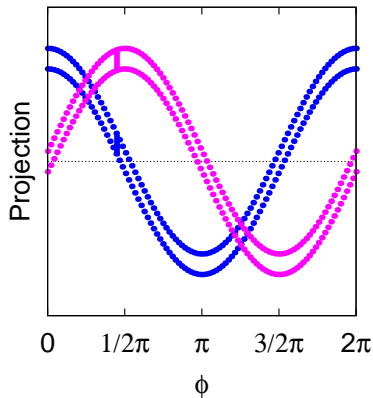
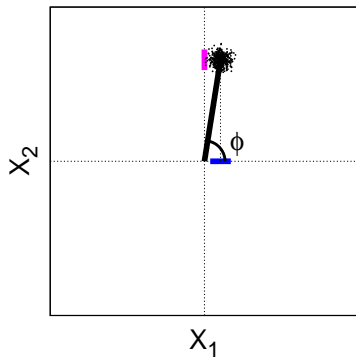
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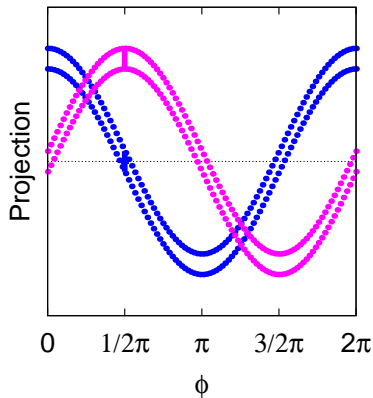
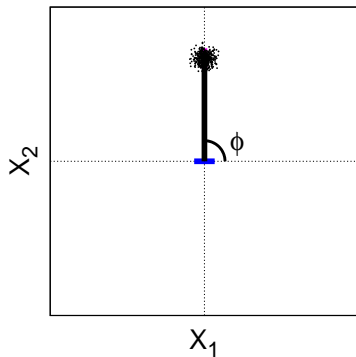
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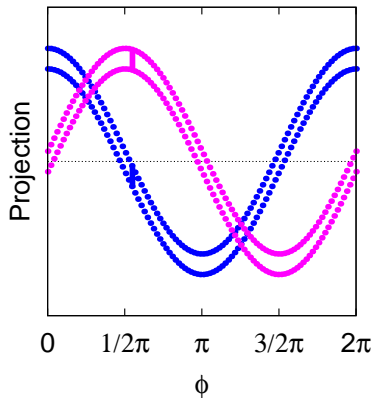
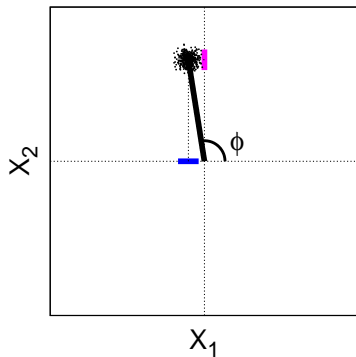
Coherent state is minimum uncertainty state

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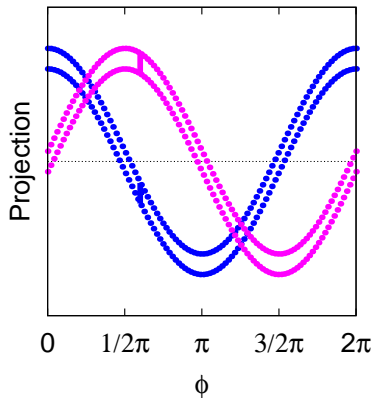
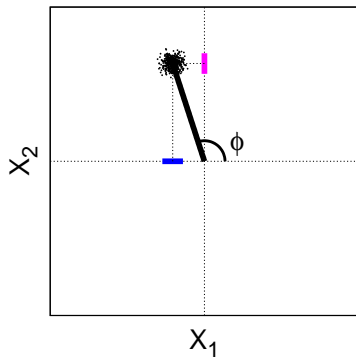
Coherent state is minimum uncertainty state

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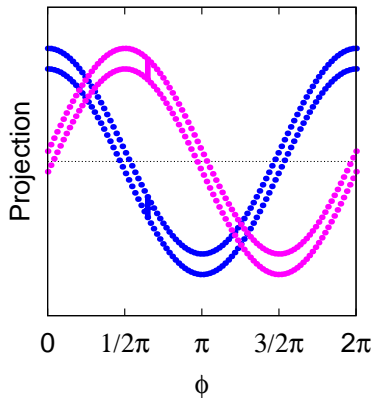
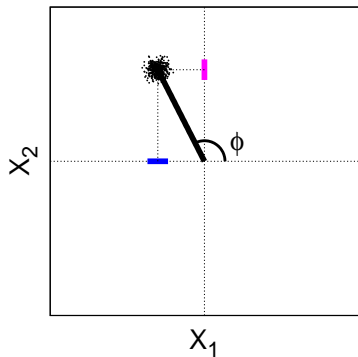
Coherent state is minimum uncertainty state

$$\Delta X_1 \Delta X_2 = 1/4$$



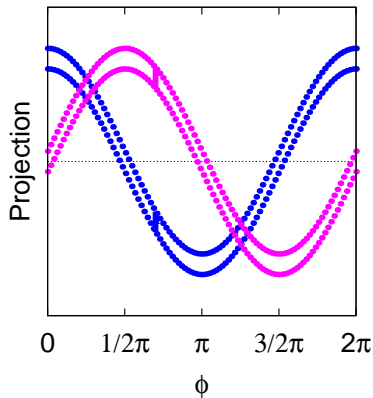
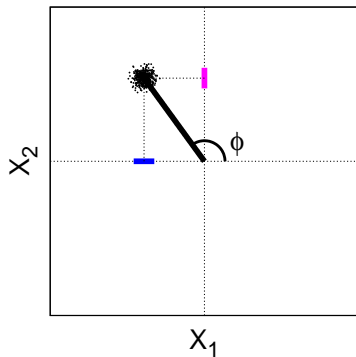
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$$\Delta X_1 \Delta X_2 = 1/4$$



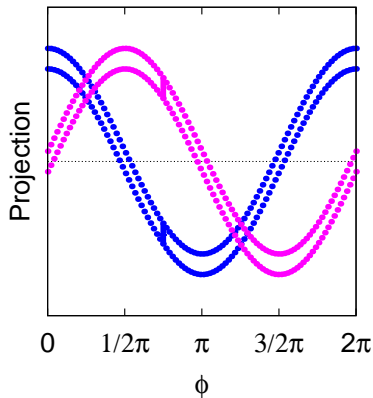
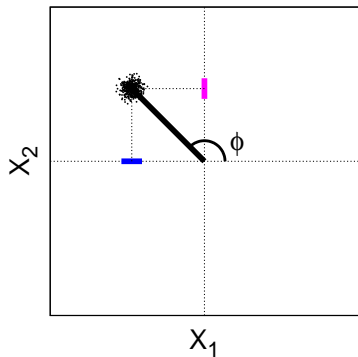
Coherent state is minimum uncertainty state

$$\Delta X_1 \Delta X_2 = 1/4$$



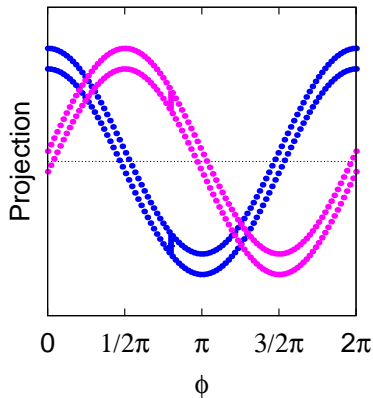
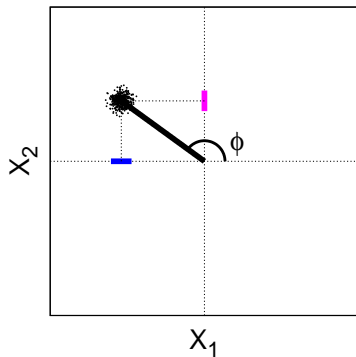
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$$\Delta X_1 \Delta X_2 = 1/4$$



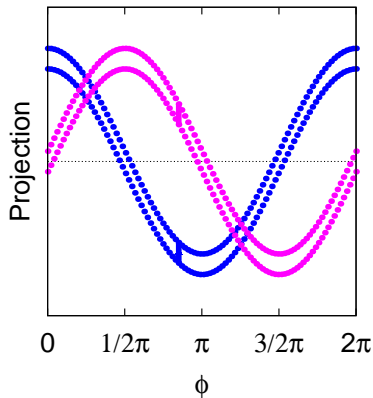
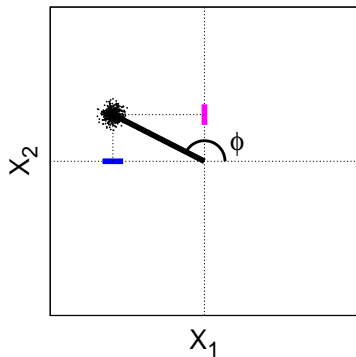
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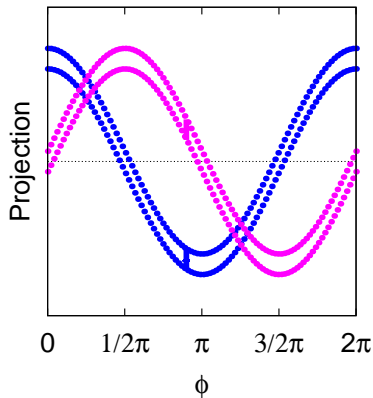
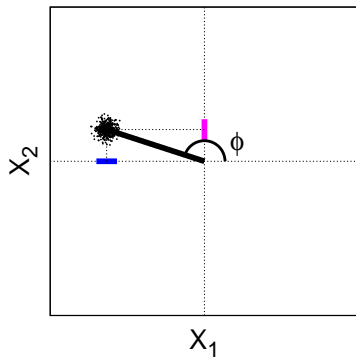
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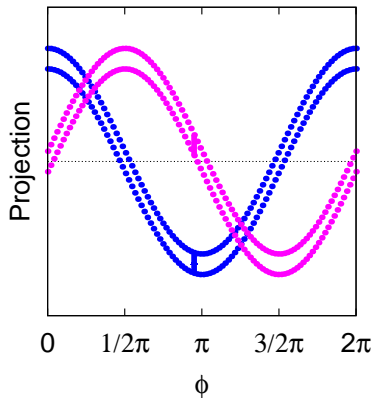
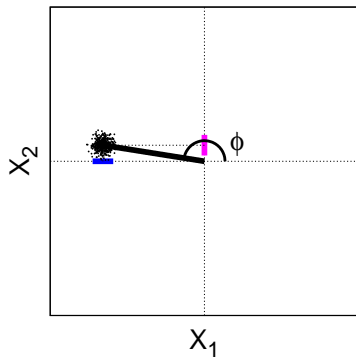
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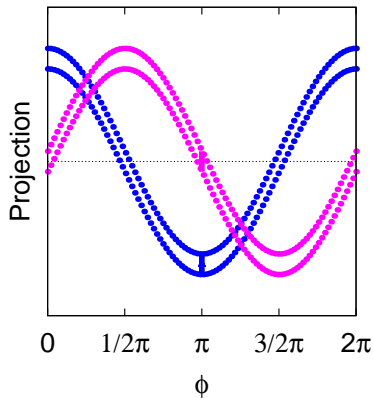
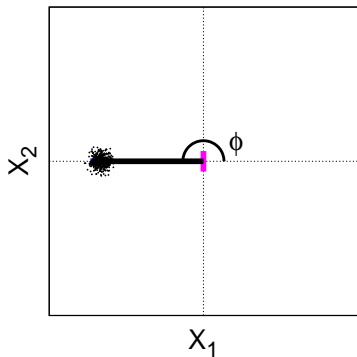
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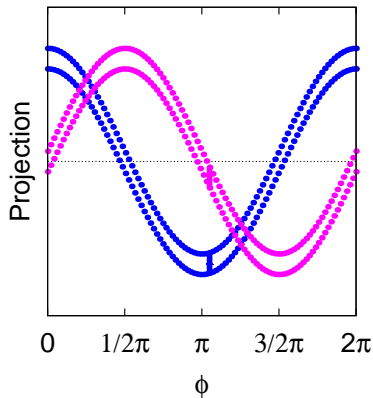
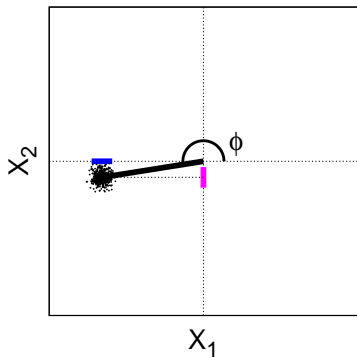
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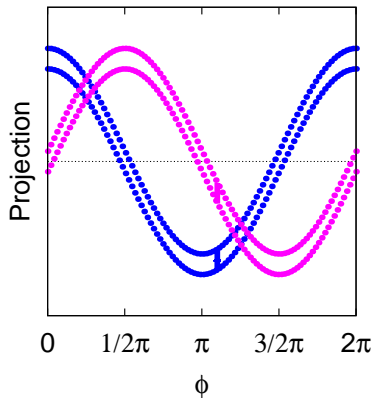
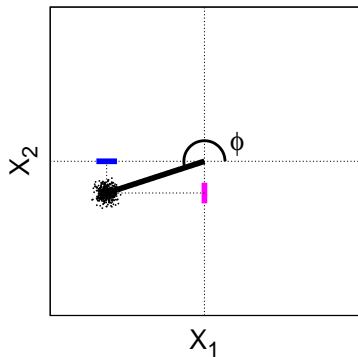
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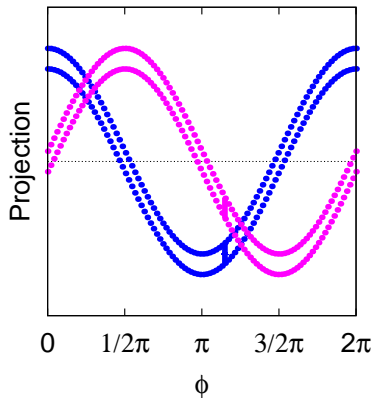
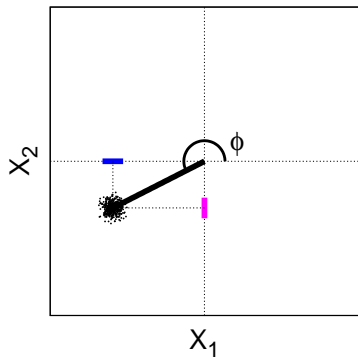
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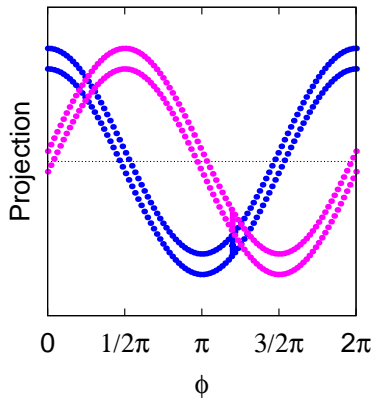
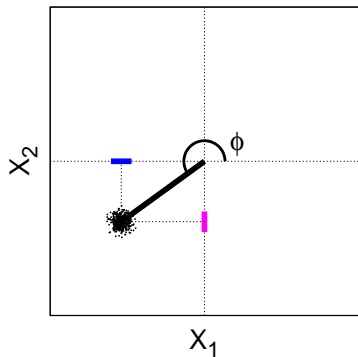
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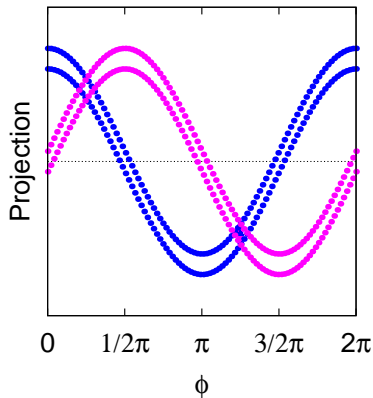
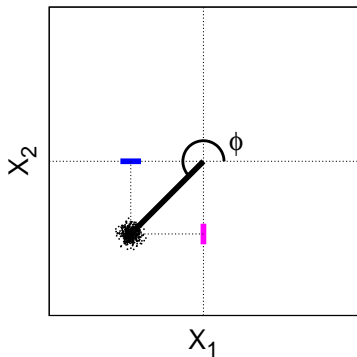
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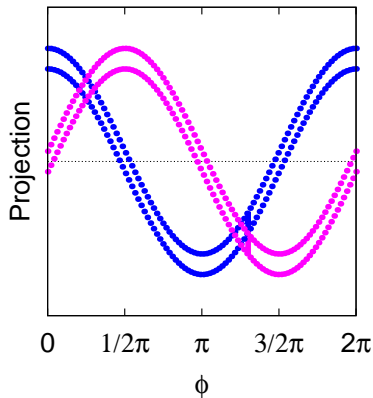
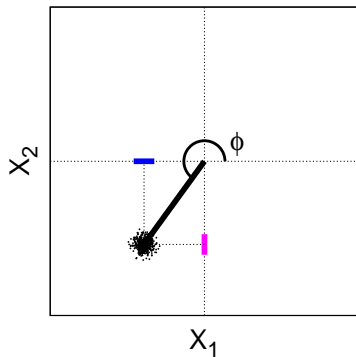
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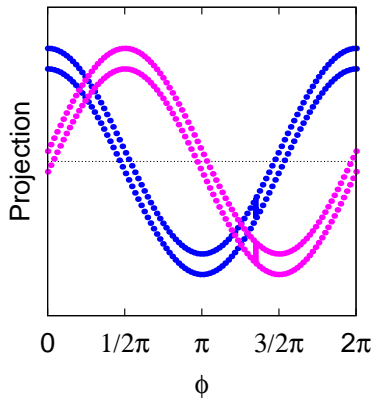
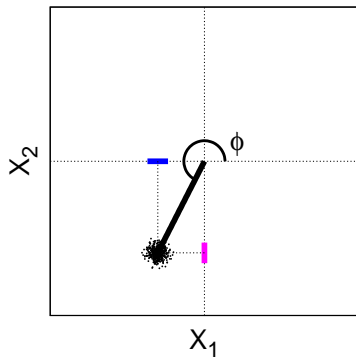
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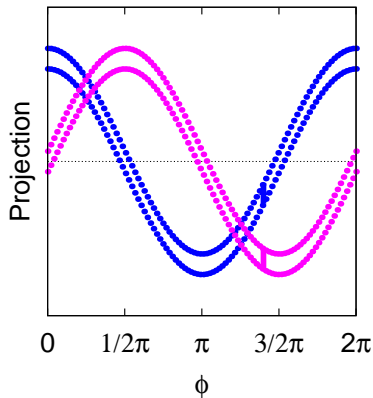
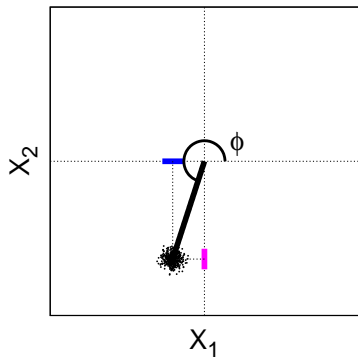
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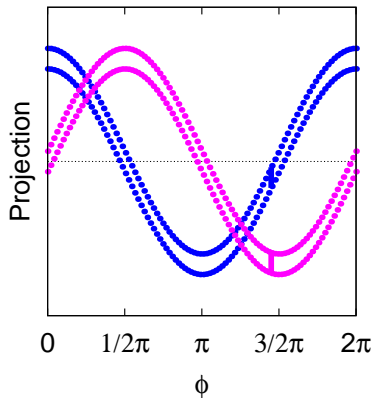
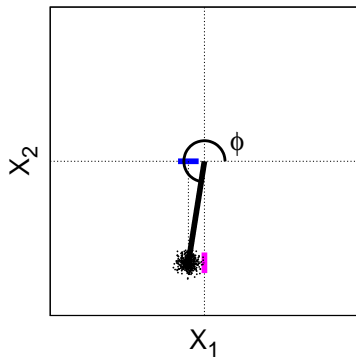
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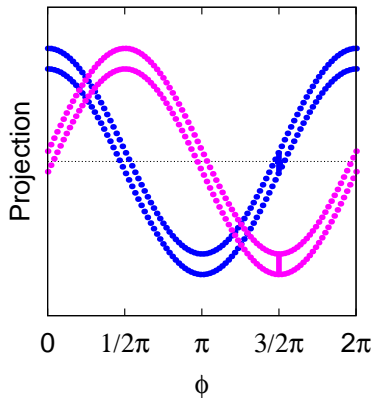
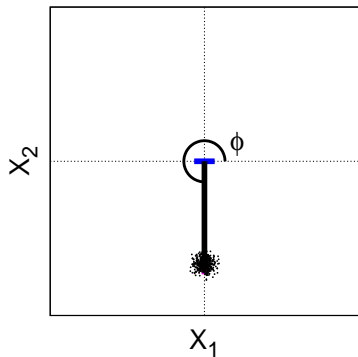
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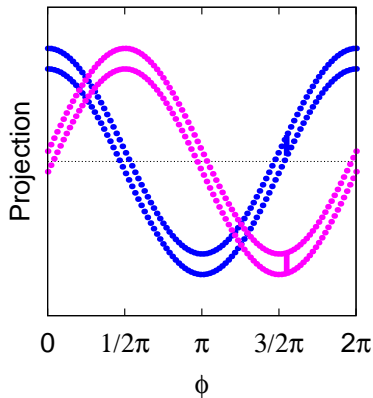
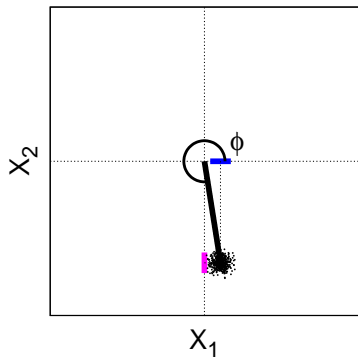
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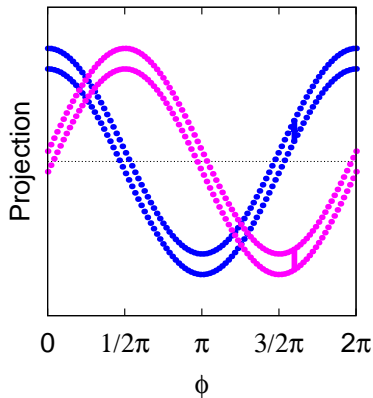
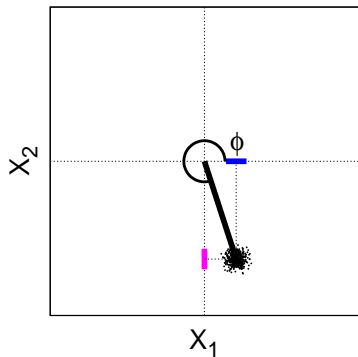
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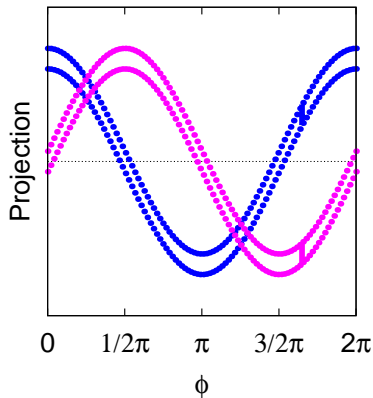
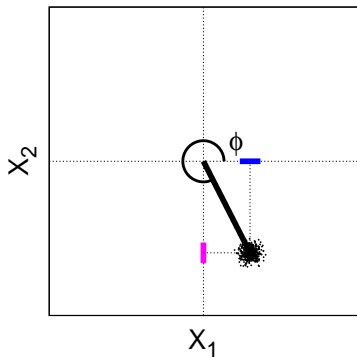
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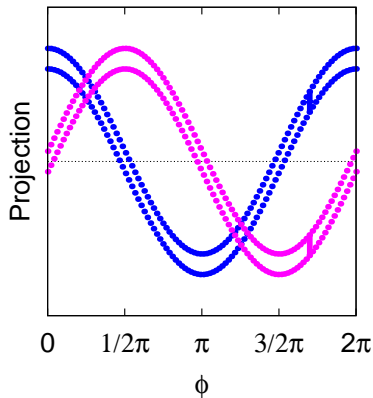
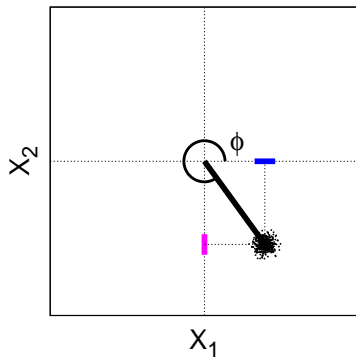
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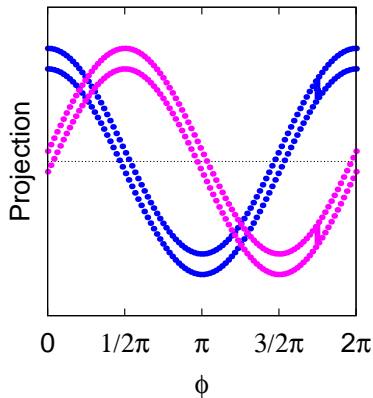
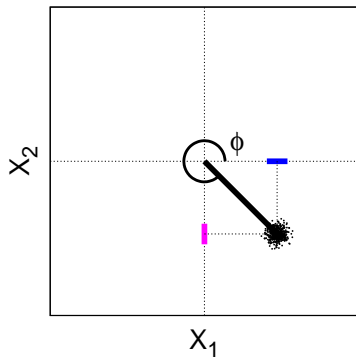
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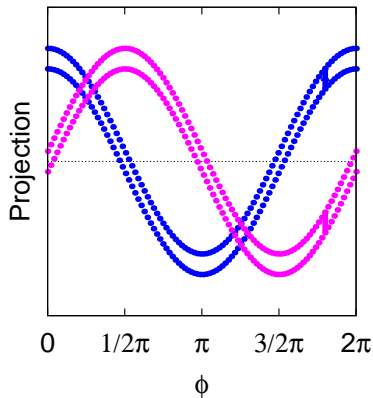
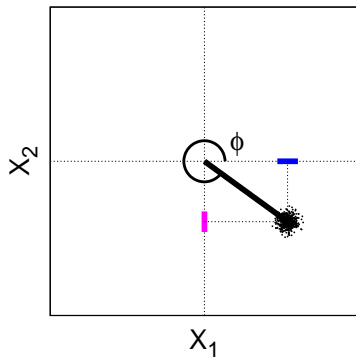
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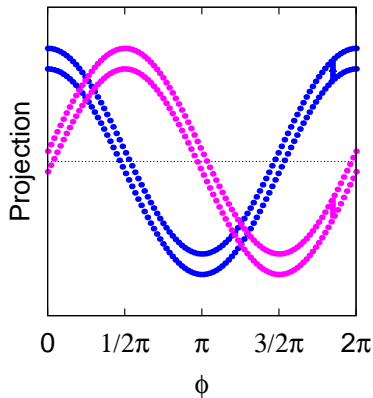
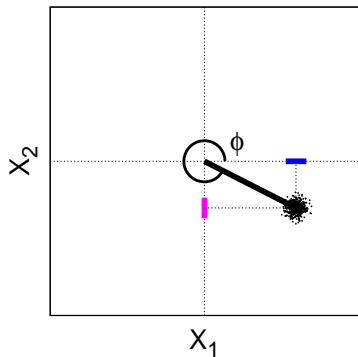
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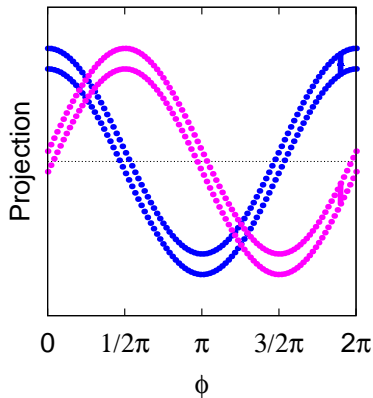
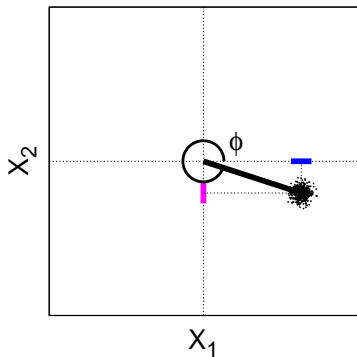
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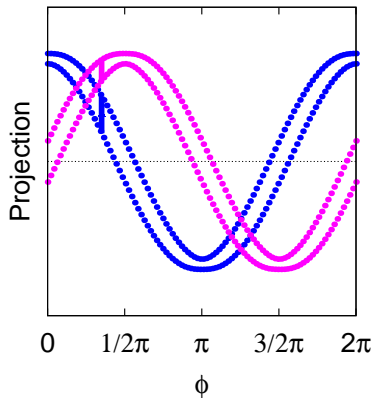
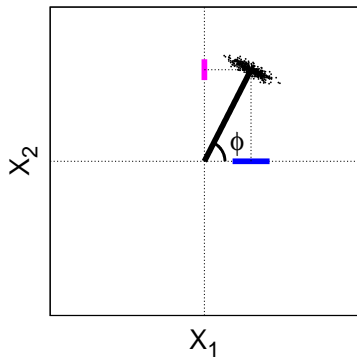
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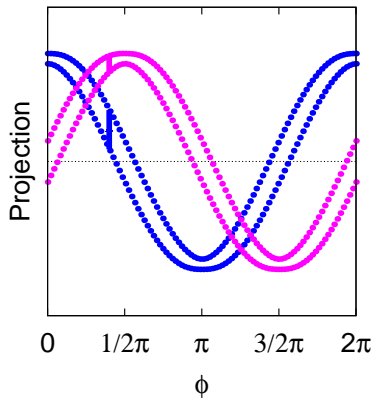
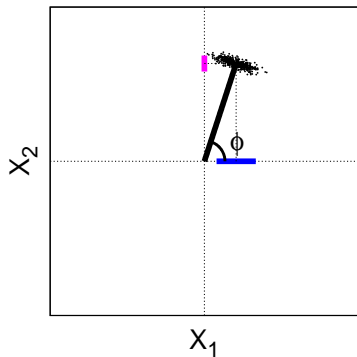
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



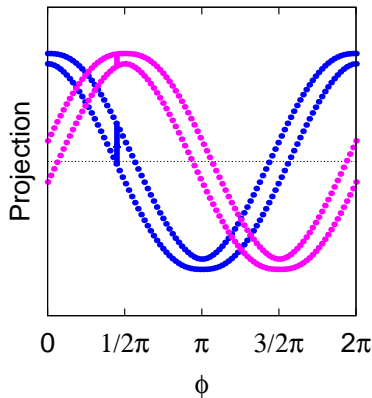
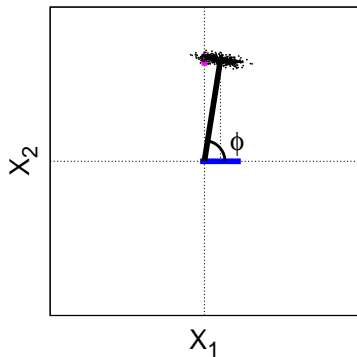
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



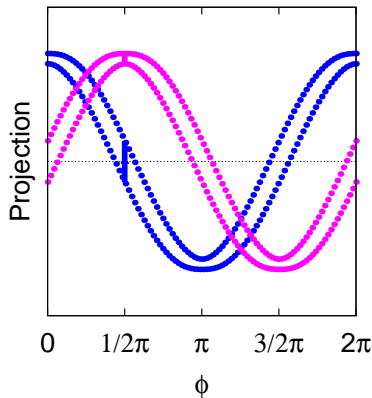
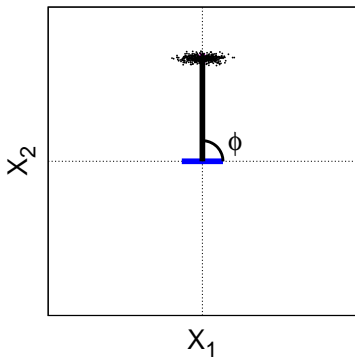
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



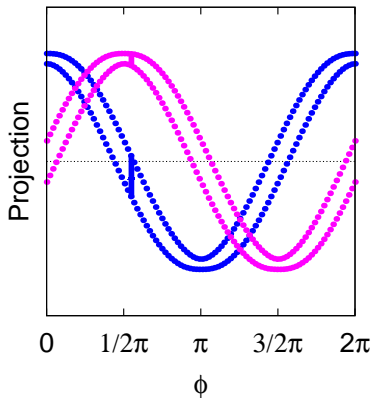
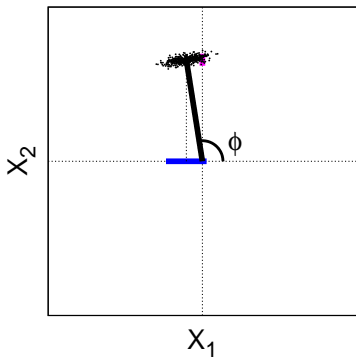
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



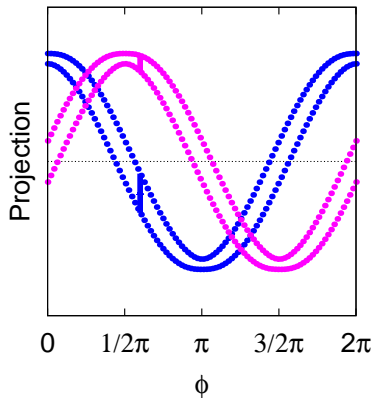
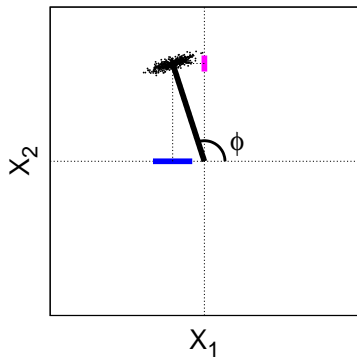
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



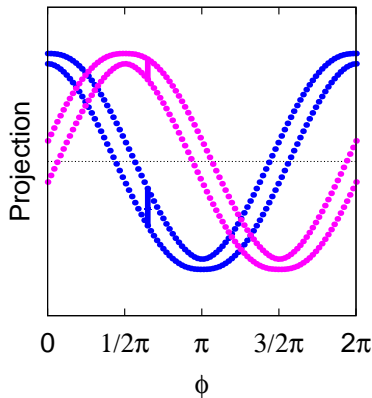
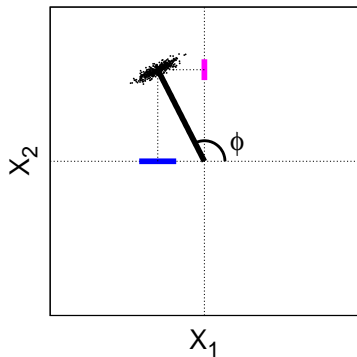
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



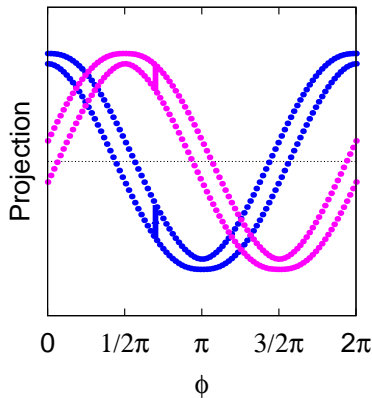
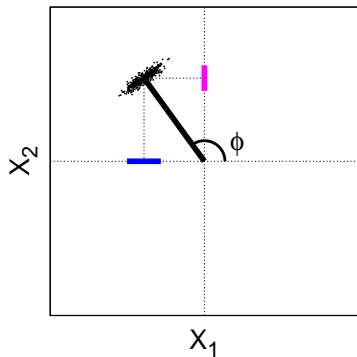
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



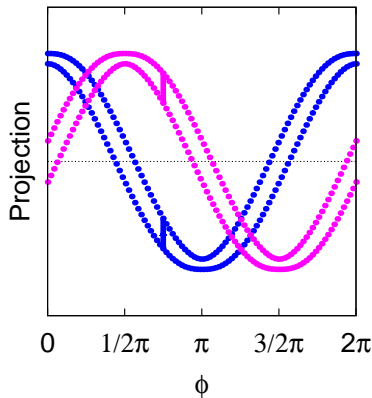
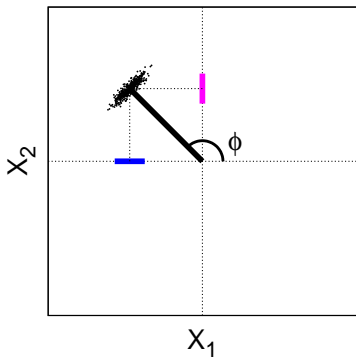
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



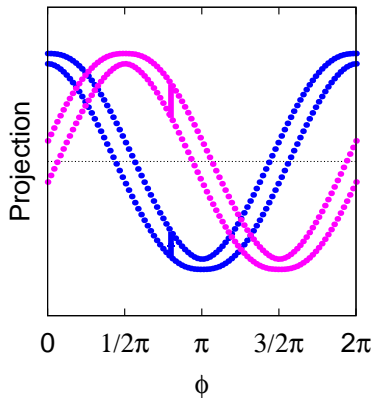
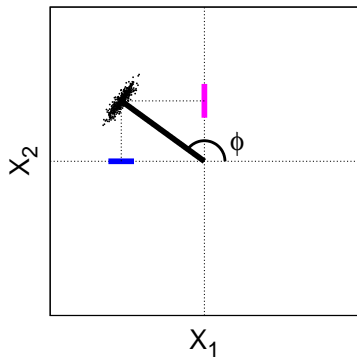
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



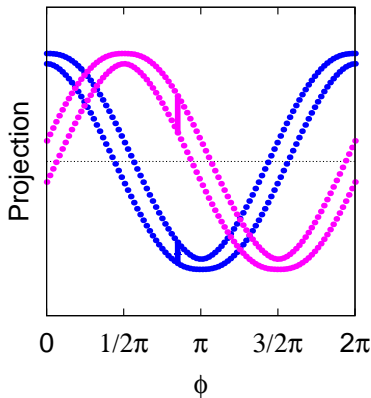
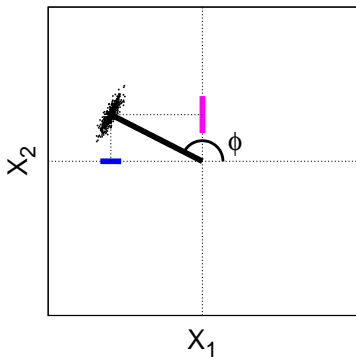
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



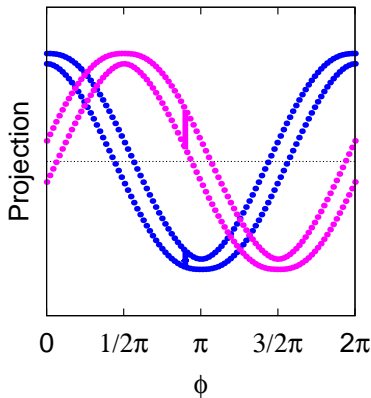
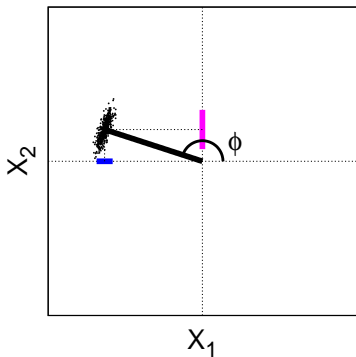
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



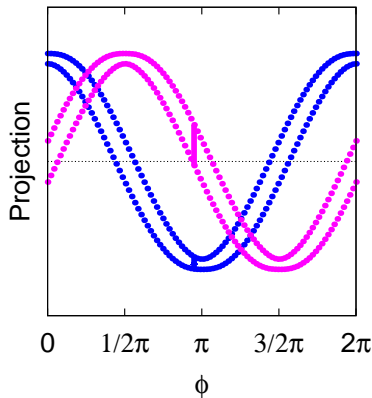
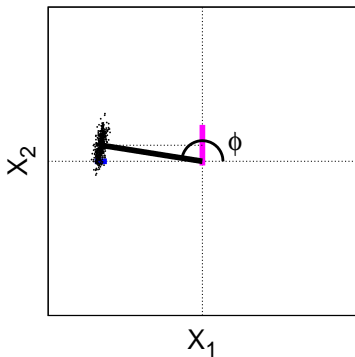
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



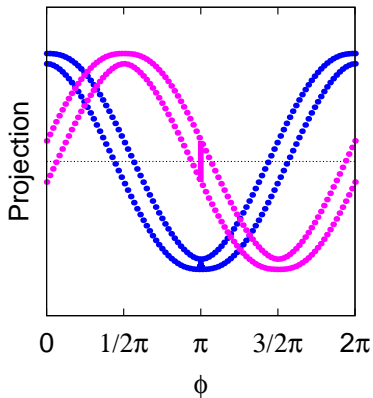
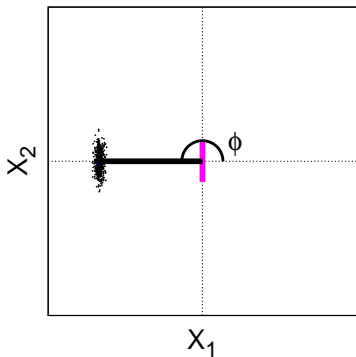
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



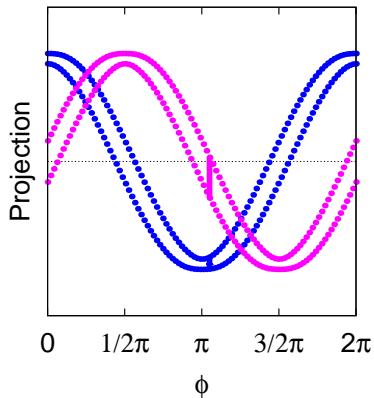
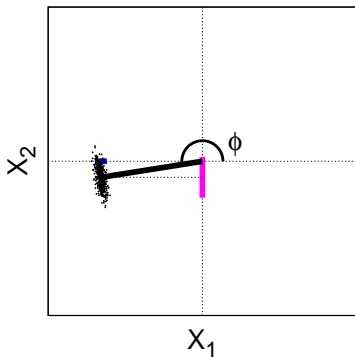
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



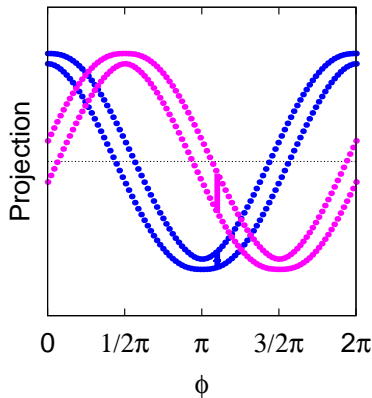
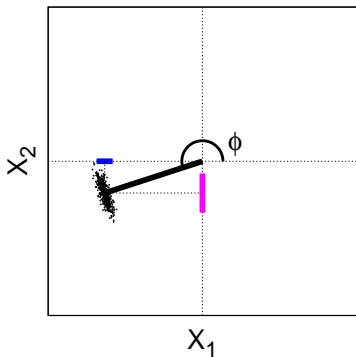
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



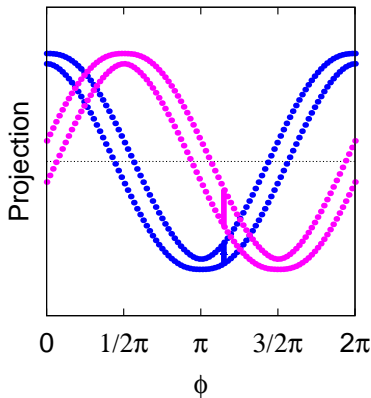
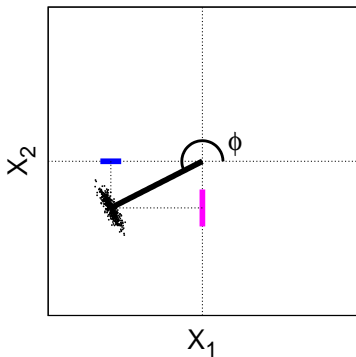
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



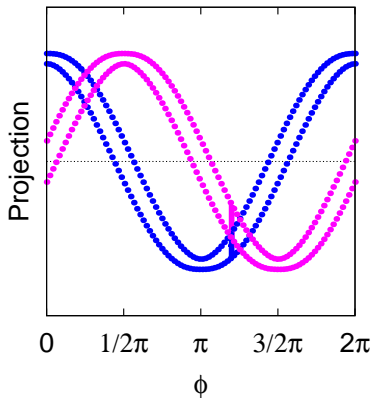
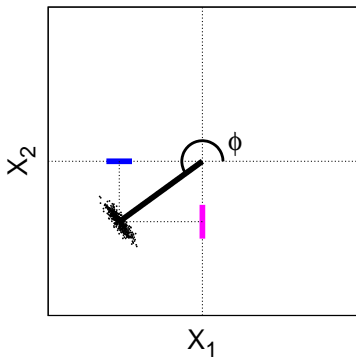
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



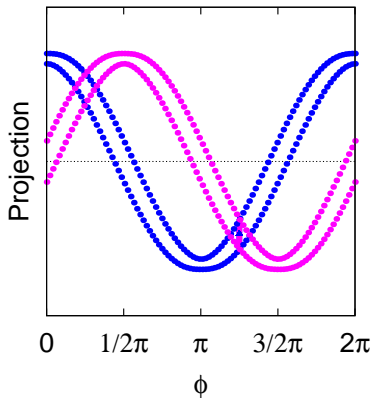
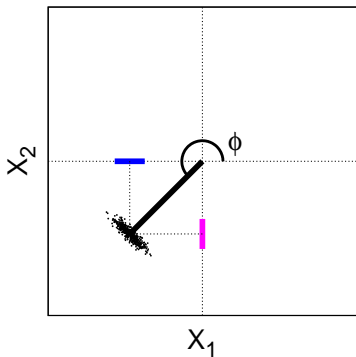
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



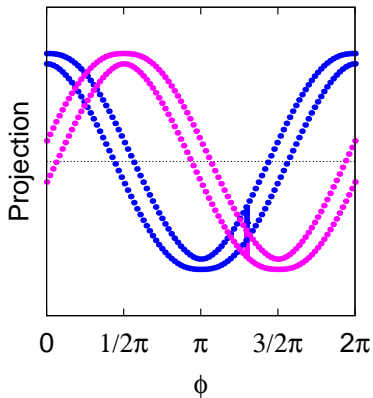
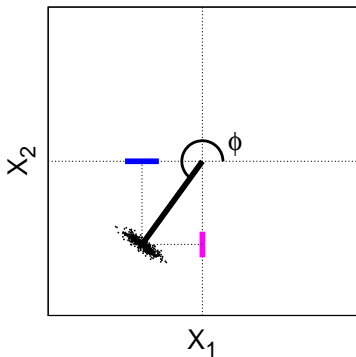
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



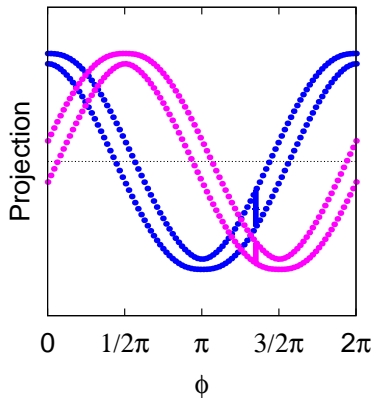
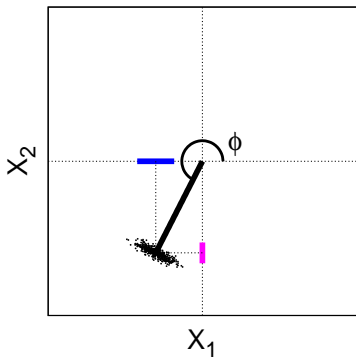
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



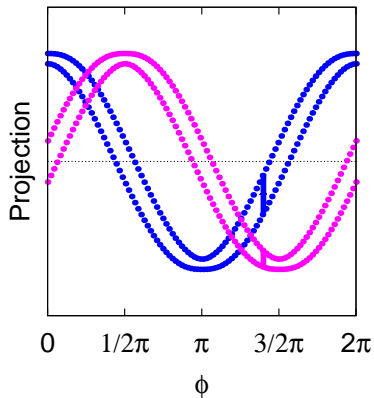
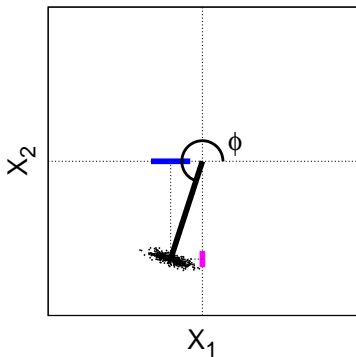
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



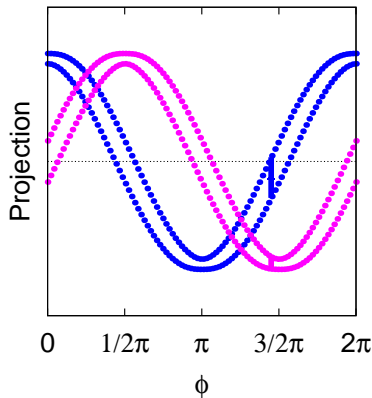
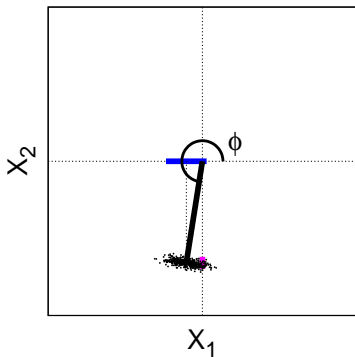
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



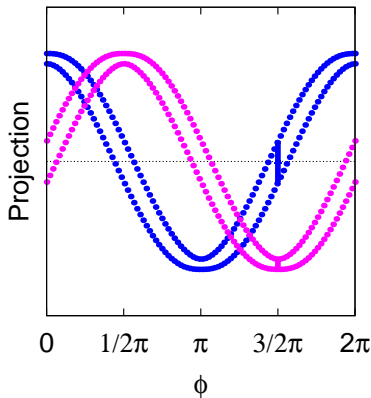
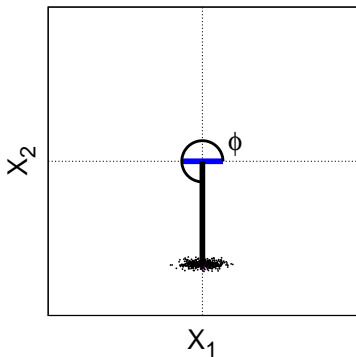
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



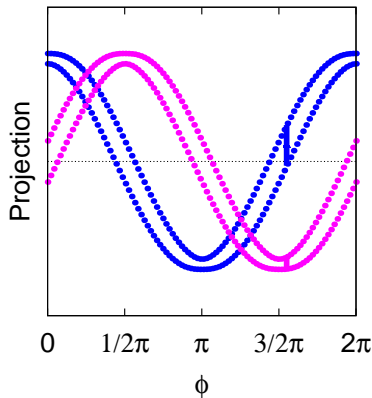
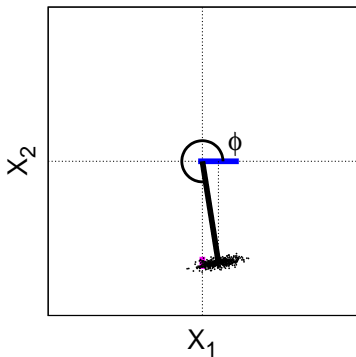
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



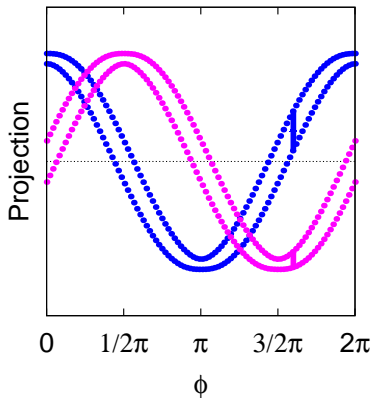
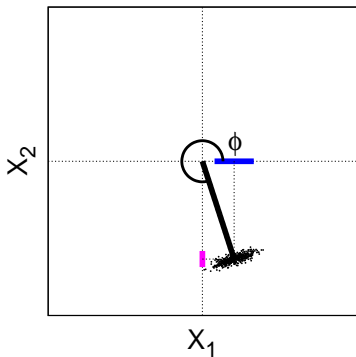
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



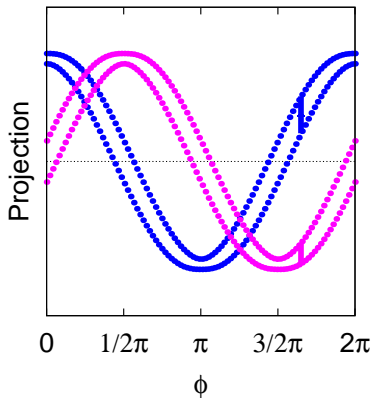
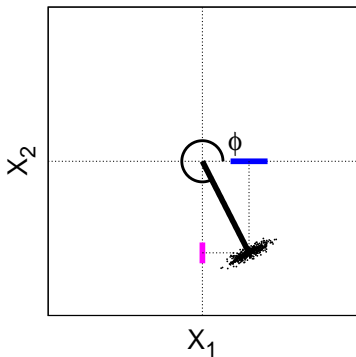
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



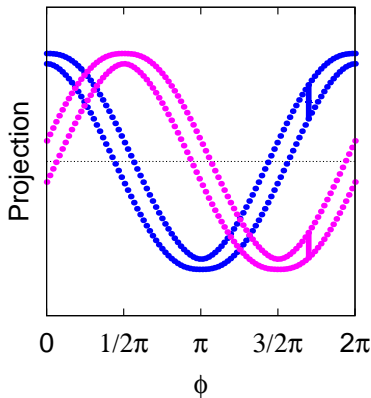
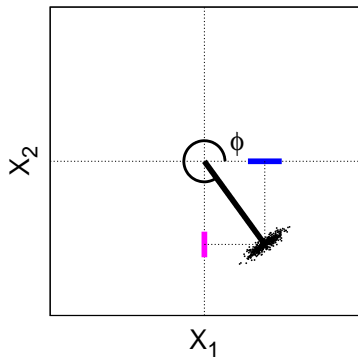
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



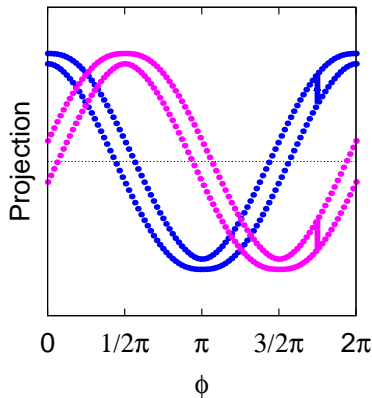
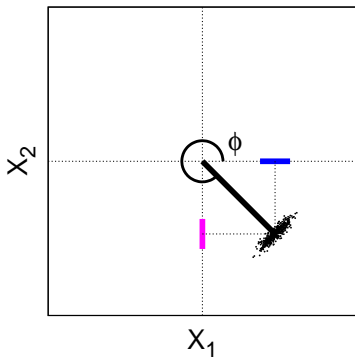
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



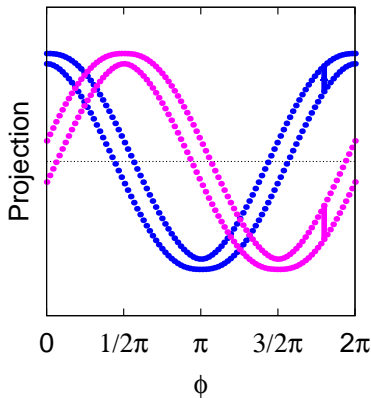
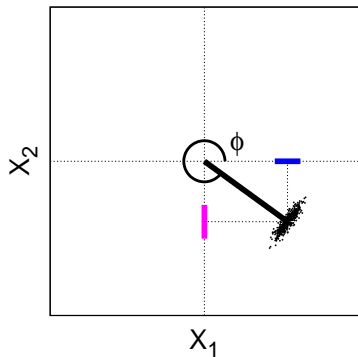
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



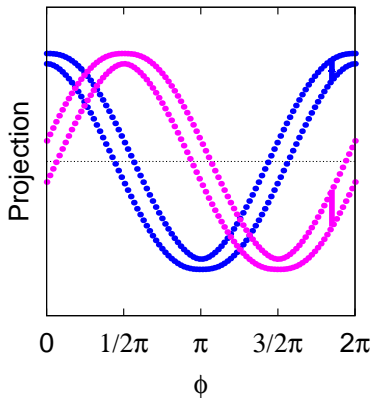
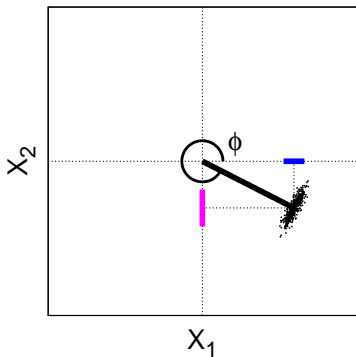
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



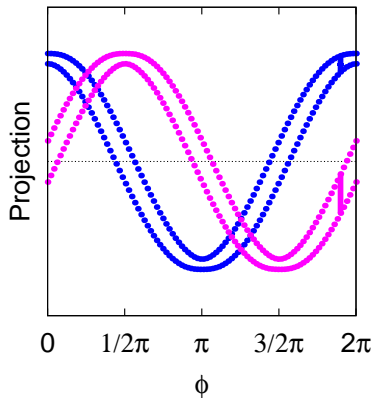
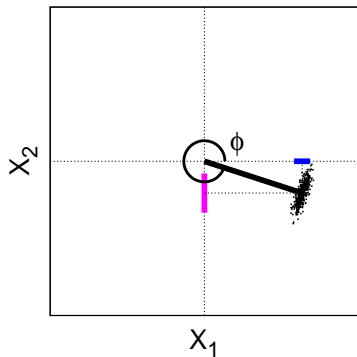
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



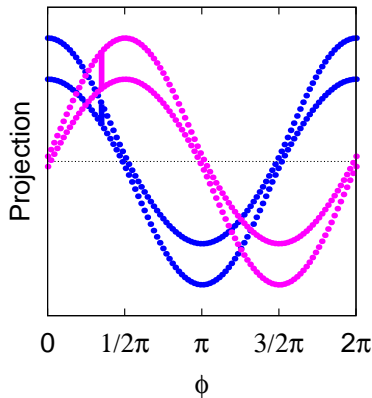
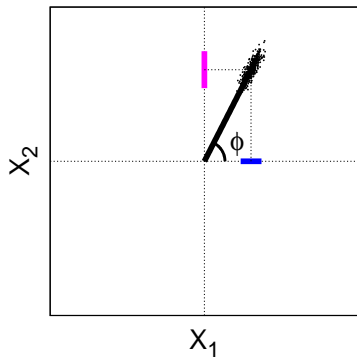
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



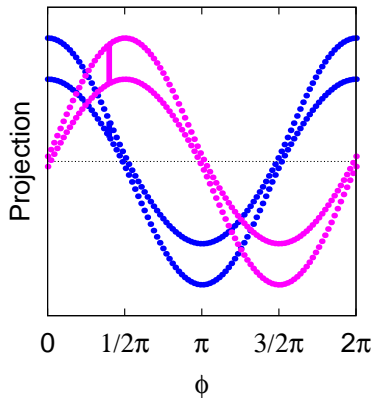
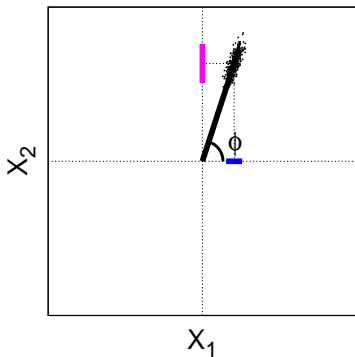
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



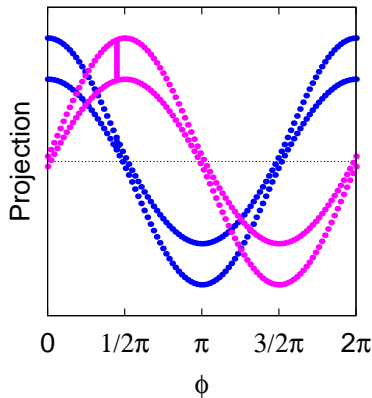
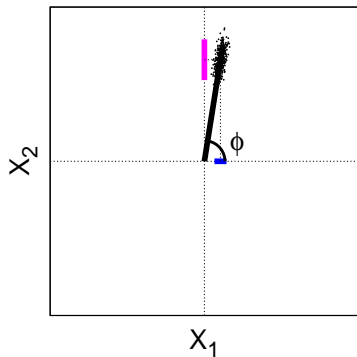
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



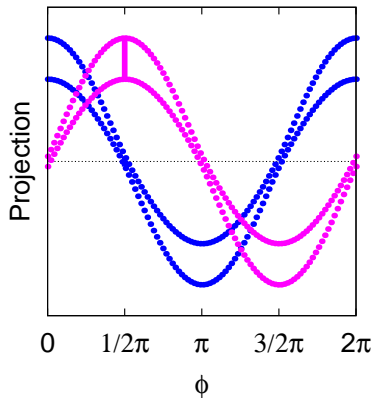
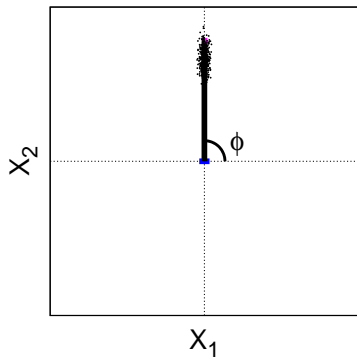
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



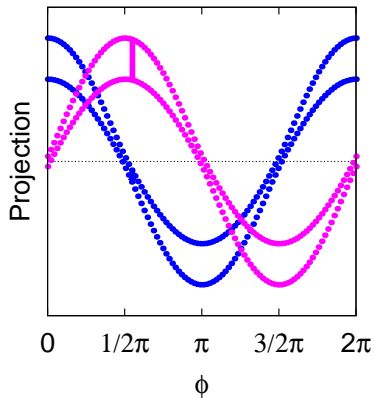
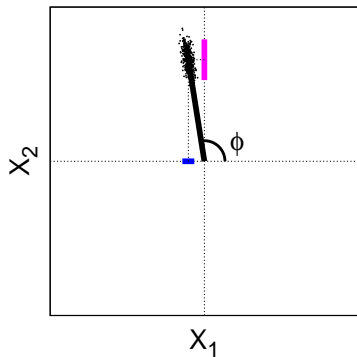
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



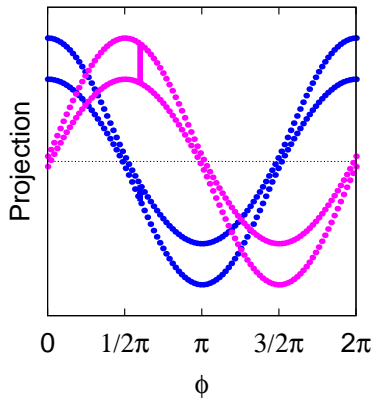
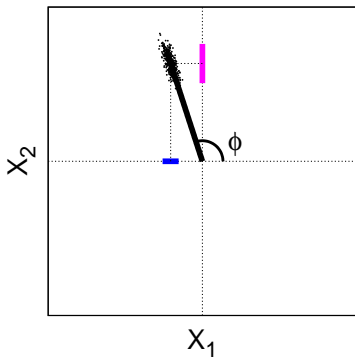
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



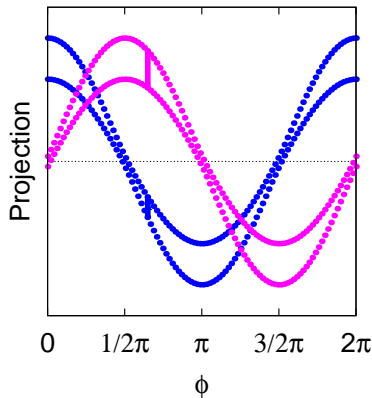
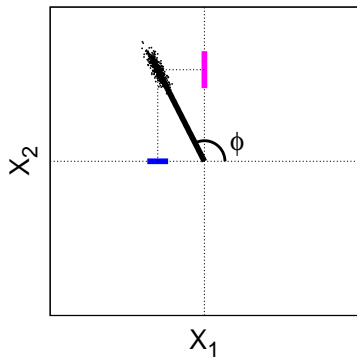
Phase squeezed states

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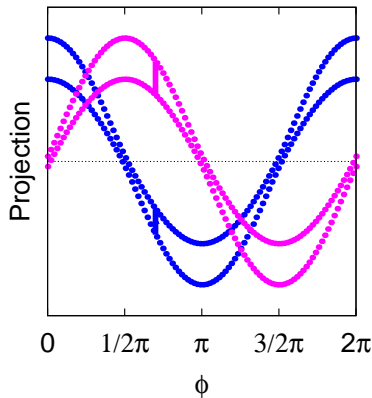
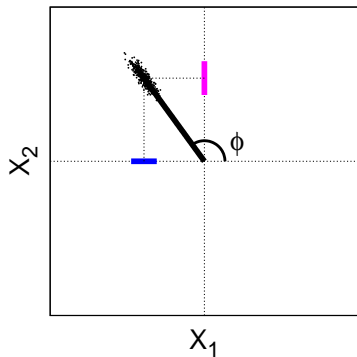
Phase squeezed states

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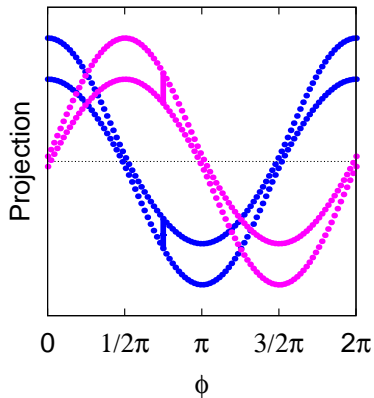
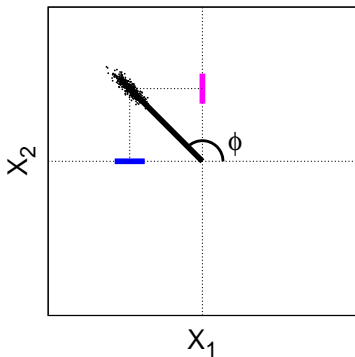
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



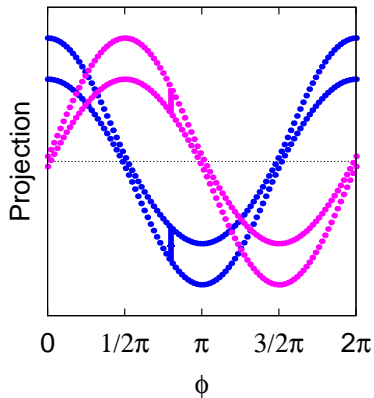
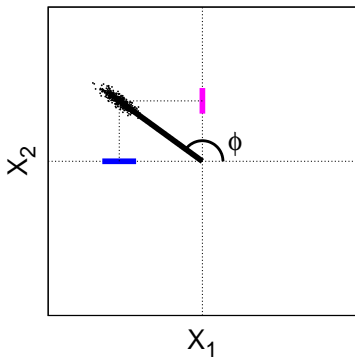
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



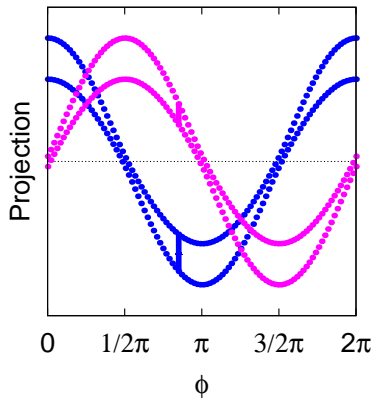
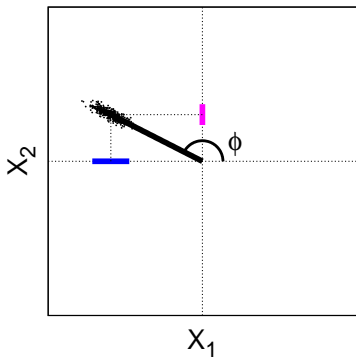
Phase squeezed states

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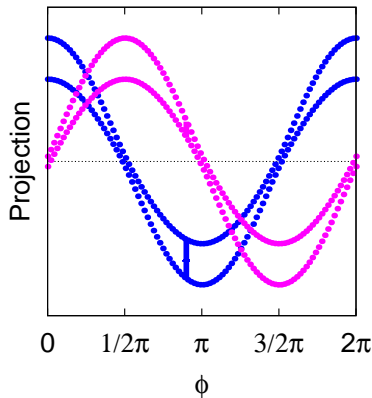
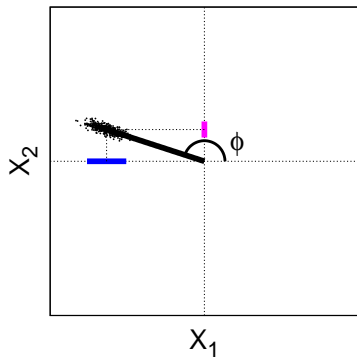
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



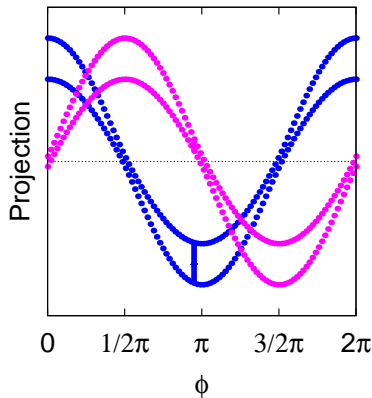
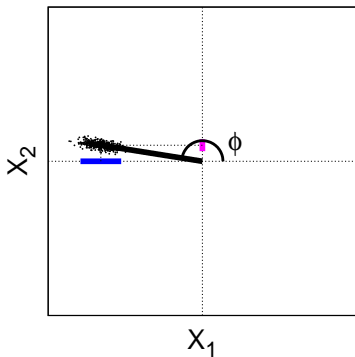
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



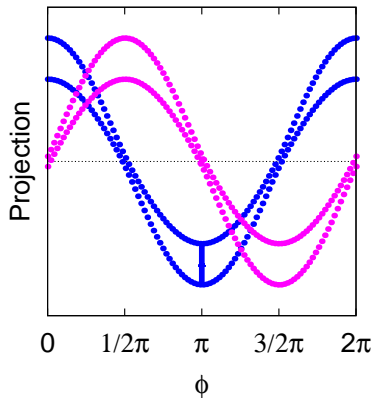
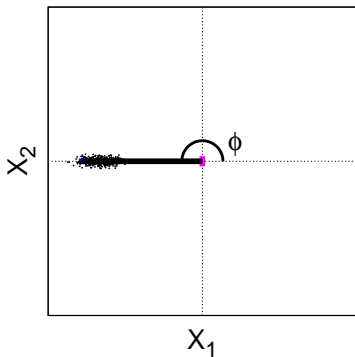
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



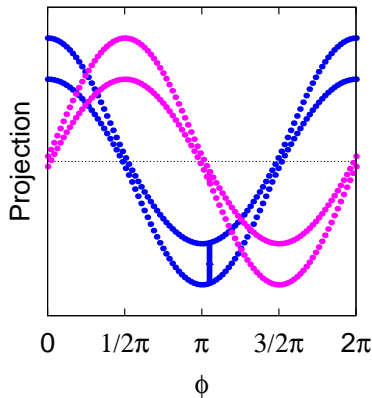
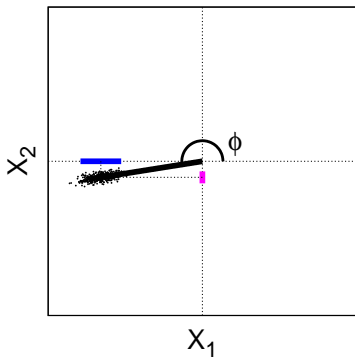
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



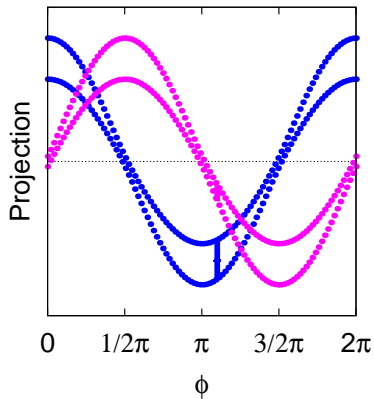
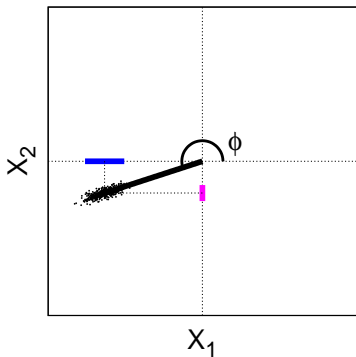
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



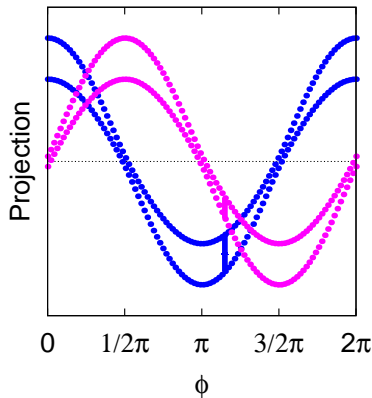
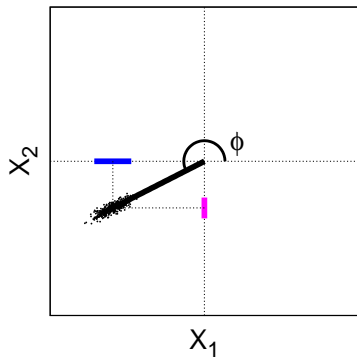
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



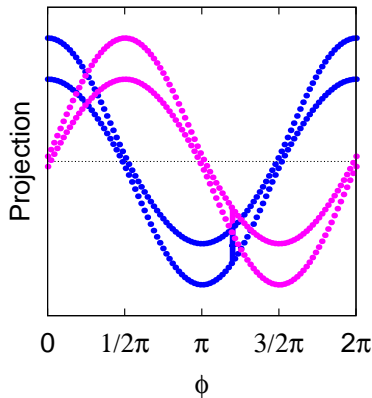
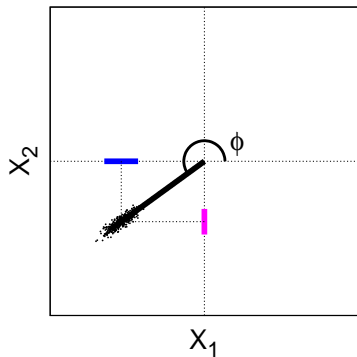
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



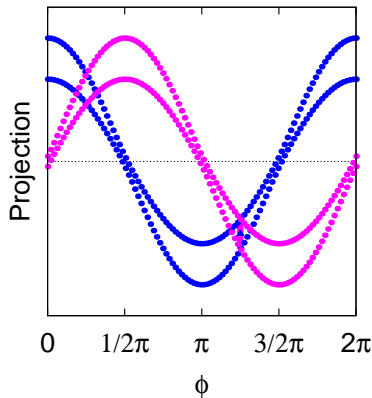
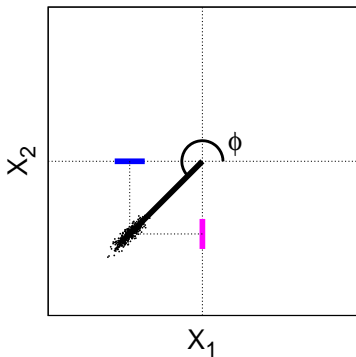
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



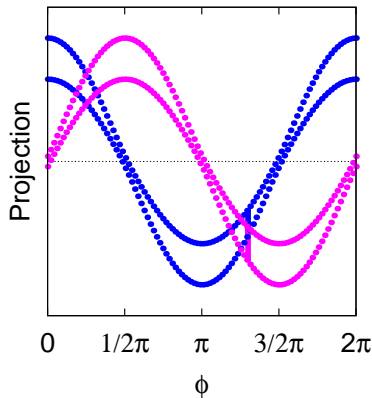
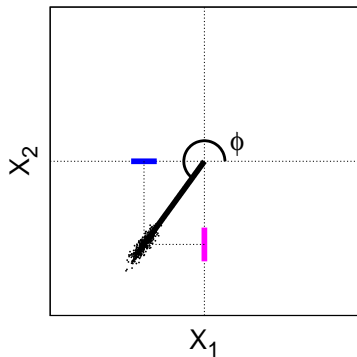
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



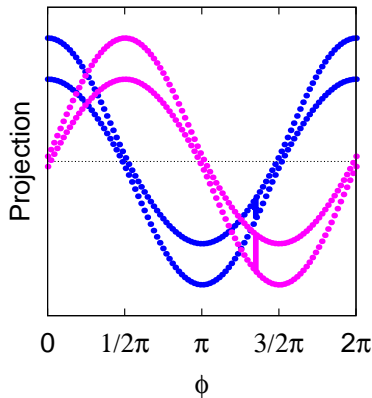
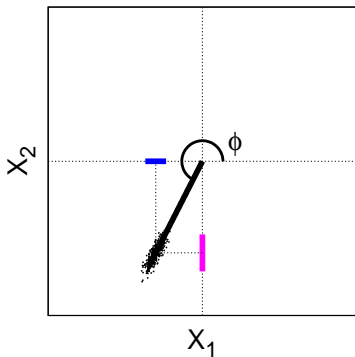
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



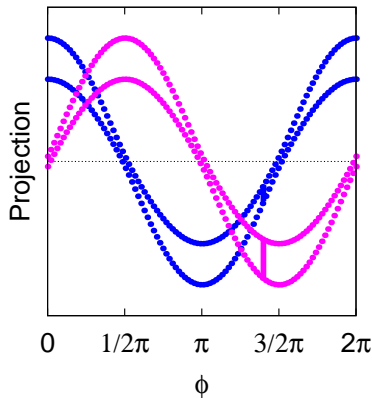
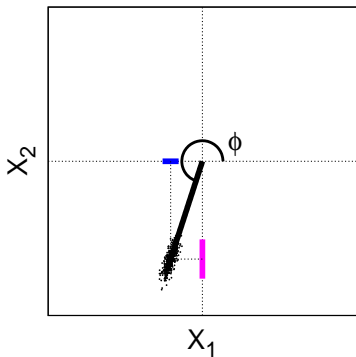
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



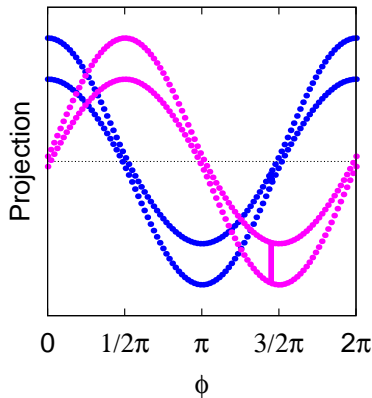
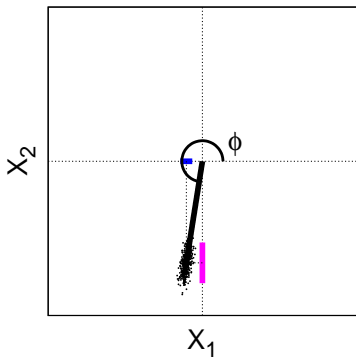
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



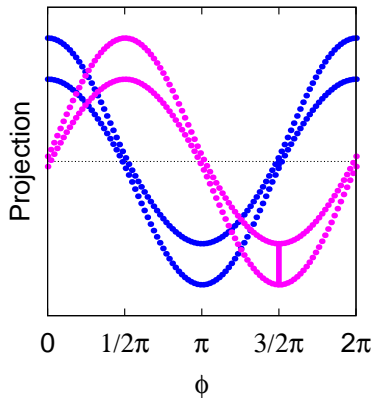
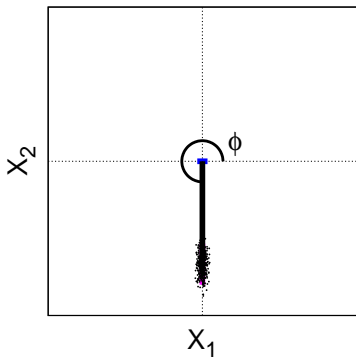
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



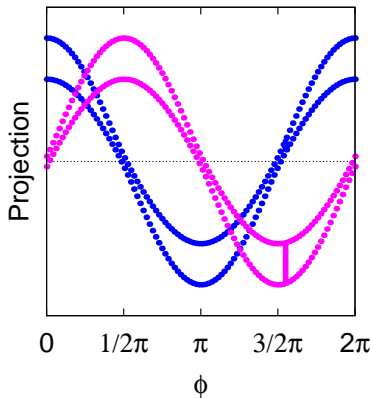
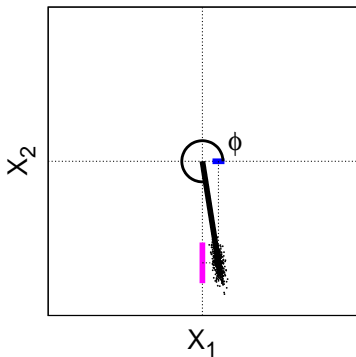
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



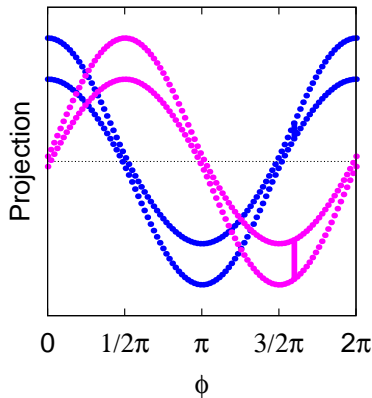
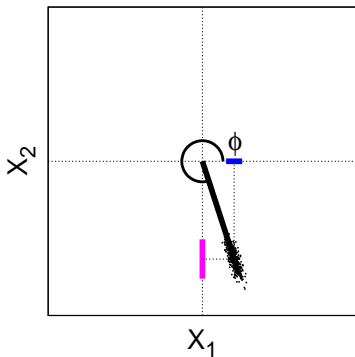
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



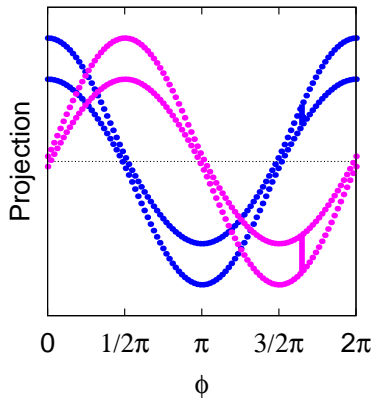
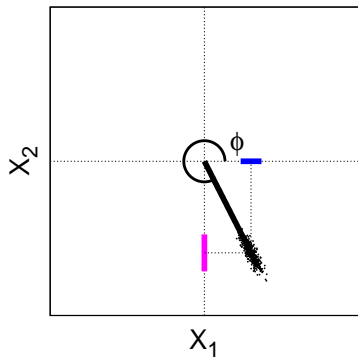
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



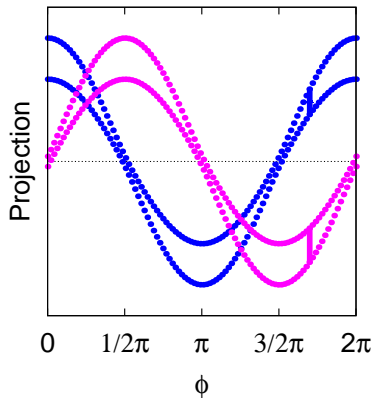
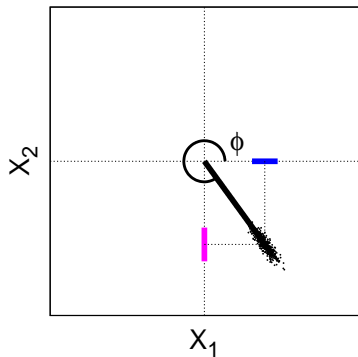
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



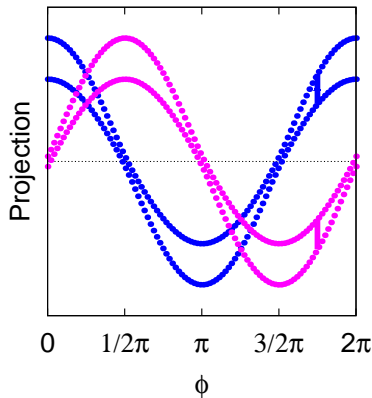
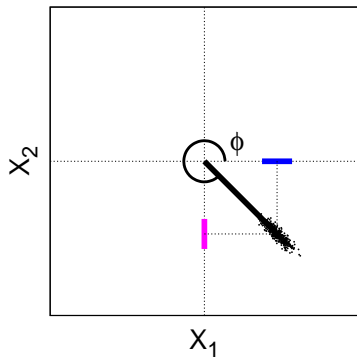
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



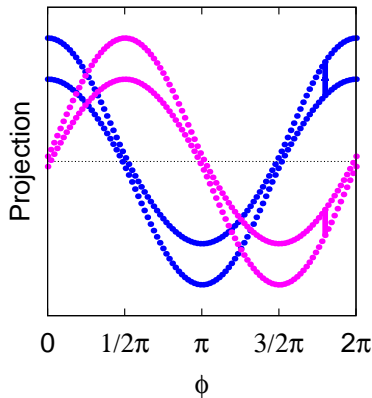
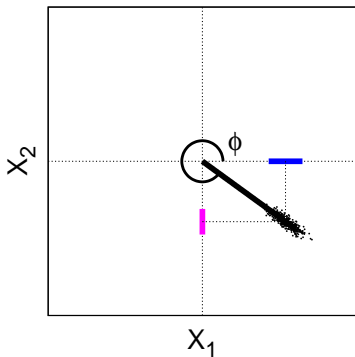
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



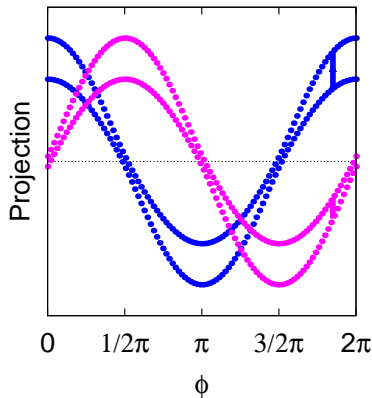
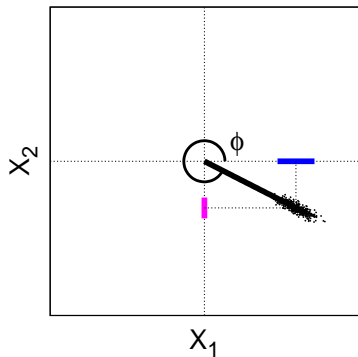
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



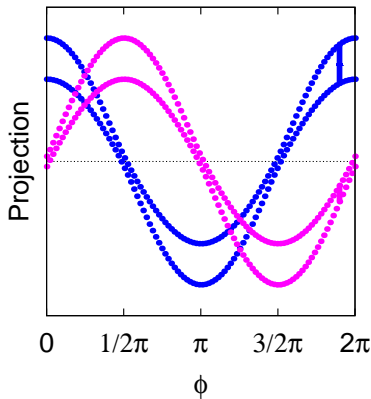
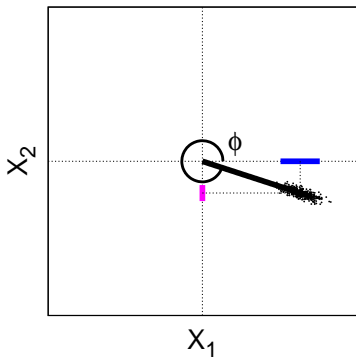
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$

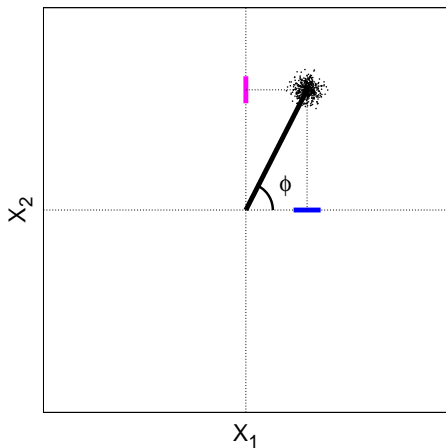


Phase squeezed states

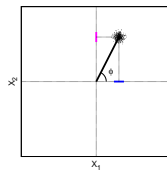
$$\Delta X_1 \Delta X_2 = 1/4$$



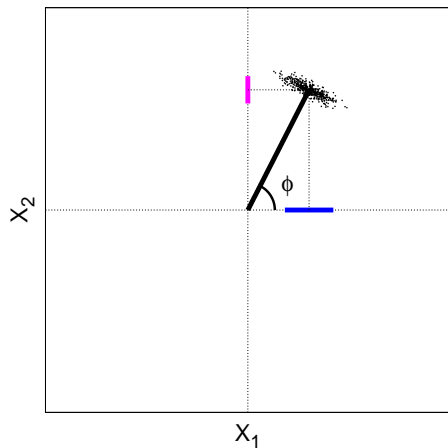
Squeezed quantum states zoo



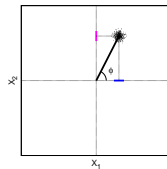
Unsqueezed
coherent



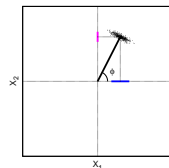
Squeezed quantum states zoo



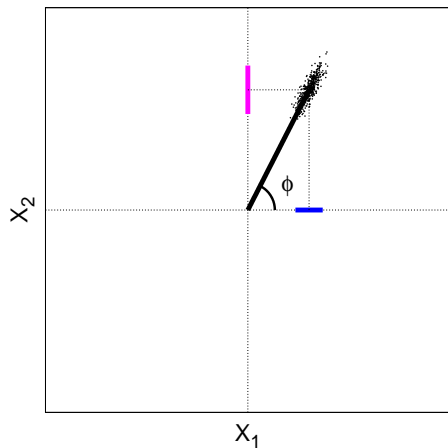
Unsqueezed
coherent



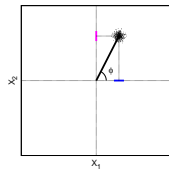
Amplitude
squeezed



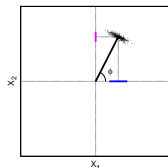
Squeezed quantum states zoo



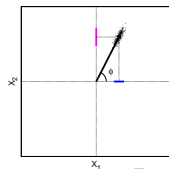
Unsqueezed
coherent



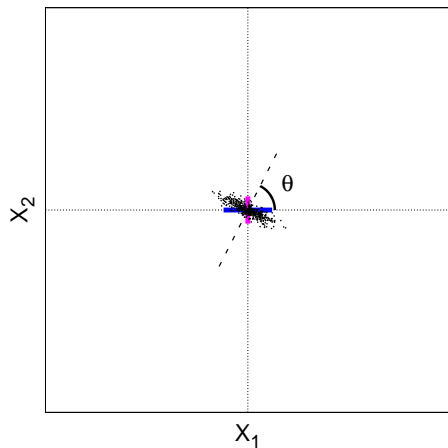
Amplitude
squeezed



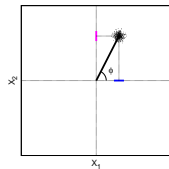
Phase
squeezed



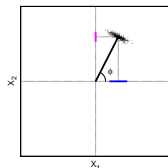
Squeezed quantum states zoo



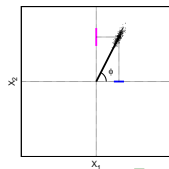
Unsqueezed
coherent



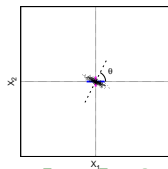
Amplitude
squeezed



Phase
squeezed

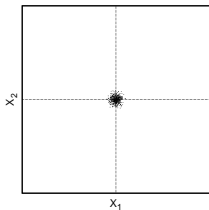


Vacuum
squeezed



Squeezed field generation recipe

Take a vacuum
state $|0\rangle$

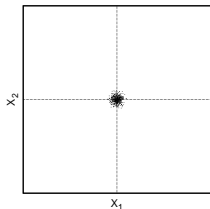


$$H = \frac{1}{2}$$

Squeezed field generation recipe

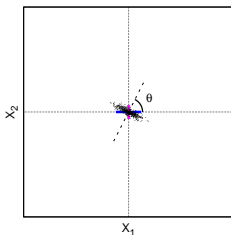
Take a vacuum state $|0\rangle$

Apply squeezing operator $|\xi\rangle = \hat{S}(\xi)|0\rangle$



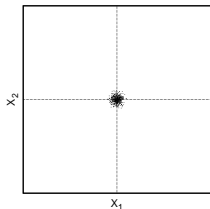
$$H = \frac{1}{2}$$

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$



Squeezed field generation recipe

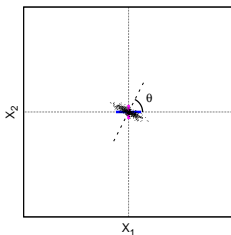
Take a vacuum state $|0\rangle$



$$H = \frac{1}{2}$$

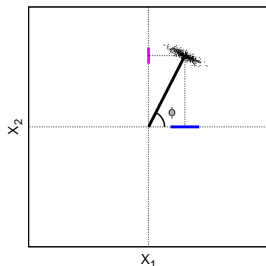
Apply squeezing operator $|\xi\rangle = \hat{S}(\xi)|0\rangle$

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$



Apply displacement operator $|\alpha, \xi\rangle = \hat{D}(\alpha)|s\rangle$

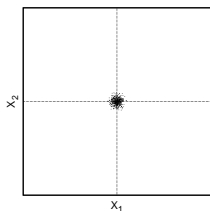
$$\hat{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$



$$\begin{aligned}\langle \alpha, \xi | X_1 | \alpha, \xi \rangle &= \text{Re}(\alpha), \\ \langle \alpha, \xi | X_2 | \alpha, \xi \rangle &= \text{Im}(\alpha)\end{aligned}$$

Squeezed field generation recipe

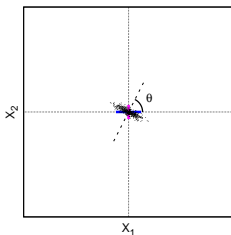
Take a vacuum state $|0\rangle$



$$H = \frac{1}{2}$$

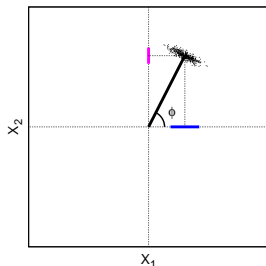
Apply squeezing operator $|\xi\rangle = \hat{S}(\xi)|0\rangle$

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$



Apply displacement operator $|\alpha, \xi\rangle = \hat{D}(\alpha)|\xi\rangle$

$$\hat{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$

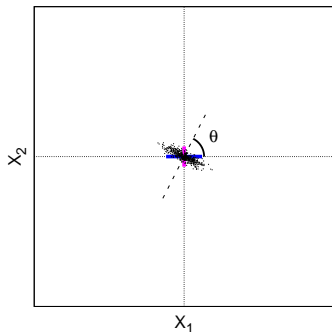


$$\langle \alpha, \xi | X_1 | \alpha, \xi \rangle = \text{Re}(\alpha),$$

$$\langle \alpha, \xi | X_2 | \alpha, \xi \rangle = \text{Im}(\alpha)$$

Notice $\Delta X_1 \Delta X_2 = \frac{1}{4}$

Squeezed state $|\xi\rangle = \hat{S}(\xi)|0\rangle$ properties



$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}, \xi = r e^{i\theta}$$

If $\theta = 0$

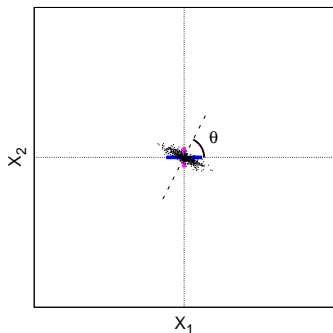
$$\langle \xi | (\Delta X_1)^2 | \xi \rangle = \frac{1}{4} e^{-2r}$$

$$\langle \xi | (\Delta X_2)^2 | \xi \rangle = \frac{1}{4} e^{2r}$$

$$\langle \xi | (\Delta X_1)^2 | \xi \rangle = \frac{1}{4} (\cosh^2 r + \sinh^2 r - 2 \sinh r \cosh r \cos \theta)$$

$$\langle \xi | (\Delta X_2)^2 | \xi \rangle = \frac{1}{4} (\cosh^2 r + \sinh^2 r + 2 \sinh r \cosh r \cos \theta)$$

Photon number of squeezed state $|\xi\rangle$



Probability to detect given number of photons $C = \langle n | \xi \rangle$ for squeezed vacuum

- even

$$C_{2m} = (-1)^m \frac{\sqrt{(2m)!}}{2^m m!} \frac{(e^{i\theta} \tanh r)^m}{\sqrt{\cosh r}}$$

- odd

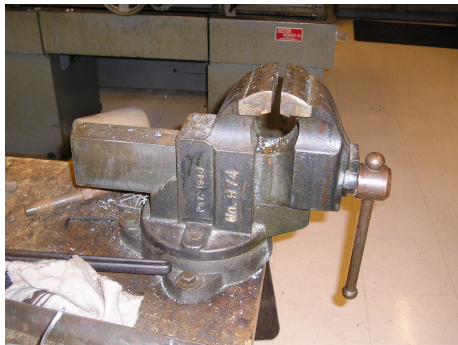
$$C_{2m+1} = 0$$

Average number of photons in general squeezed state

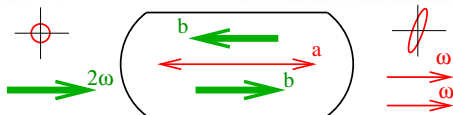
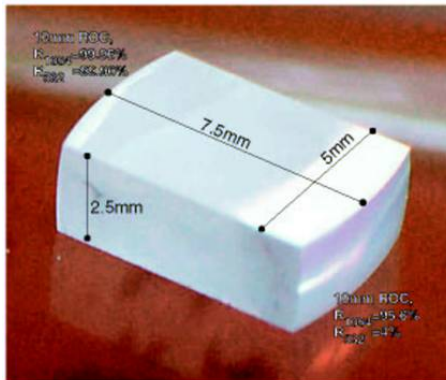
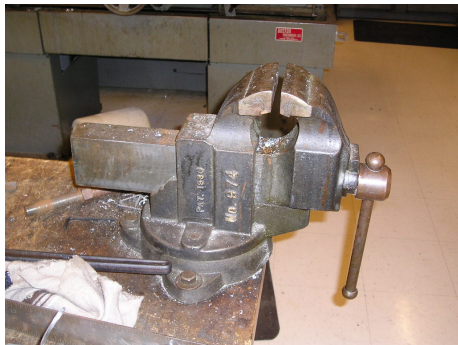
$$\langle \alpha, \xi | a^\dagger a | \alpha, \xi \rangle = \alpha + \sinh^2 r$$

Tools for squeezing

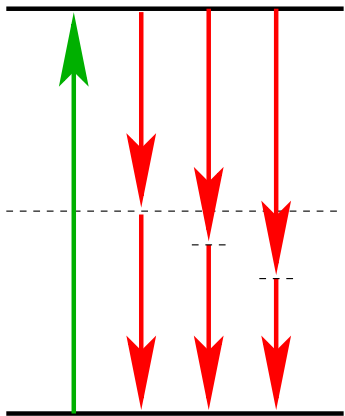
Tools for squeezing



Tools for squeezing



Two photon squeezing picture

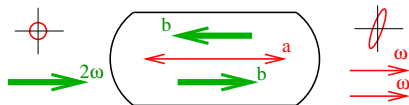


Squeezing operator

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$

Parametric down-conversion in crystal

$$\hat{H} = i\hbar\chi^{(2)}(a^2 b^\dagger - a^{\dagger 2} b)$$

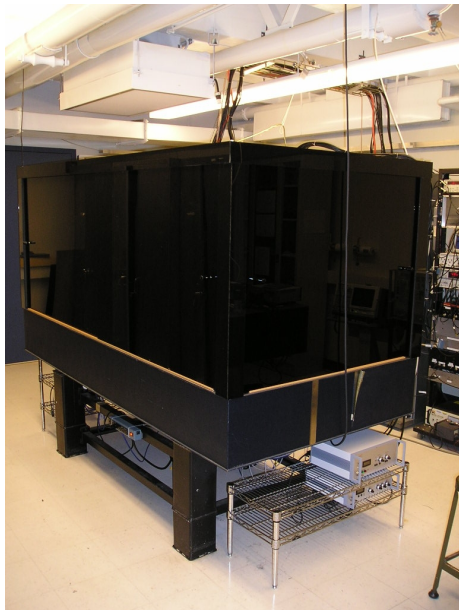


Squeezing

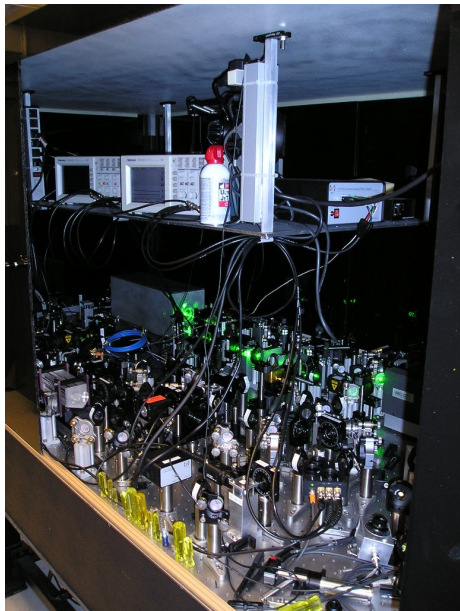
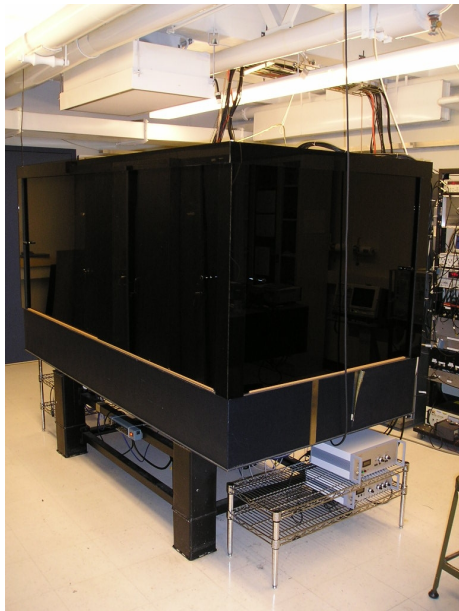
result of correlation of upper and lower sidebands

Squeezer appearance

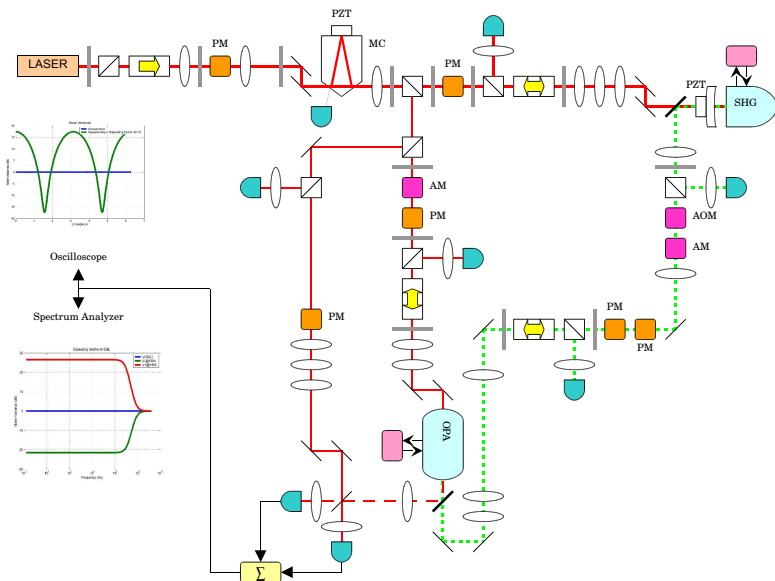
Squeezer appearance



Squeezer appearance



Crystal squeezing setup scheme



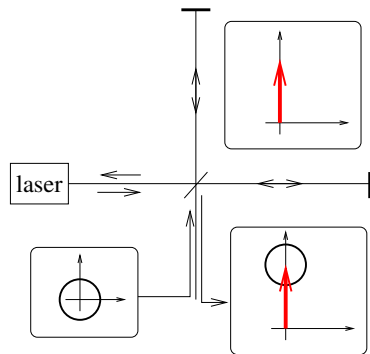
Possible squeezing applications

- improvements any shot noise limited optical sensors
- noiseless signal amplification
- secure communications (you would notice eavesdropper)
- photon pair generation, entanglement, true single photon sources
- interferometers sensitivity boost (for example gravitational wave antennas)
- light free measurements
- quantum memory probe and information carrier

Squeezing and interferometer

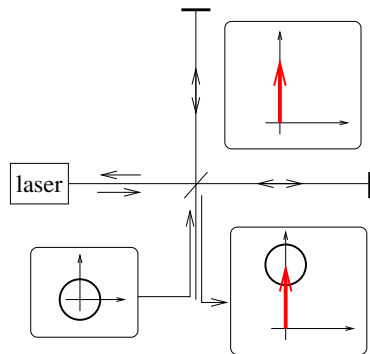
Squeezing and interferometer

Vacuum input

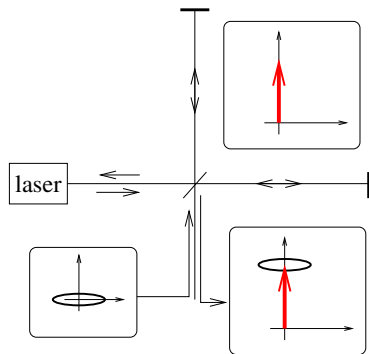


Squeezing and interferometer

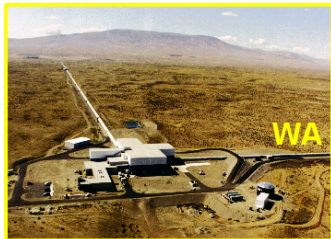
Vacuum input



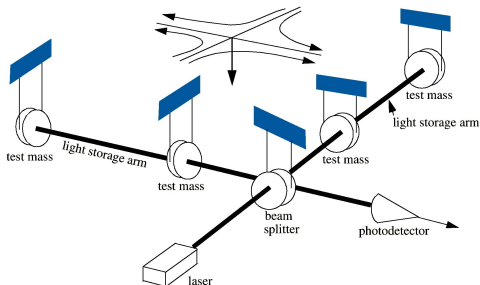
Squeezed input



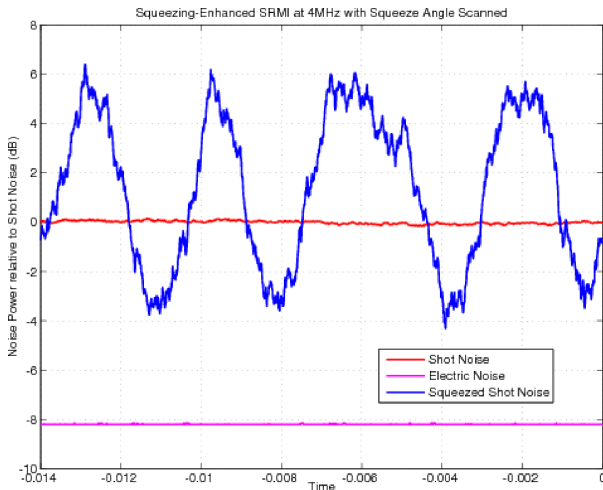
Laser Interferometer Gravitational-wave Observatory



- $L = 4 \text{ km}$
- $h \sim 10^{-21}$
- $\Delta L \sim 10^{-18} \text{ m}$
- $\Delta \phi \sim 10^{-10} \text{ rad}$

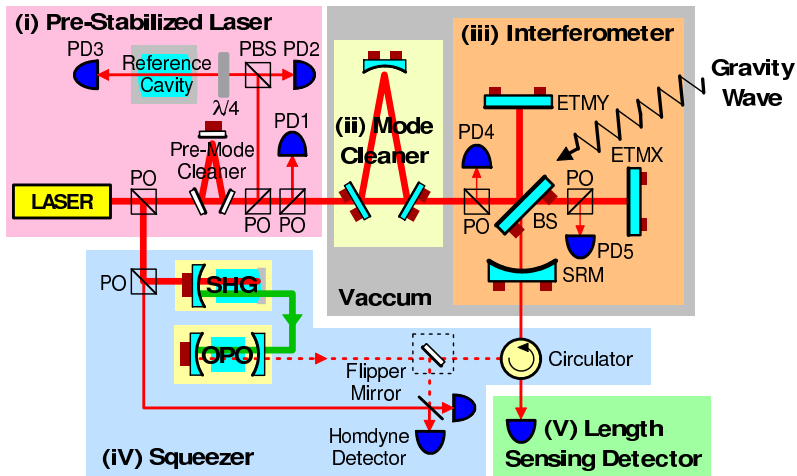


Squeezing level vs time (unlocked)

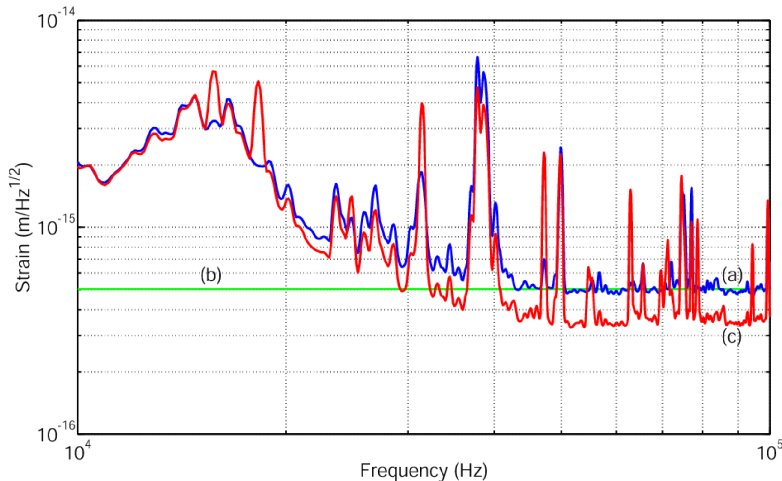


“A quantum-enhanced prototype gravitational-wave detector”,
Nature Physics, **4**, 472-476, (2008).

GW 40m detector and squeezer

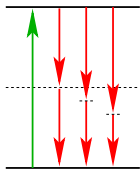


GW 40m detector with 4dB of squeezed vacuum

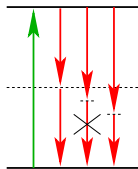
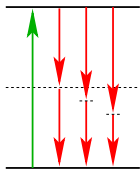


Signal to noise improvement by factor of 1.43

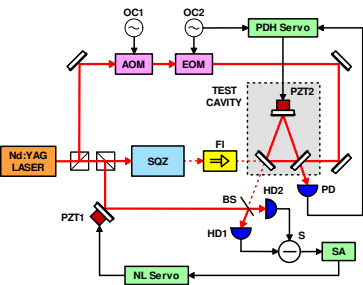
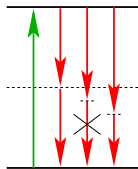
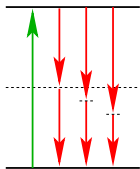
Cavity parameters with squeezing



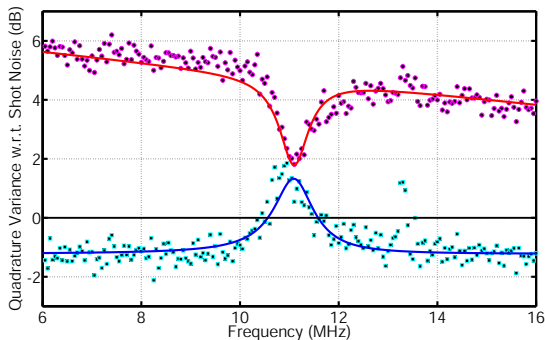
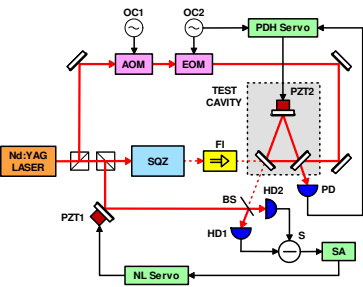
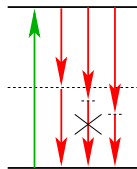
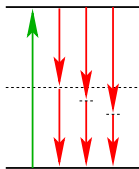
Cavity parameters with squeezing



Cavity parameters with squeezing

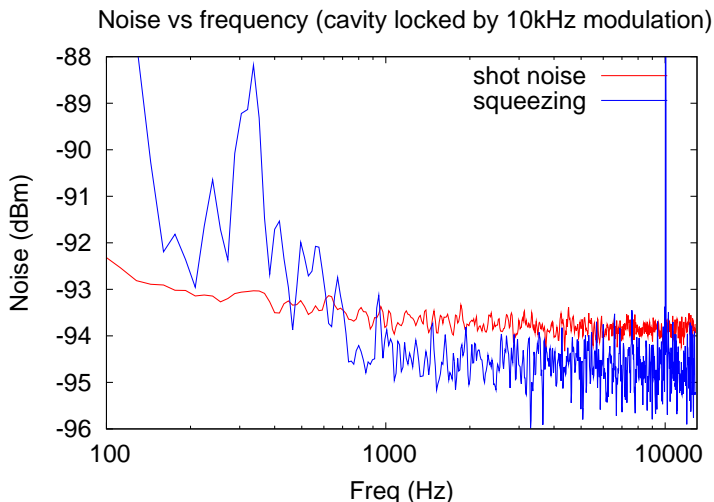


Cavity parameters with squeezing



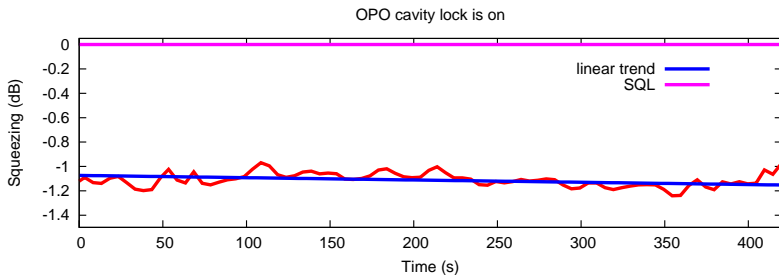
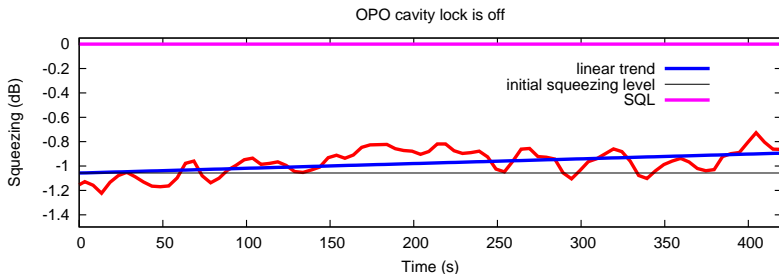
“Noninvasive measurements of cavity parameters by use of squeezed vacuum”, *Physical Review A*, **74**, 033817, (2006).

Low frequency squeezing with light free noise lock



“Quantum noise locking”,
J. Opt. B: Quantum Semiclass. Opt., 7, S421, (2005).

Squeezing level vs time (homodyne angle lock is on)



Summary for crystal squeezing

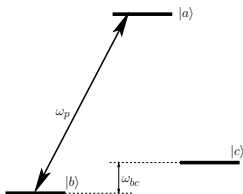
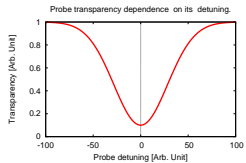
Pros

- mainstream: many different nonlinear crystals available
- so far the best squeezers
 - maximum squeezing value detected **11.5 dB at 1064 nm**
 - Moritz Mehmet, Henning Vahlbruch, Nico Lastzka, Karsten Danzmann, and Roman Schnabel, "Observation of squeezed states with strong photon-number oscillations", Phys. Rev. A **81**, 013814 (2010)
- well understood

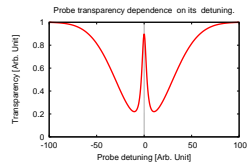
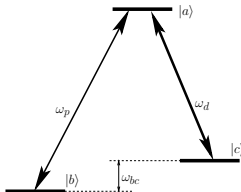
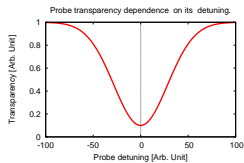
Cons

- crystals have limited transparency window
- thus squeezing is hard to generate at visible wavelength
 - at 795 nm only 4-6 dB squeezing is reported
- this limits applications of such squeezers for spectroscopy

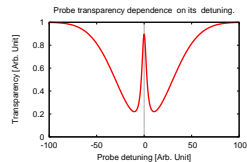
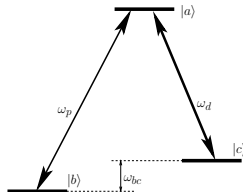
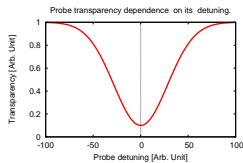
Quantum memory with atomic ensembles



Quantum memory with atomic ensembles

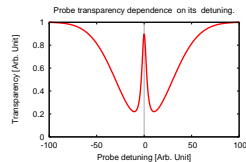
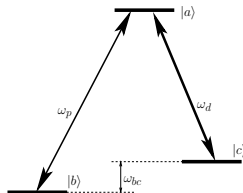
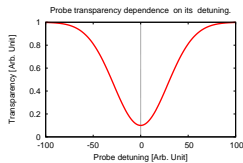


Quantum memory with atomic ensembles



Storage and retrieval

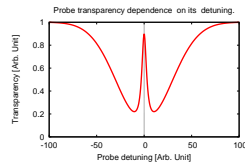
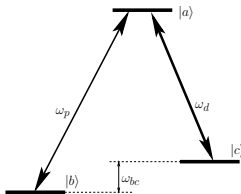
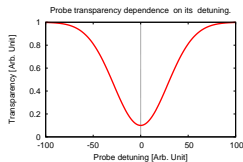
Quantum memory with atomic ensembles



Storage and retrieval

- single photon

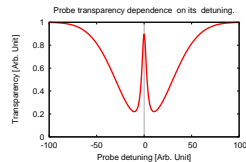
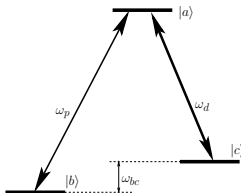
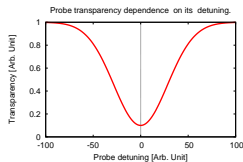
Quantum memory with atomic ensembles



Storage and retrieval

- single photon
- squeezed state (Furusawa and Lvovsky PRL **100** 2008)

Quantum memory with atomic ensembles

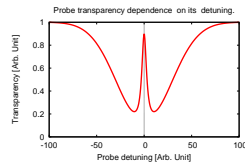
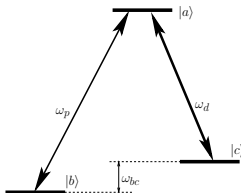
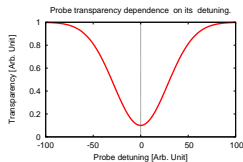


Storage and retrieval

- single photon
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Squeezed state requirements for a quantum memory probe

Quantum memory with atomic ensembles



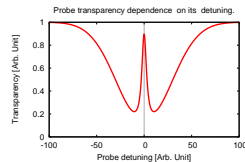
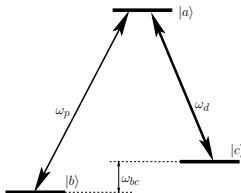
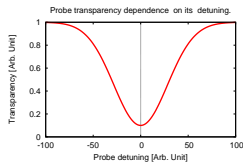
Storage and retrieval

- single photon
- squeezed state (Furusawa and Lvovsky PRL **100** 2008)

Squeezed state requirements for a quantum memory probe

- squeezing carrier at atomic wavelength (780nm, 795nm)
- squeezing within narrow resonance window at frequencies ($< 100\text{kHz}$)

Quantum memory with atomic ensembles



Storage and retrieval

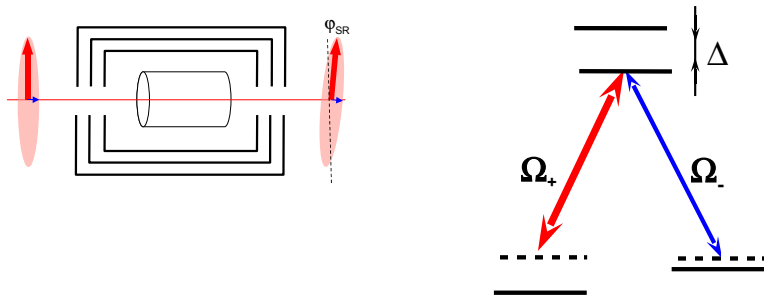
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Squeezed state requirements for a quantum memory probe

- squeezing carrier at atomic wavelength (780nm, 795nm)
- squeezing within narrow resonance window at frequencies ($< 100\text{kHz}$)

Traditional nonlinear crystal based squeezers are capable of it, but they are **extremely technically challenging** especially at short wave length.

Self-rotation of elliptical polarization in atomic medium



A.B. Matsko et al., PRA 66, 043815 (2002): theoretically prediction of 4-6 dB noise suppression

$$a_{out} = a_{in} + \frac{igL}{2}(a_{in}^\dagger - a_{in}) \quad (2)$$

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- **Yes!** J. Ries, B. Brezger, and A. I. Lvovsky, Experimental vacuum squeezing in rubidium vapor via self-rotation, PRA **68**, 025801 (2003).
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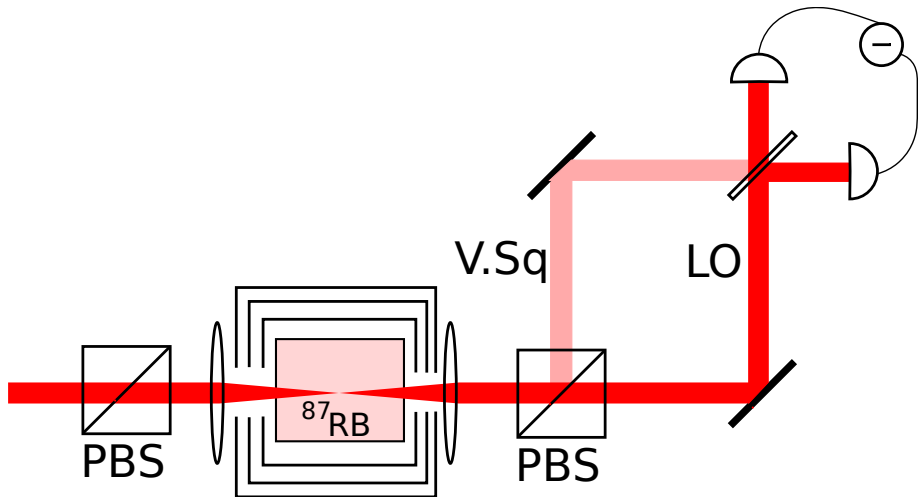
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- **Definitely** Eugeny E. Mikhailov et al. Optics Letters, Issue 11, **33**, 1213-1215, (2008).
- **Definitely** Eugeny E. Mikhailov et al. JMO , Issues 18&19, **56**, 1985-1992, (2009).
- **Definitely** Philippe Grangier et al. Optics Express, **18**, Issue 5, pp. 4198-4205 (2010)
 - 1.4 dB of squeezing

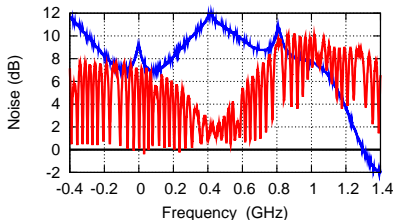
Setup



Noise contrast vs detuning in hot ^{87}Rb vacuum cell

$$F_g = 2 \rightarrow F_e = 1, 2$$

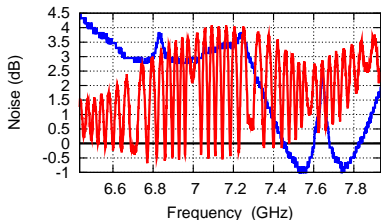
Noise vs detuning



Transmission — PSR noise

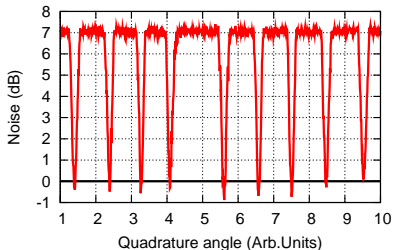
$$F_g = 1 \rightarrow F_e = 1, 2$$

Noise vs detuning

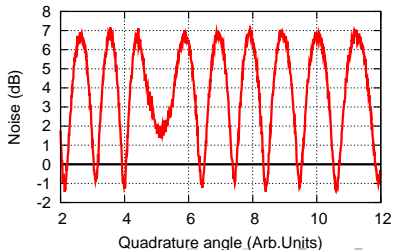


Transmission — PSR noise

Noise vs quadrature angle

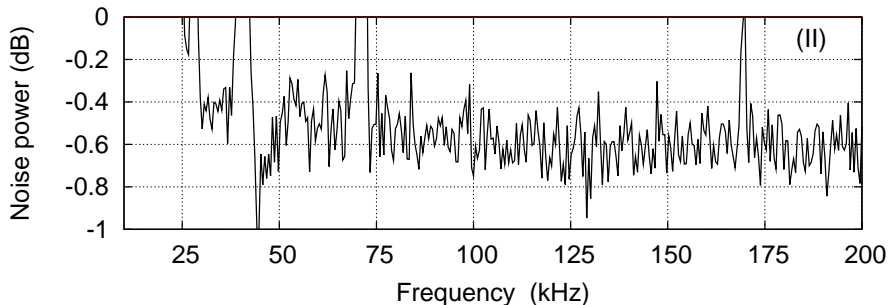


Noise vs quadrature angle



Low frequency squeezing vs power in ^{87}Rb at 795 nm

^{87}Rb cell + 2.5Torr Ne, $T=63.3^\circ\text{C}$ $P=1.5$ mW

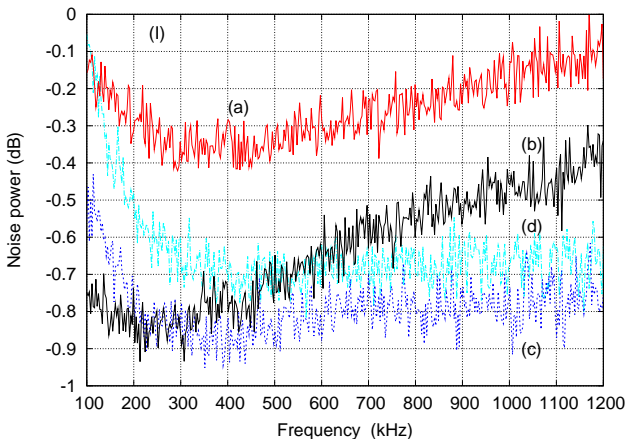


Eugeniy E. Mikhailov, Irina Novikova: Optics Letters, Issue 11, 33, 1213-1215, (2008).

Low frequency squeezing vs detuning in ^{87}Rb at 795 nm

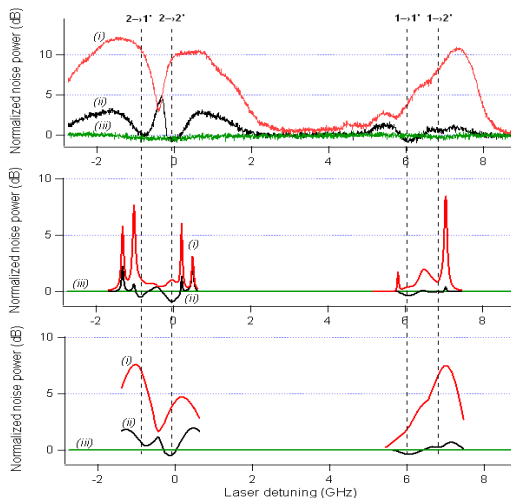
^{87}Rb cell + 2.5Torr Ne, $T=63.3^\circ\text{C}$

(a) $P=1.0$ mW, (b) $P=1.5$ mW, (c) $P=4.2$ mW, (d) $P=6.6$ mW



Squeezing theory and experiment

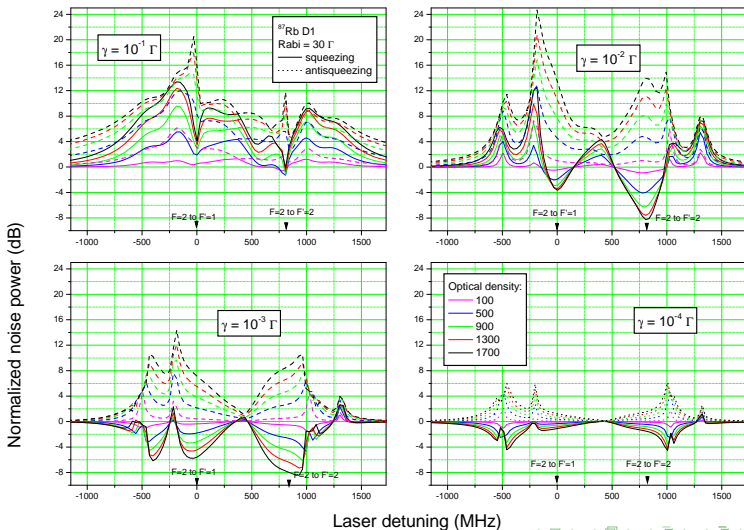
- ^{87}Rb cell
- no buffer gas
- density $2 \cdot 10^{11} \text{ cm}^{-3}$
- laser power 6 mW
- beam size 0.2 mm



E.E. Mikhailov, A. Lezama, T. Noel and I. Novikova,
J. Mod. Opt. **56**, 1985 (2009).

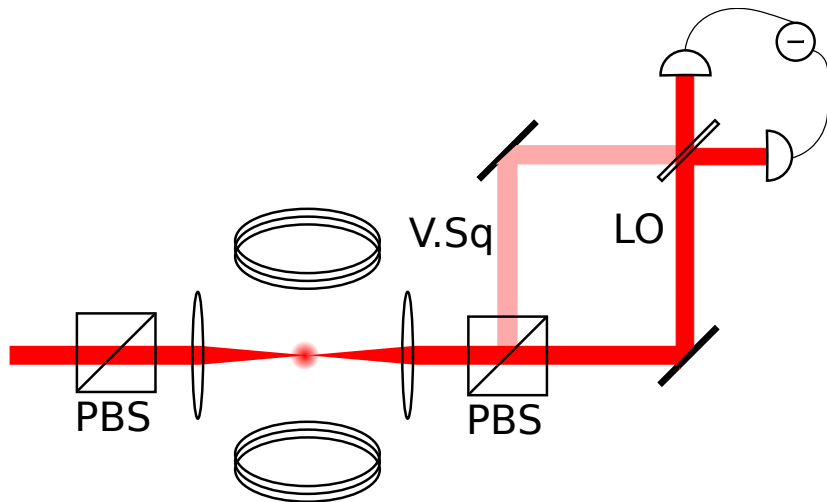
Theoretical prediction for MOT squeezing with ^{87}Rb

$F_g = 2 \rightarrow F_e = 1, 2$ high optical density is very important

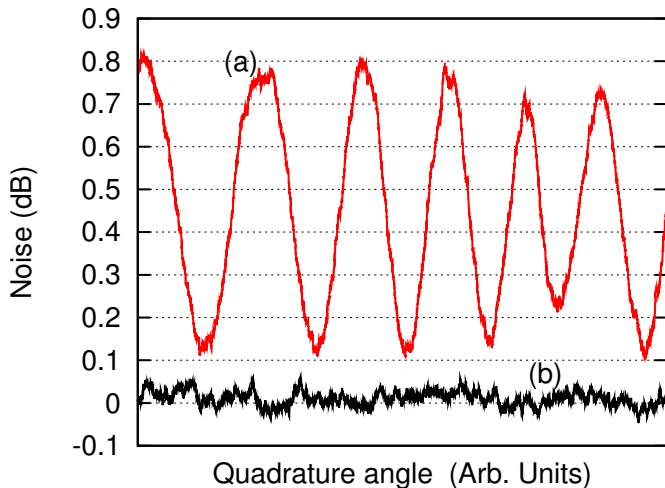


MOT squeezer

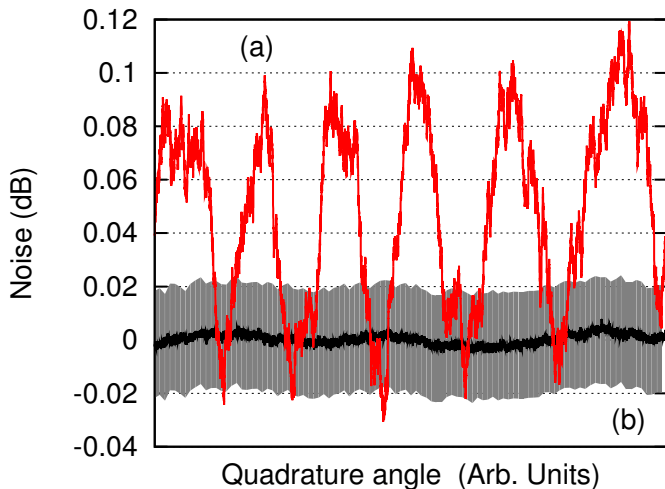
Cloud size = 1 mm, $T = 200 \mu\text{K}$, $N = 7 \times 10^9 \text{ 1/cm}^3$,
OD = 2, beam size = 0.1 mm, 10^5 interacting atoms



Noise contrast in MOT with ^{87}Rb $F_g = 2 \rightarrow F_e = 1$



Squeezing in MOT with ^{87}Rb $F_g = 2 \rightarrow F_e = 1$





Support from



Summary

- Squeezing is exiting
- many applications benefit from squeezing
- there is still a lot of interesting physics to do