

Squeezed light, generation and applications.

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What is quantum optics

Classical/Geometrical optics

- light is a ray
- which propagates straight
- cannot explain diffraction and interference

Semiclassical optics

- light is a wave
- color (wavelength/frequency) is important
- amplitude (a) and phase are important, $E(t) = ae^{i(kz - \omega t)}$
- cannot explain residual measurements noise

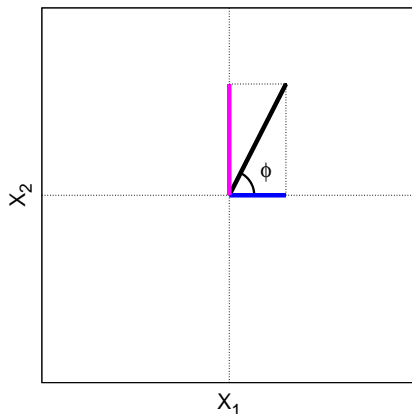
Quantum optics

- light consists of photons: $N = a^\dagger a$
- detector measures quadratures: $X_1 = (a^\dagger + a)/2$ and $X_2 = i(a^\dagger - a)/2$
- amplitude and phase cannot be measured precisely:
 $\langle \Delta X_1^2 \rangle \langle \Delta X_2^2 \rangle \geq 1$

Classical field

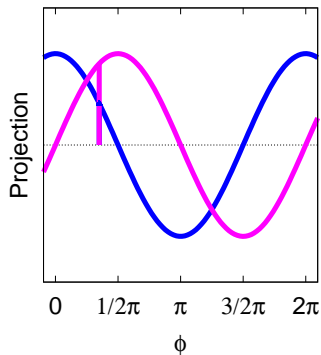
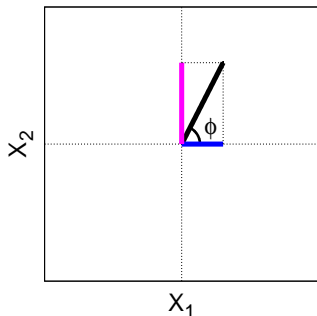
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Detectors sense the **real** part of the field (X_1) but there is a way to see X_2 as well



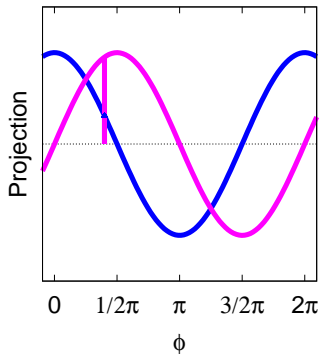
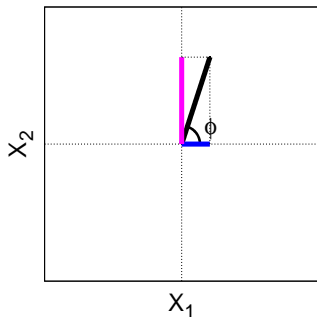
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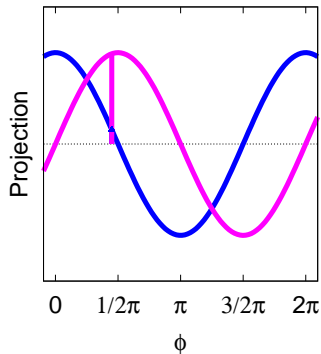
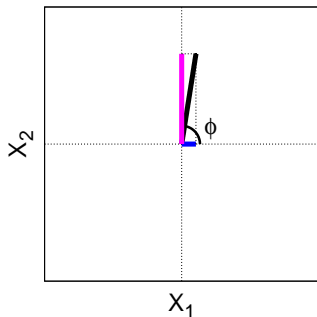
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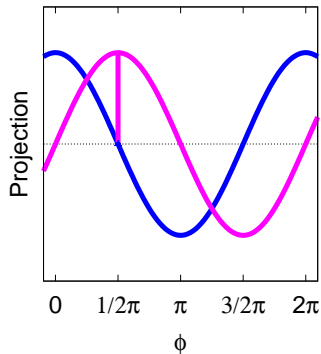
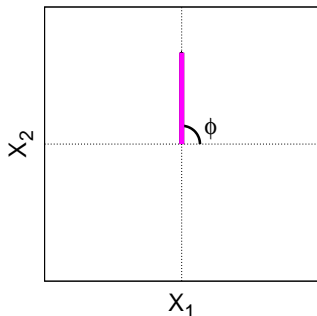
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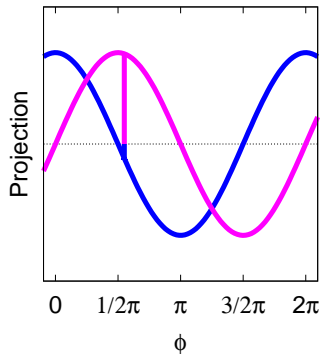
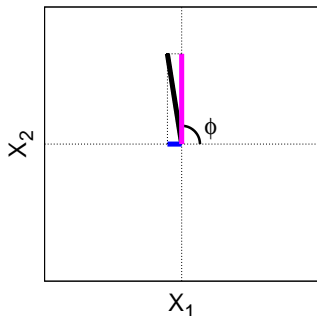
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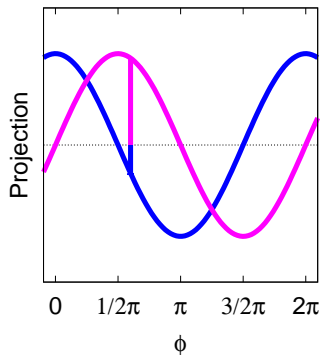
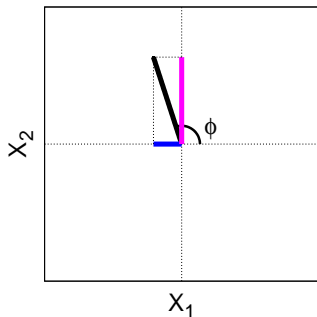
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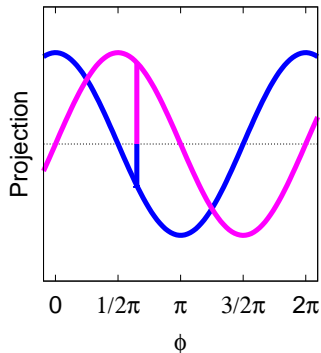
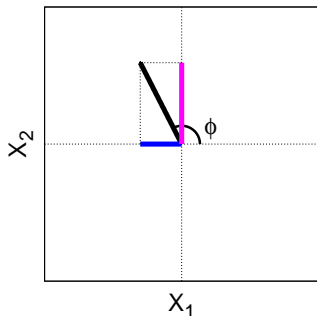
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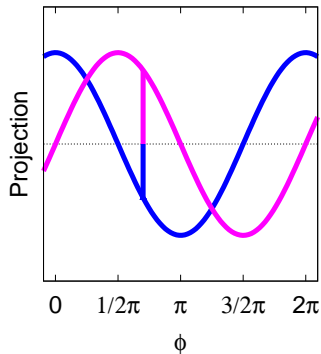
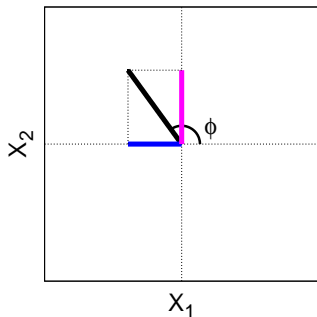
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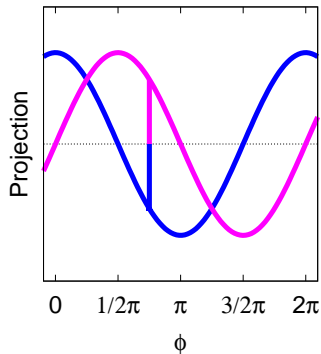
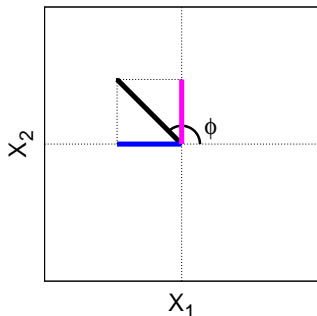
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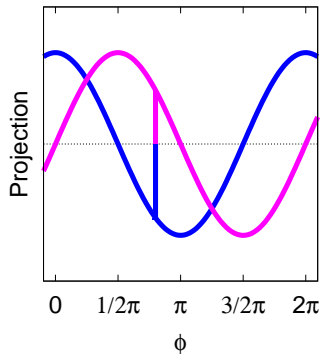
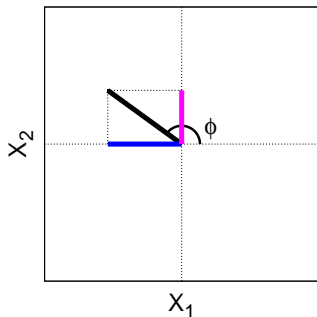
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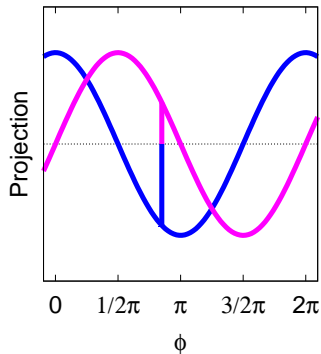
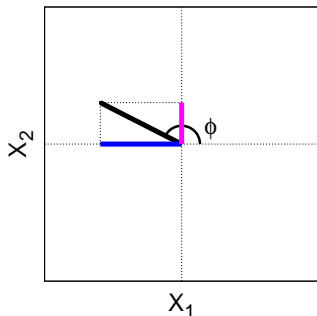
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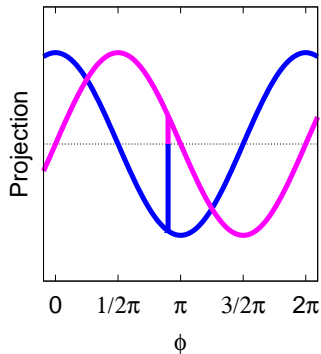
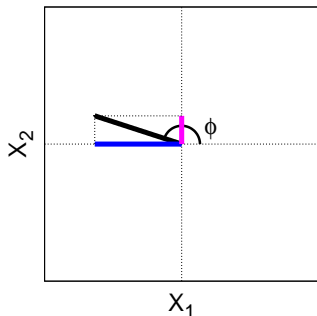
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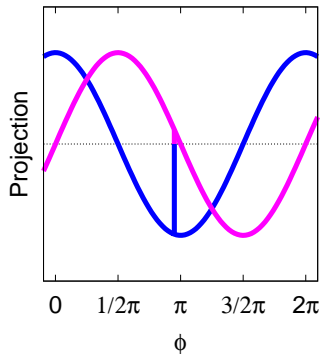
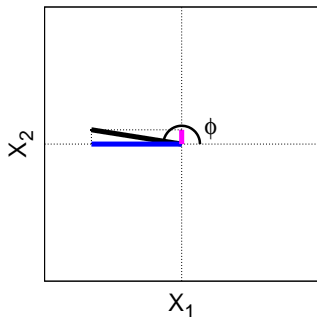
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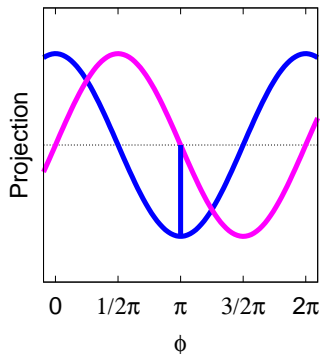
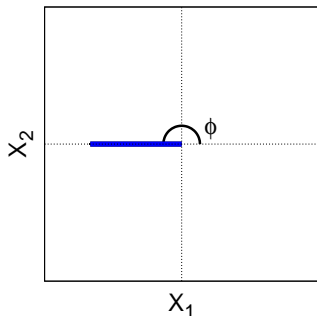
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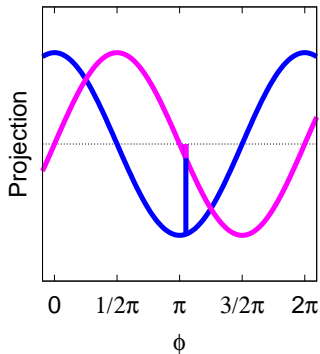
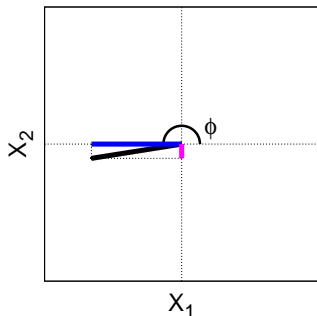
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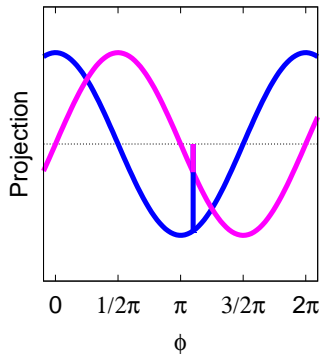
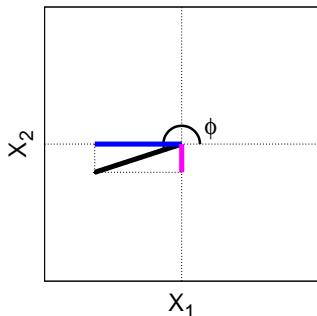
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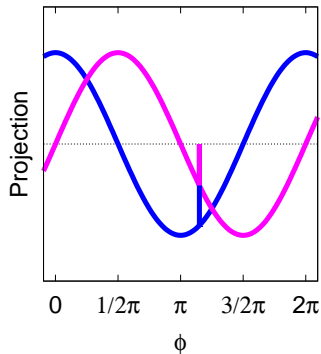
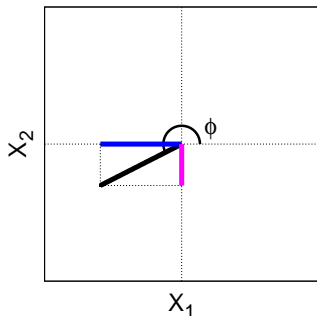
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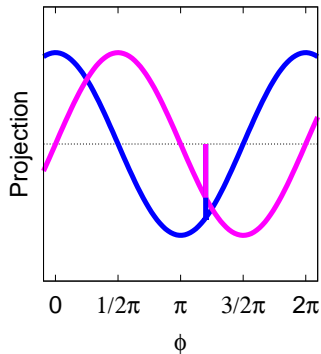
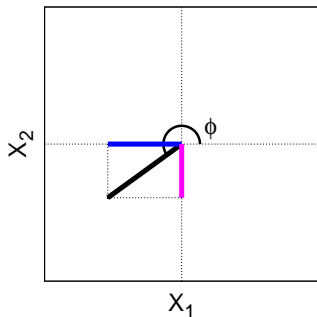
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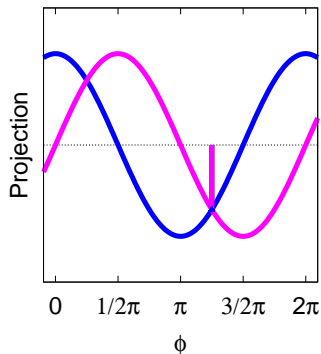
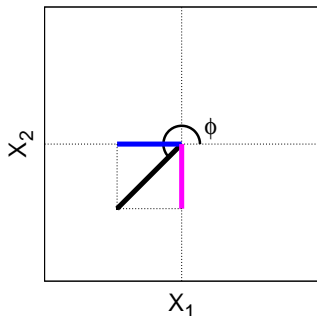
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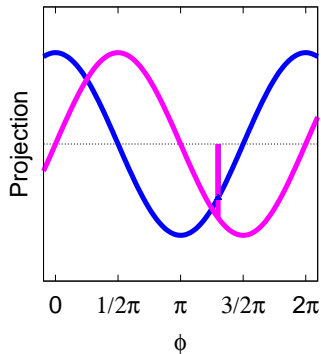
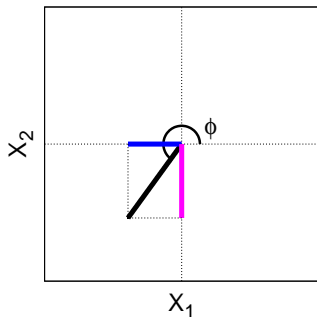
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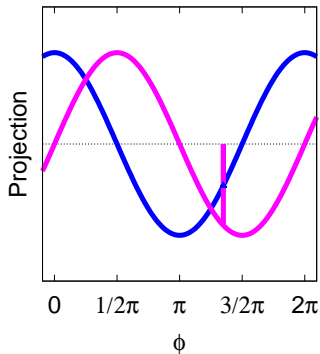
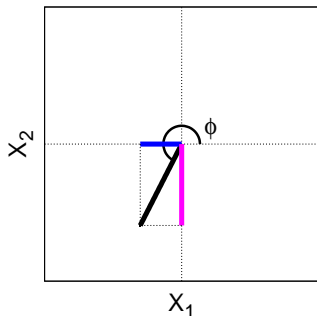
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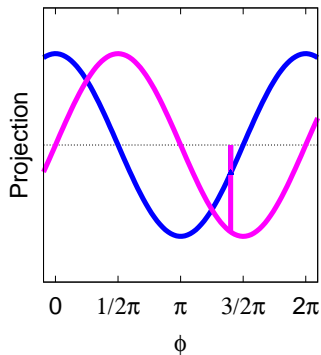
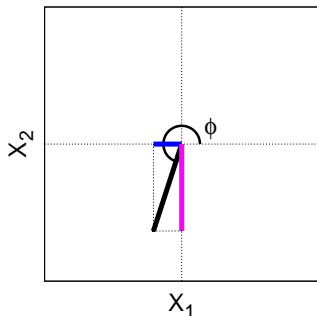
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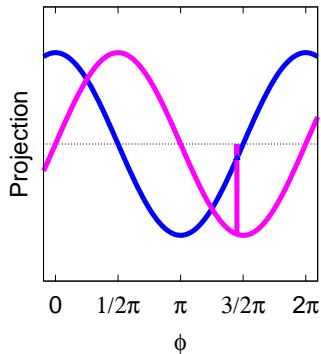
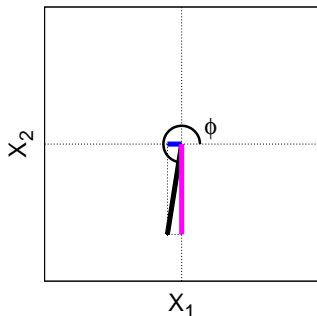
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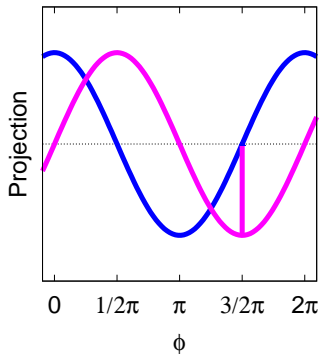
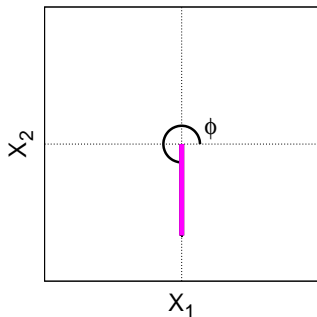
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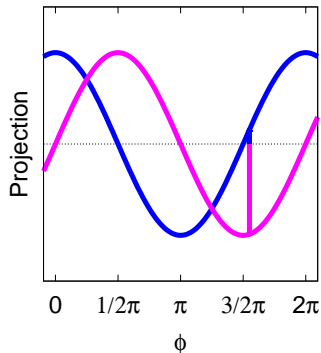
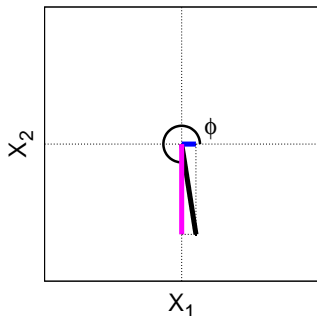
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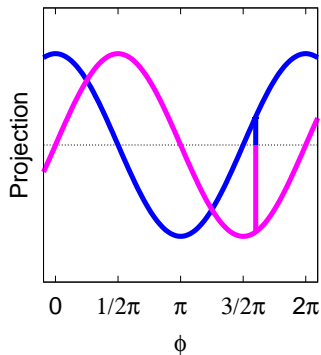
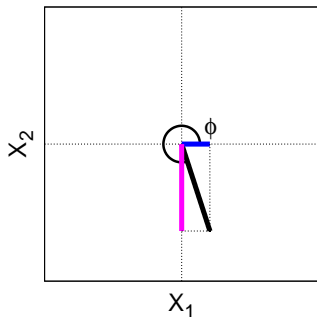
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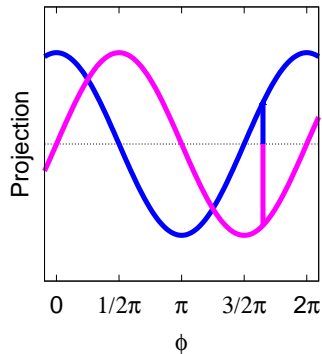
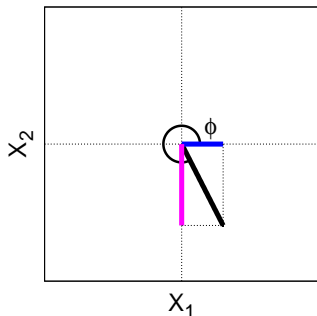
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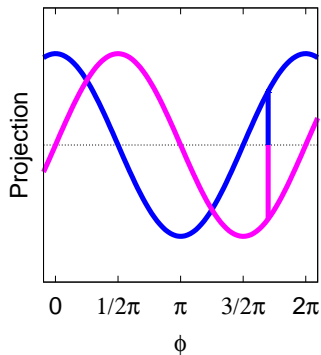
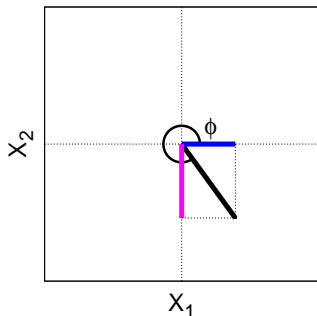
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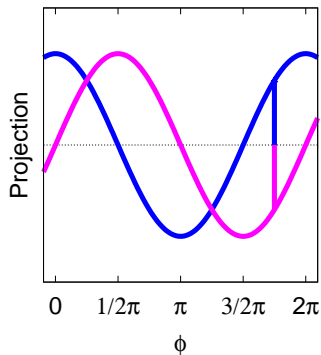
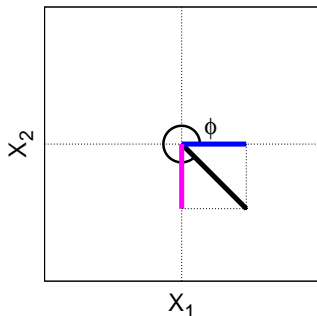
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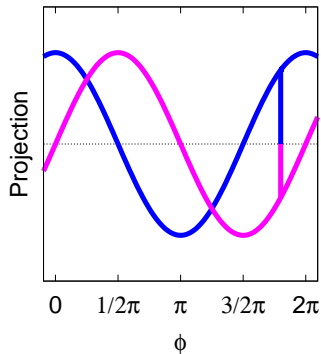
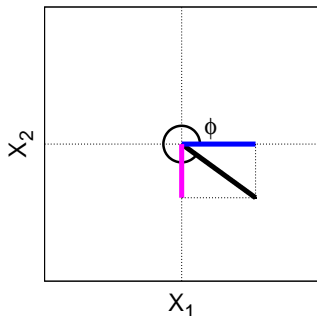
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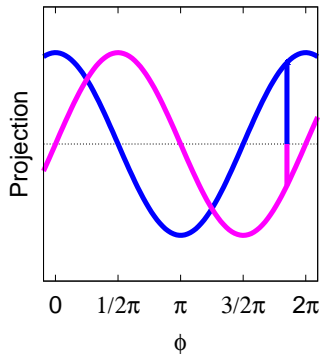
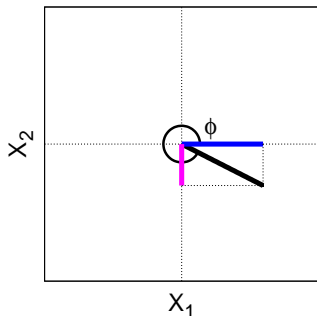
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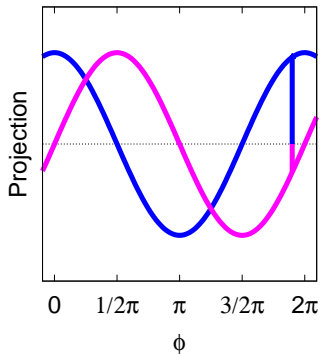
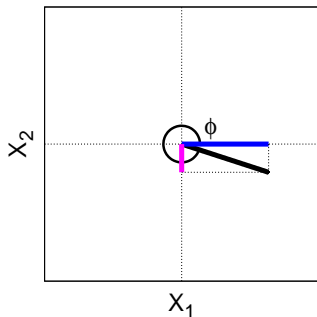
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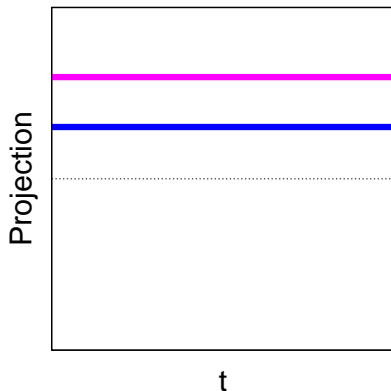
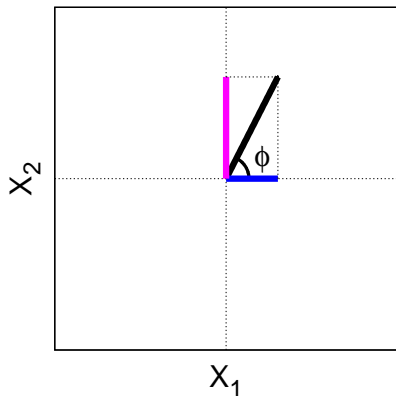
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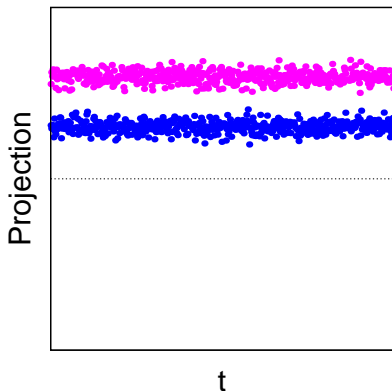
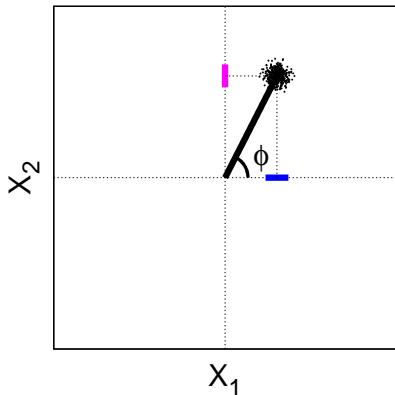
Classical quadratures vs time in a rotating frame

$$E(\phi) = |a|e^{-i\phi} = |a|\cos(\phi) + i|a|\sin(\phi) = X_1 + iX_2, \quad \phi = \omega t - kz$$



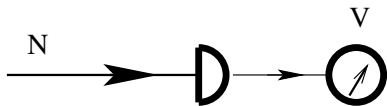
Reality check quadratures vs time

$$E(\phi) = |a|e^{-i\phi} = |a|\cos(\phi) + i|a|\sin(\phi) = X_1 + iX_2, \quad \phi = \omega t - kz$$



Detector quantum noise

Simple photodetector

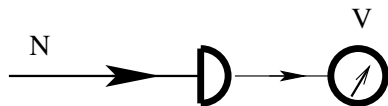


$$V \sim N$$

$$\Delta V \sim \sqrt{N}$$

Detector quantum noise

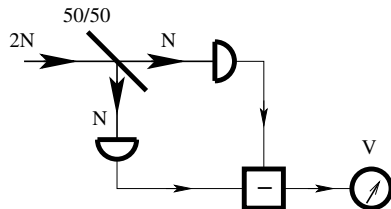
Simple photodetector



$$V \sim N$$

$$\Delta V \sim \sqrt{N}$$

Balanced photodetector



$$V = 0$$

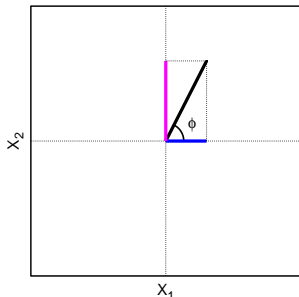
$$\Delta V \sim \sqrt{N}$$

Transition from classical to quantum field

Classical analog

- Field amplitude a
- Field real part
 $X_1 = (a^* + a)/2$
- Field imaginary part
 $X_2 = i(\hat{a}^* - a)/2$

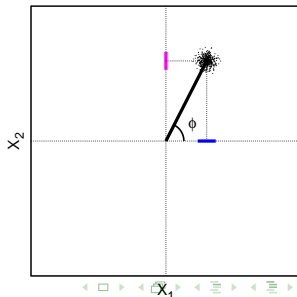
$$E(\phi) = |a|e^{-i\phi} = X_1 + iX_2$$



Quantum approach

- Field operator \hat{a}
- Amplitude quadrature
 $\hat{X}_1 = (\hat{a}^\dagger + \hat{a})/2$
- Phase quadrature
 $\hat{X}_2 = i(\hat{a}^\dagger - \hat{a})/2$

$$\hat{E}(\phi) = \hat{X}_1 + i\hat{X}_2$$



Heisenberg uncertainty principle and its optics equivalent

Heisenberg uncertainty principle

$$\Delta p \Delta x \geq \hbar/2$$

The more precisely the POSITION is determined, the less precisely the MOMENTUM is known, and vice versa



Heisenberg uncertainty principle and its optics equivalent



Heisenberg uncertainty principle

$$\Delta p \Delta x \geq \hbar/2$$

The more precisely the POSITION is determined, the less precisely the MOMENTUM is known, and vice versa

Optics equivalent

$$\Delta \phi \Delta N \geq 1$$

The more precisely the PHASE is determined, the less precisely the AMPLITUDE is known, and vice versa

Heisenberg uncertainty principle and its optics equivalent



Heisenberg uncertainty principle

$$\Delta p \Delta x \geq \hbar/2$$

The more precisely the POSITION is determined, the less precisely the MOMENTUM is known, and vice versa

Optics equivalent

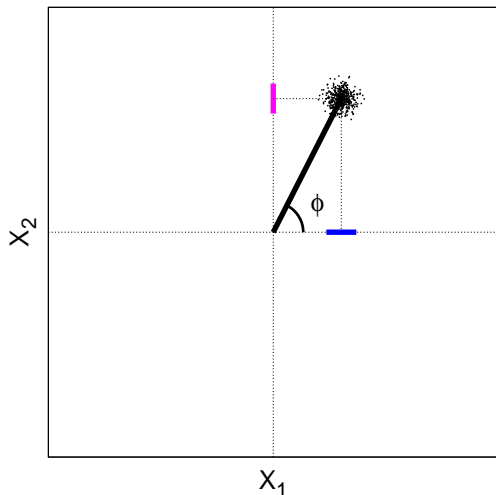
$$\Delta \phi \Delta N \geq 1$$

The more precisely the PHASE is determined, the less precisely the AMPLITUDE is known, and vice versa

Optics equivalent strict definition

$$\langle \Delta X_1 \rangle \langle \Delta X_2 \rangle \geq 1/4$$

Quantum optics summary



Light consist of photons

- $\hat{N} = a^\dagger a$

Commutator relationship

- $[a, a^\dagger] = 1$

- $[X_1, X_2] = i/2$

Detectors measure

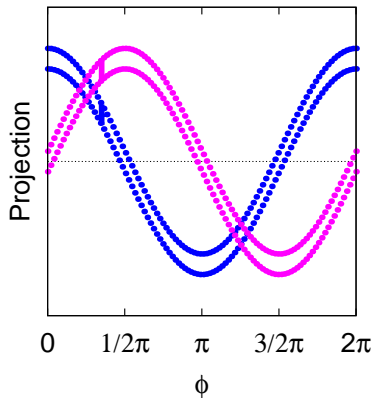
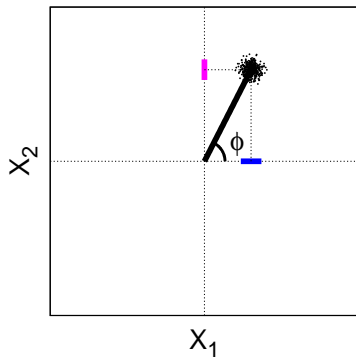
- number of photons N
- Quadratures \hat{X}_1 and \hat{X}_2

Uncertainty relationship

- $\Delta X_1 \Delta X_2 \geq 1/4$

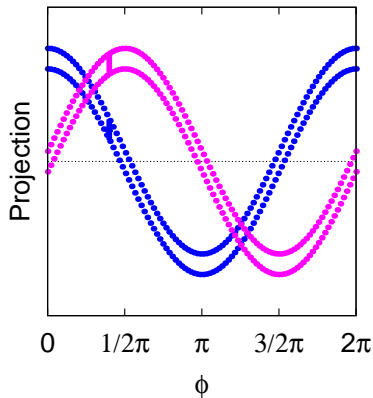
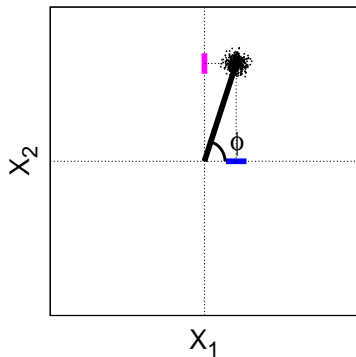
Coherent state is minimum uncertainty state

$$\Delta X_1 \Delta X_2 = 1/4$$



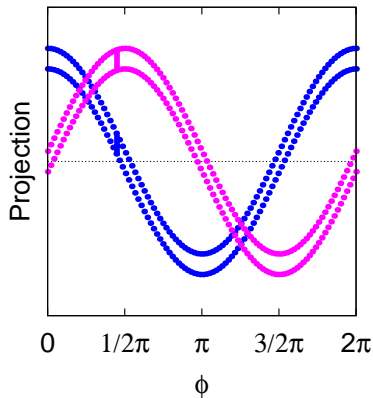
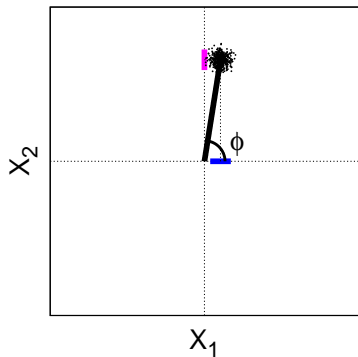
Coherent state is minimum uncertainty state

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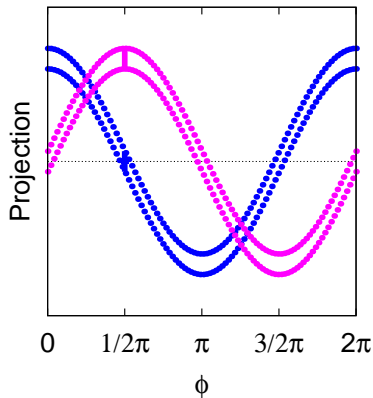
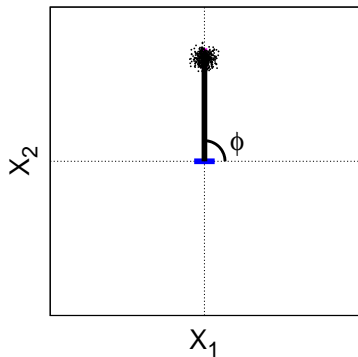
Coherent state is minimum uncertainty state

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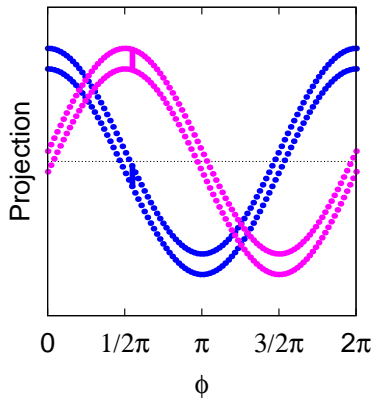
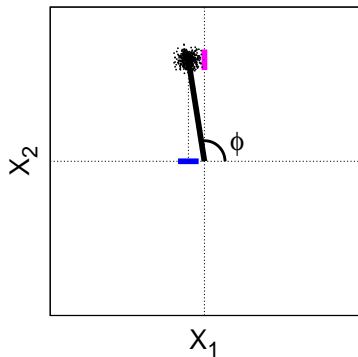
Coherent state is minimum uncertainty state

$$\Delta X_1 \Delta X_2 = 1/4$$



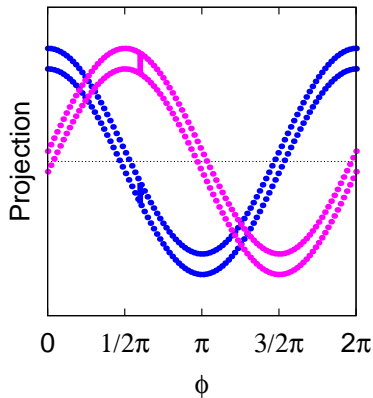
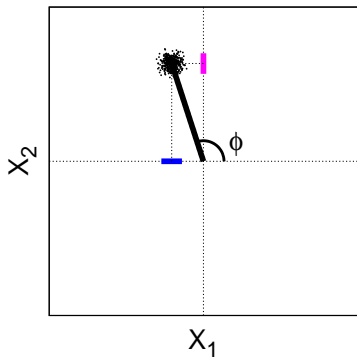
Coherent state is minimum uncertainty state

$$\Delta X_1 \Delta X_2 = 1/4$$



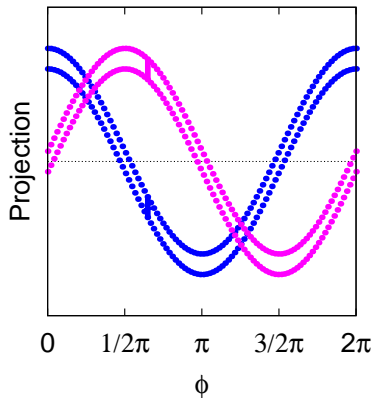
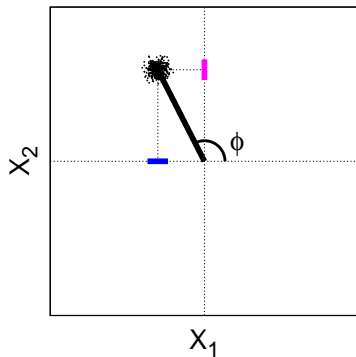
Coherent state is minimum uncertainty state

$$\Delta X_1 \Delta X_2 = 1/4$$



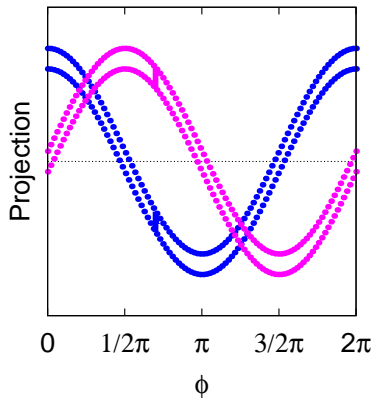
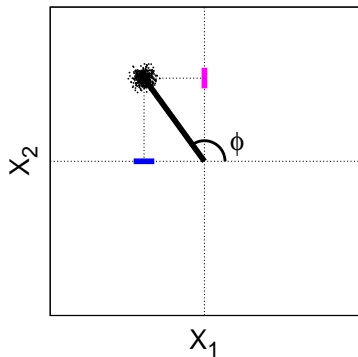
Coherent state is minimum uncertainty state

$$\Delta X_1 \Delta X_2 = 1/4$$



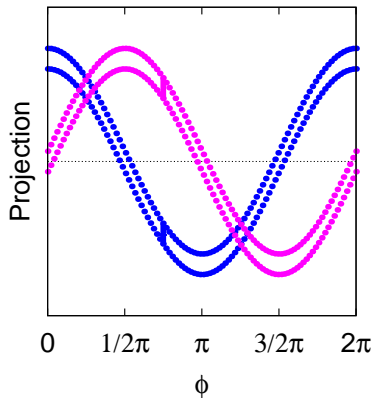
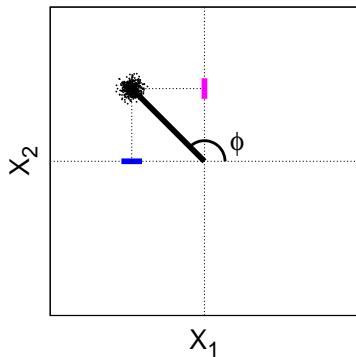
Coherent state is minimum uncertainty state

$$\Delta X_1 \Delta X_2 = 1/4$$



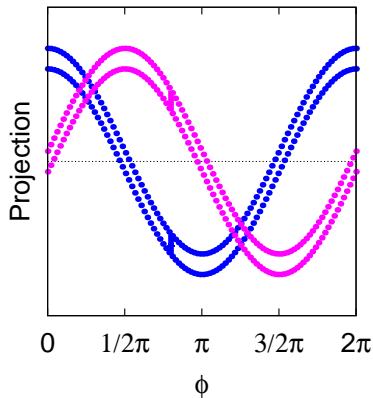
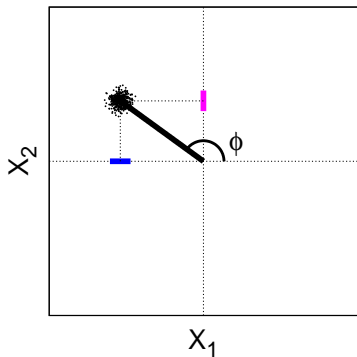
Coherent state is minimum uncertainty state

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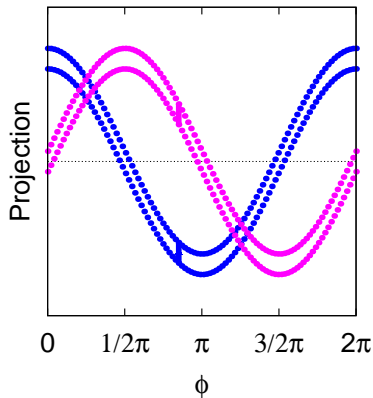
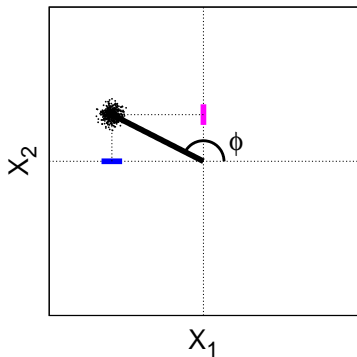
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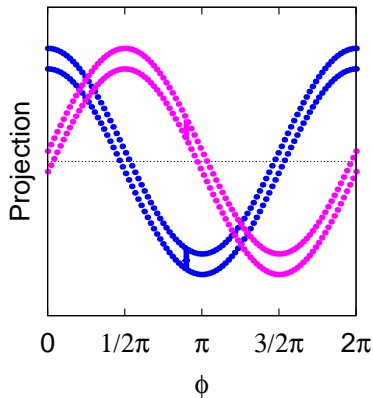
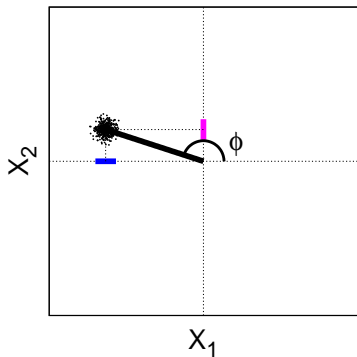
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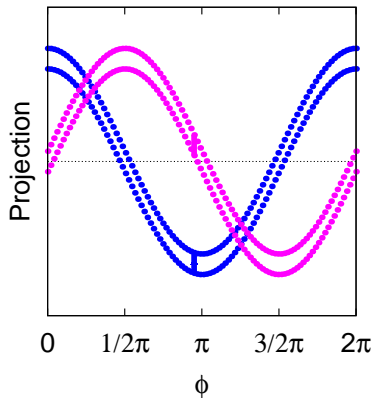
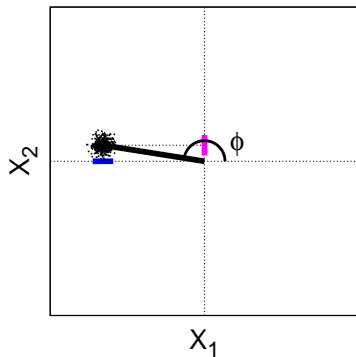
Coherent state is minimum uncertainty state

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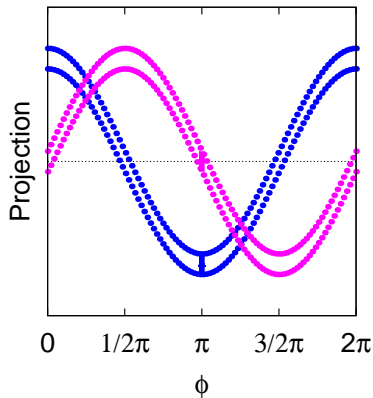
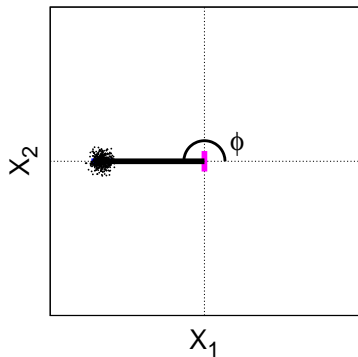
Coherent state is minimum uncertainty state

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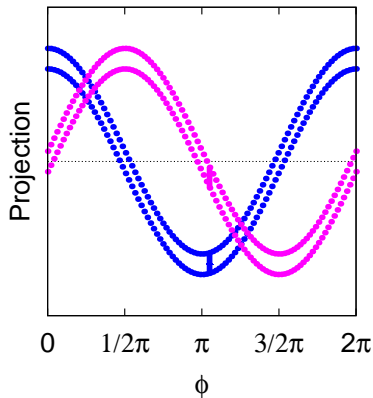
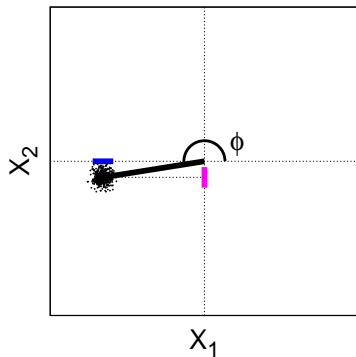
Coherent state is minimum uncertainty state

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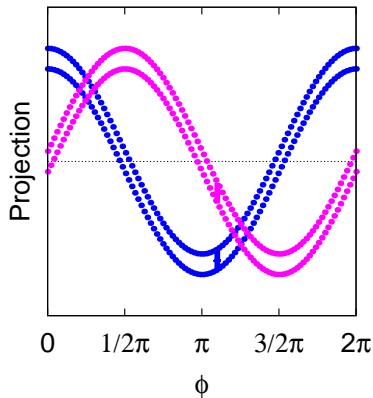
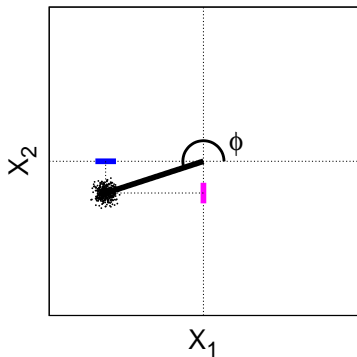
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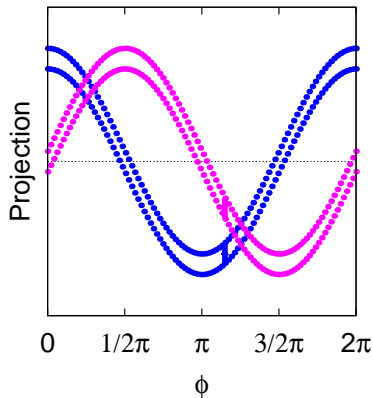
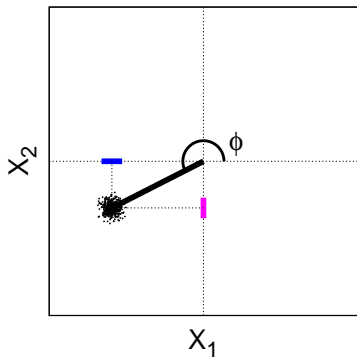
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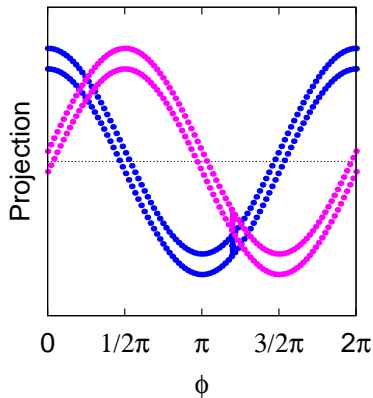
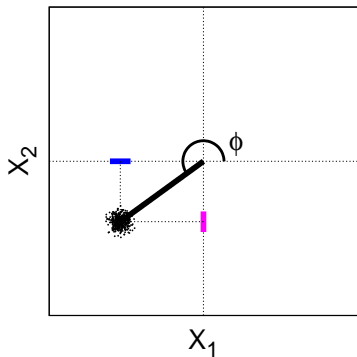
Coherent state is minimum uncertainty state

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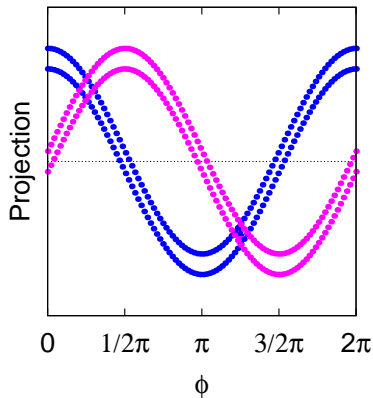
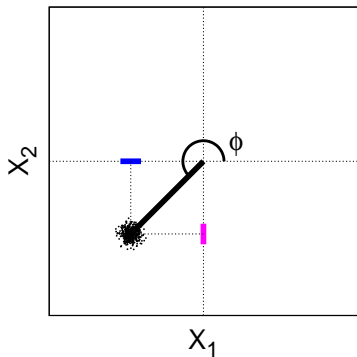
Coherent state is minimum uncertainty state

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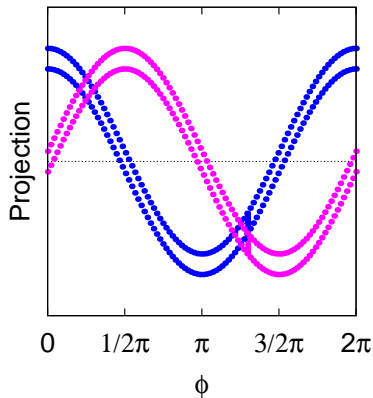
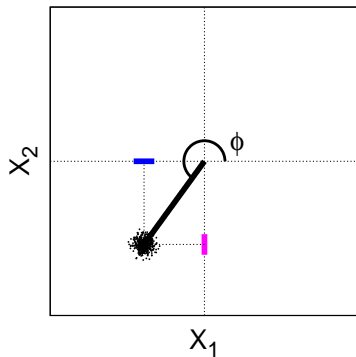
Coherent state is minimum uncertainty state

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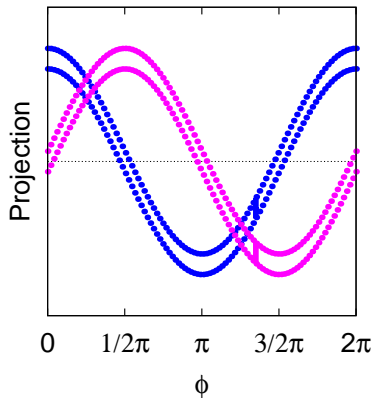
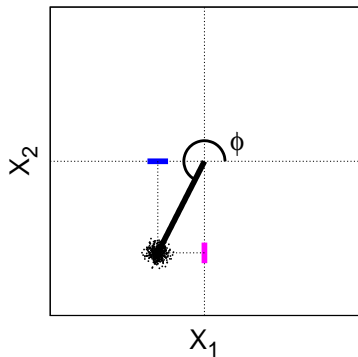
Coherent state is minimum uncertainty state

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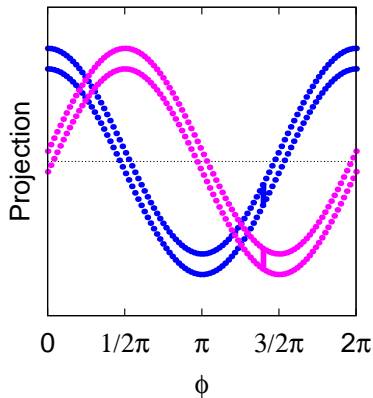
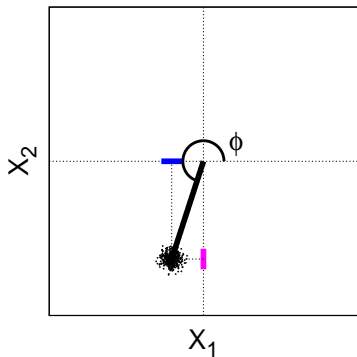
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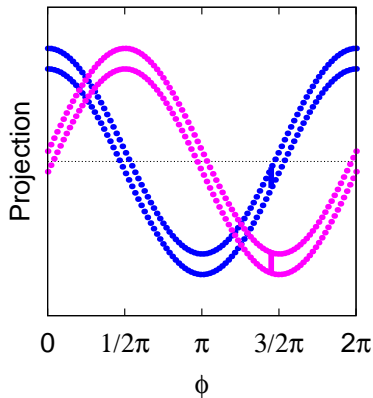
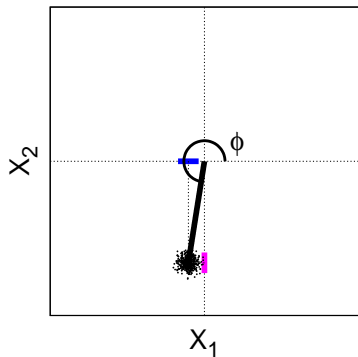
Coherent state is minimum uncertainty state

$$\Delta X_1 \Delta X_2 = 1/4$$



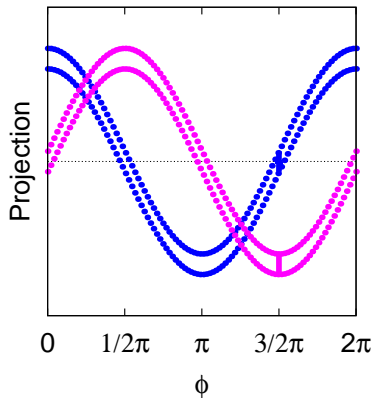
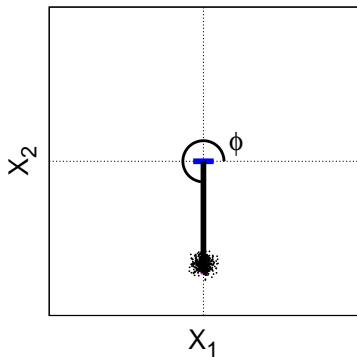
Coherent state is minimum uncertainty state

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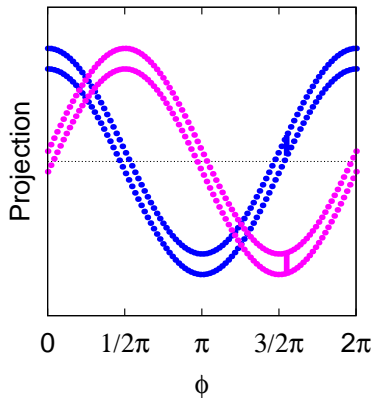
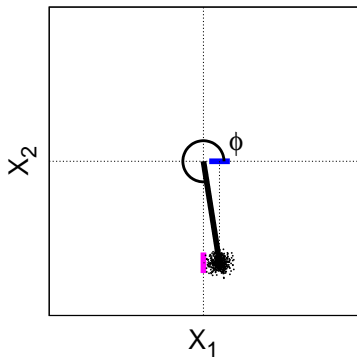
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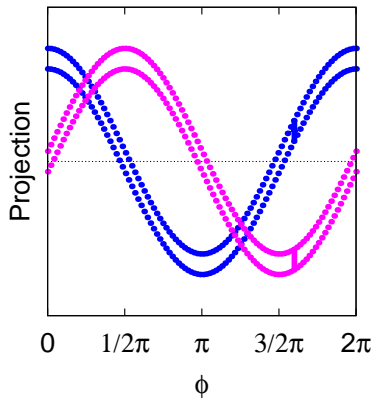
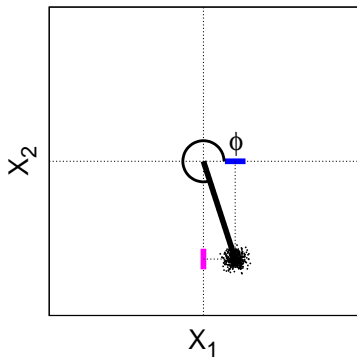
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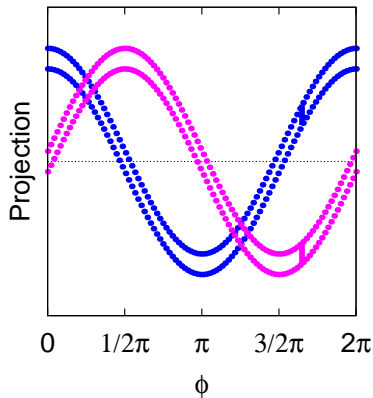
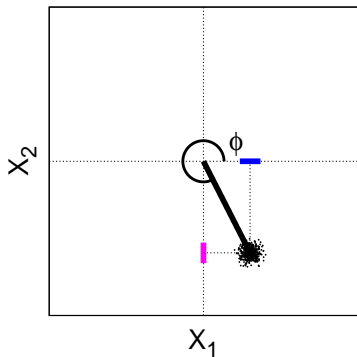
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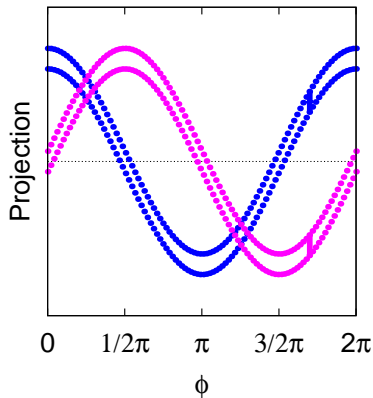
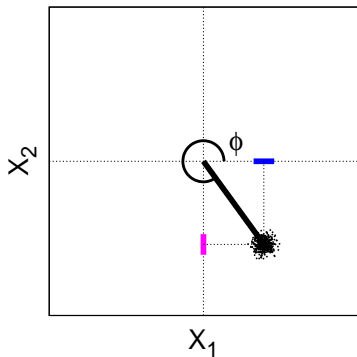
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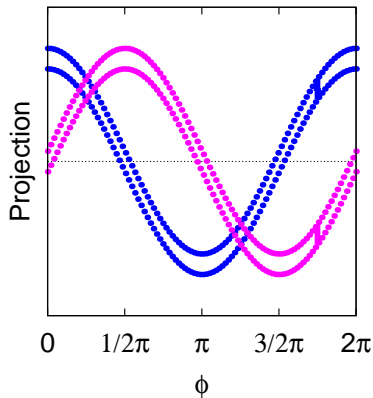
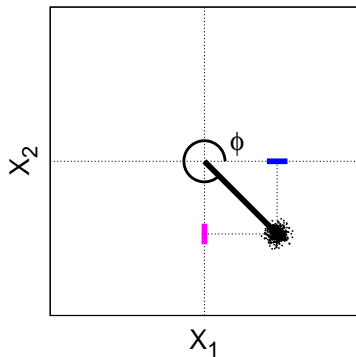
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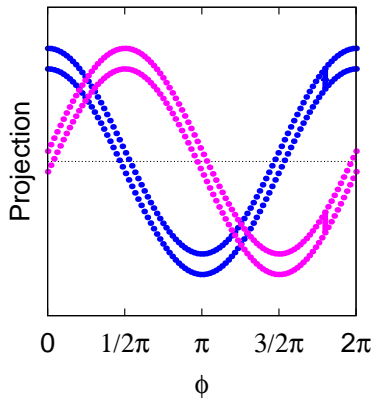
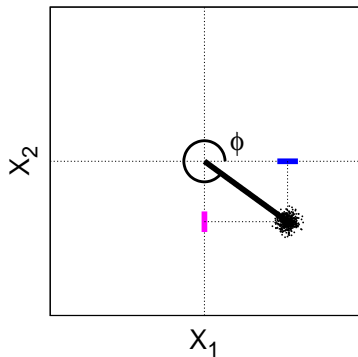
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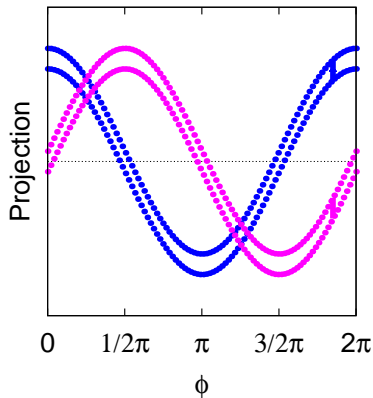
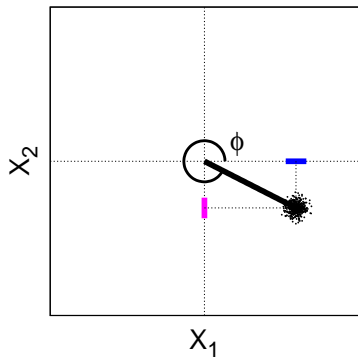
Coherent state is minimum uncertainty state

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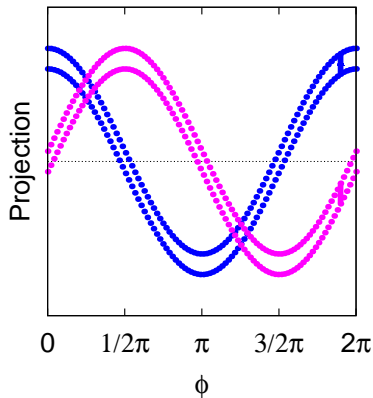
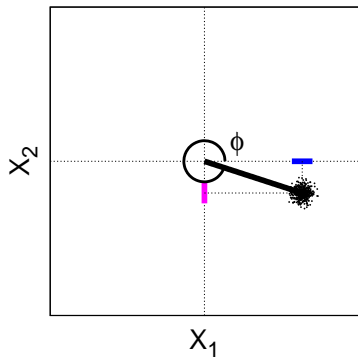
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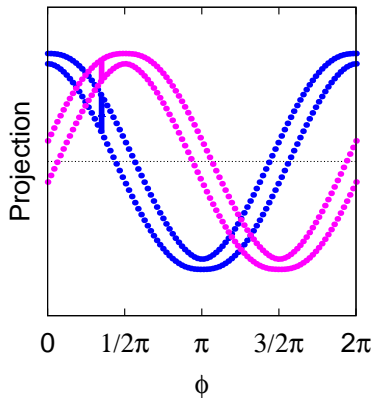
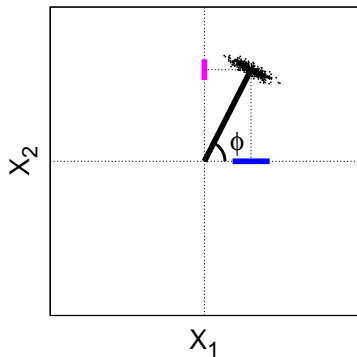
Coherent state is minimum uncertainty state

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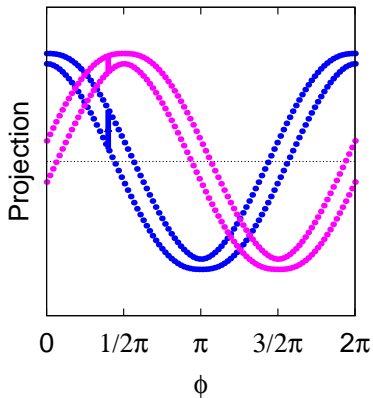
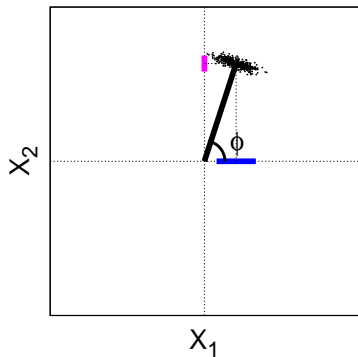
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



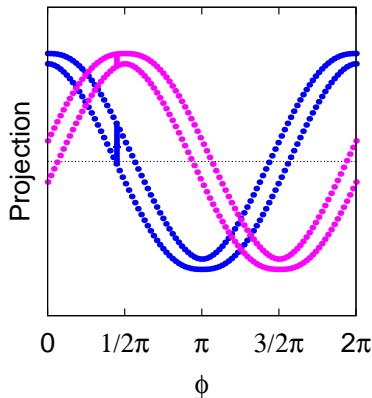
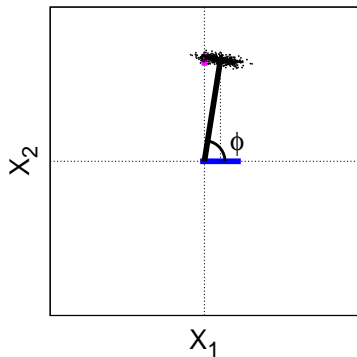
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



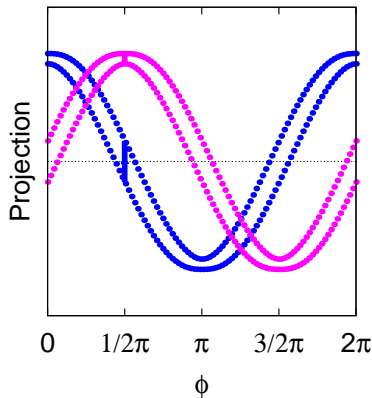
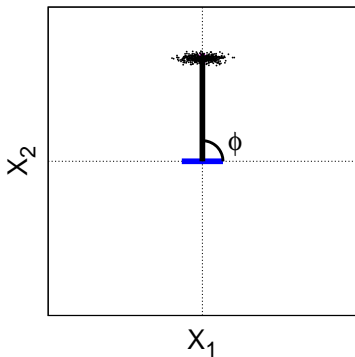
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



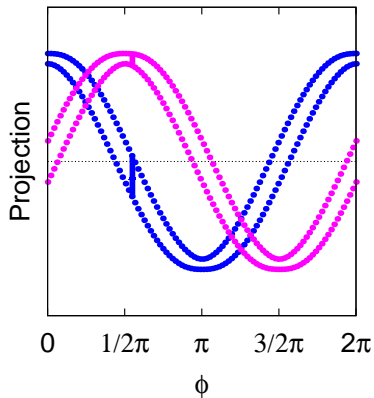
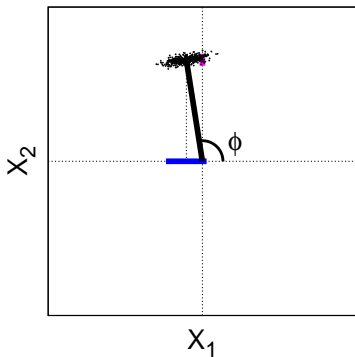
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



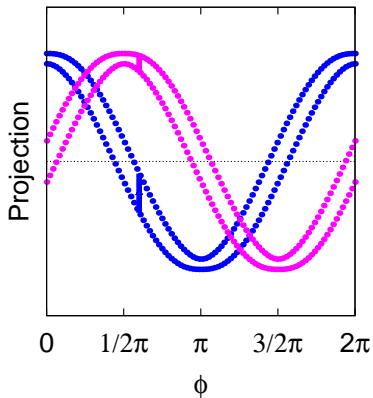
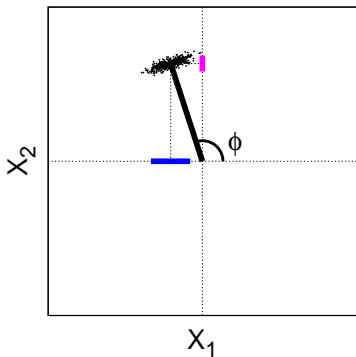
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



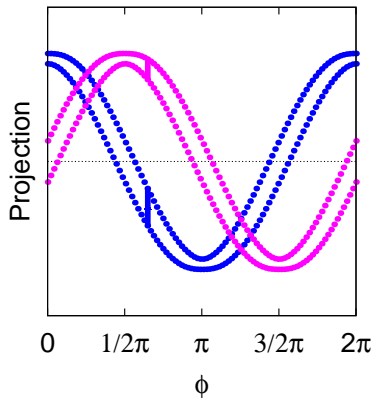
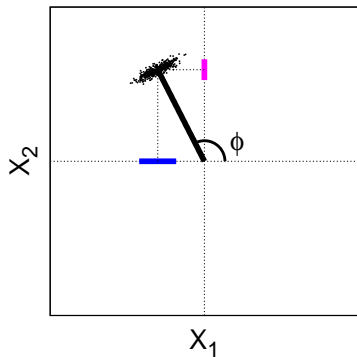
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



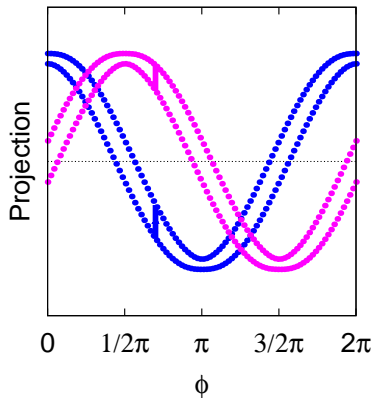
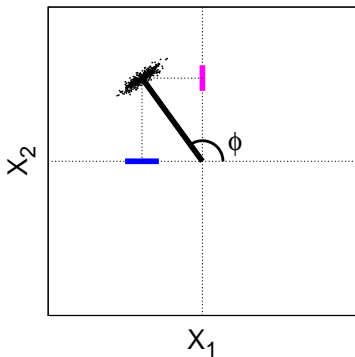
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



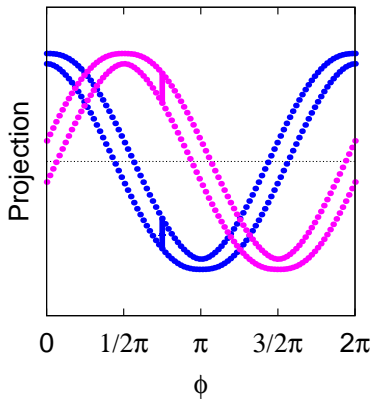
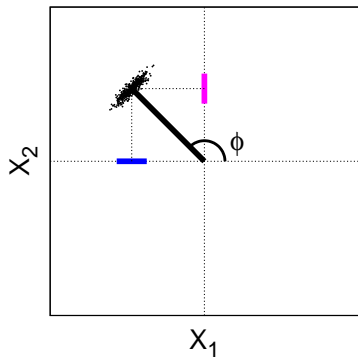
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



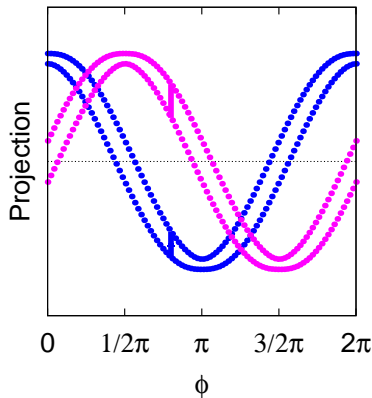
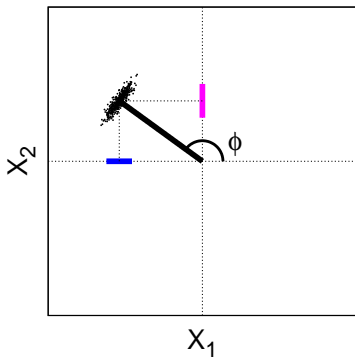
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



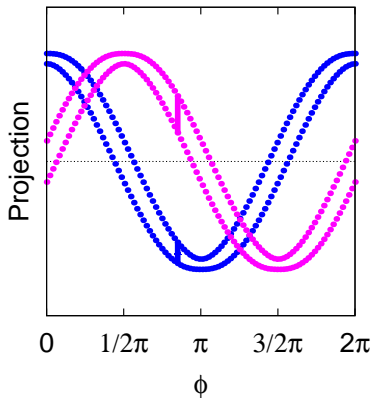
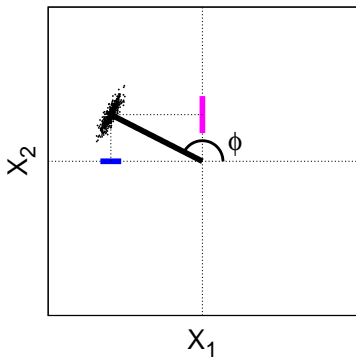
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



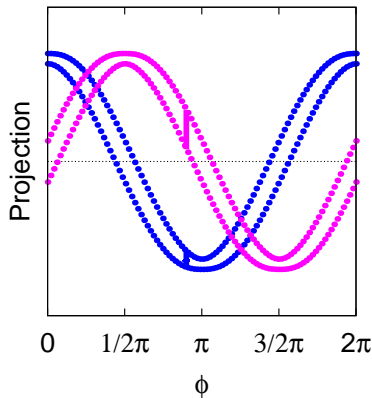
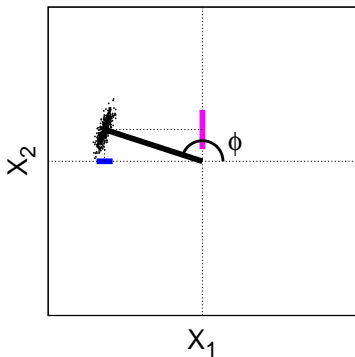
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



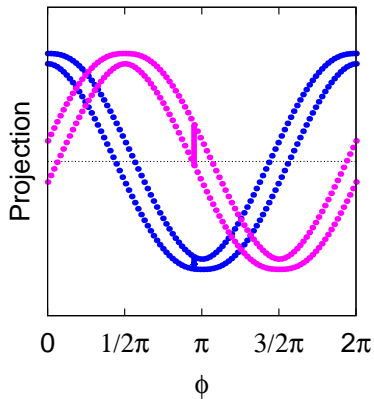
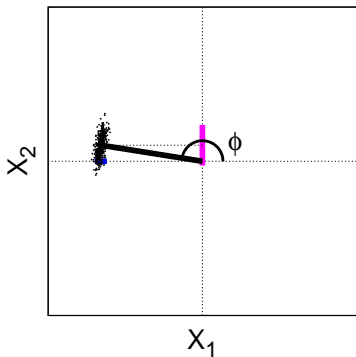
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



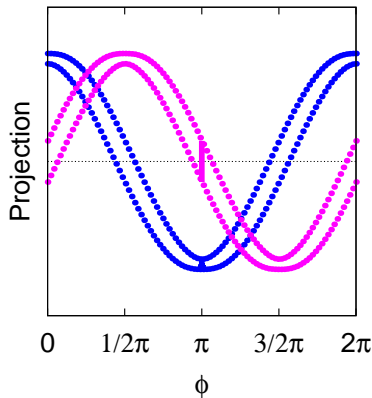
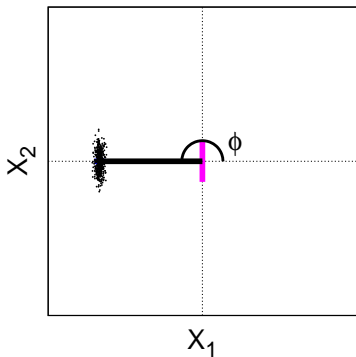
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



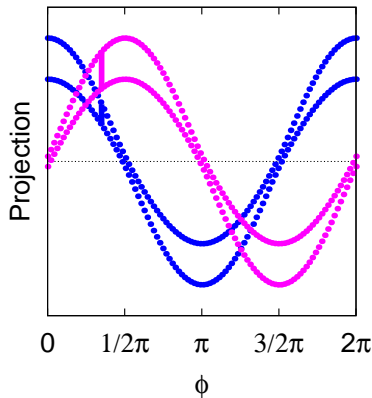
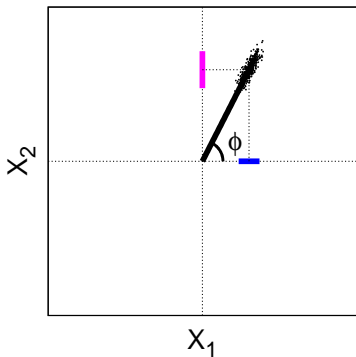
Amplitude squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



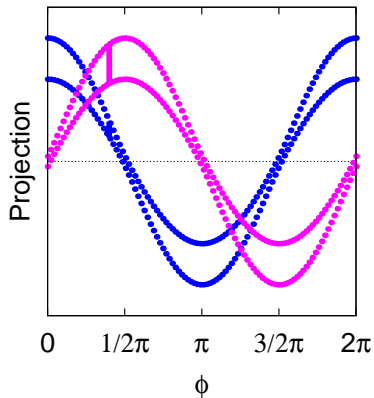
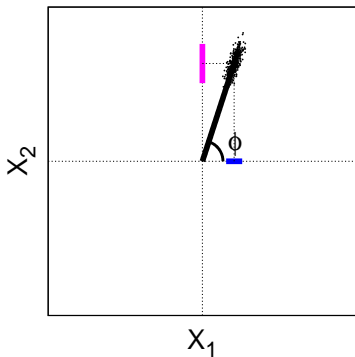
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



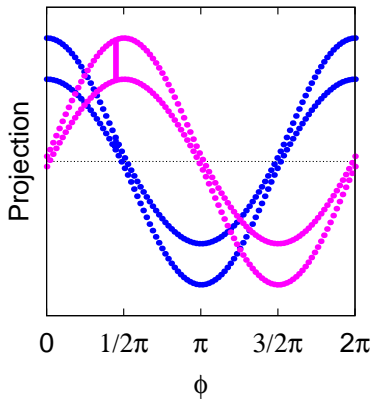
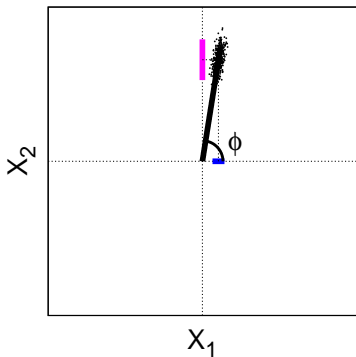
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



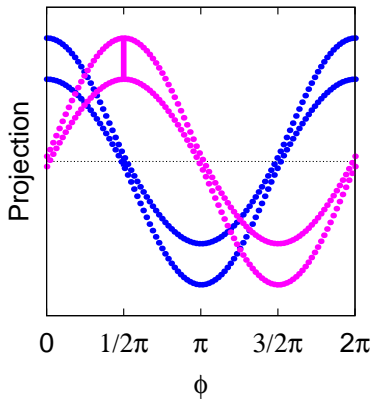
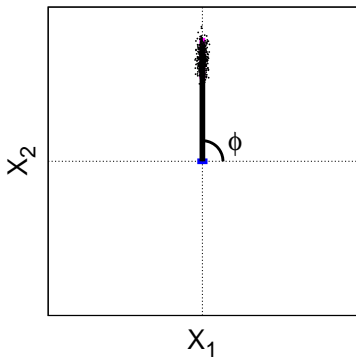
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



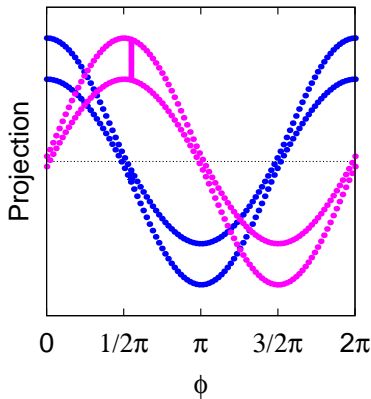
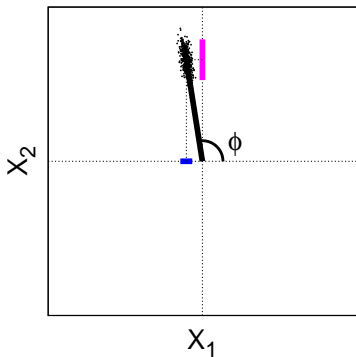
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



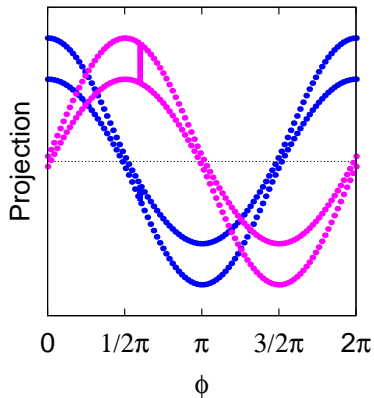
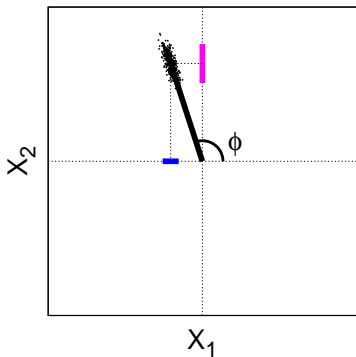
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



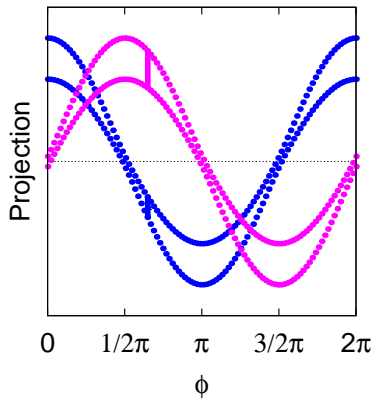
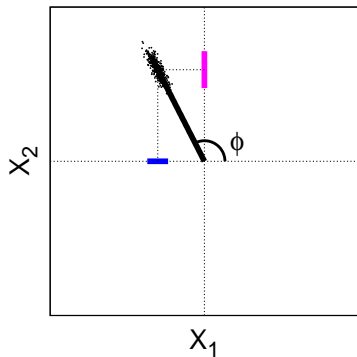
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



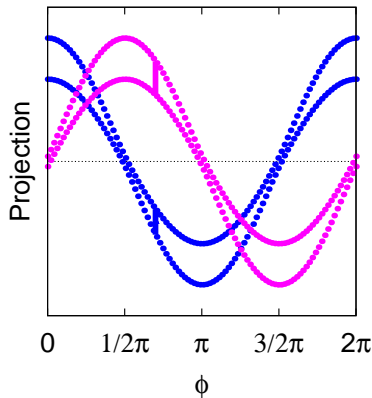
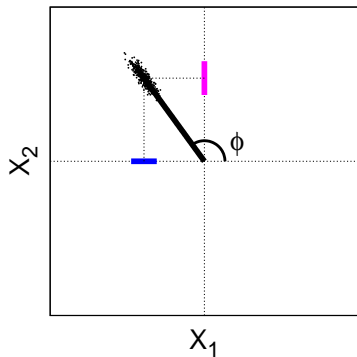
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



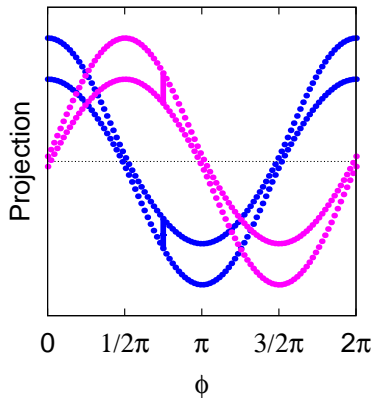
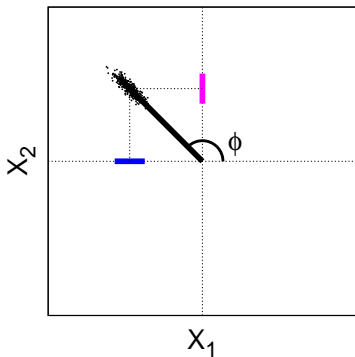
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



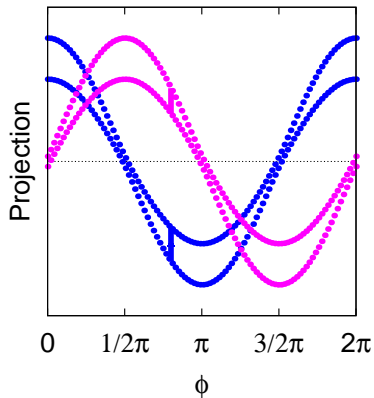
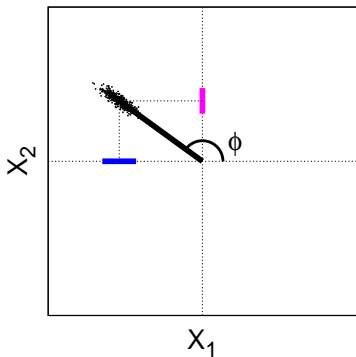
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



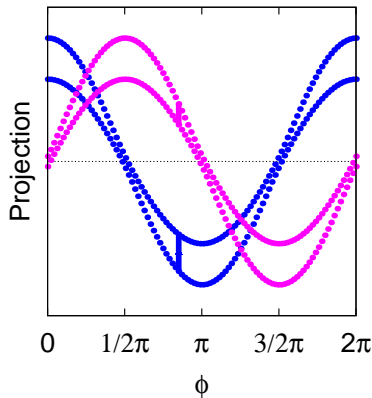
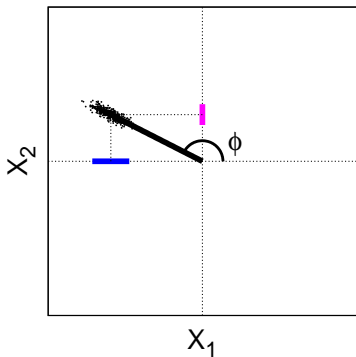
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



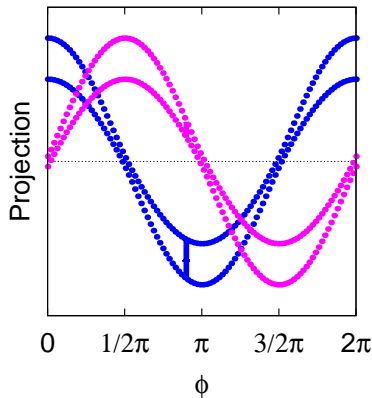
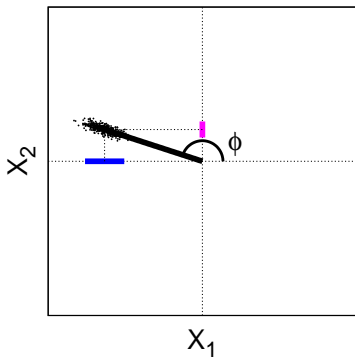
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$



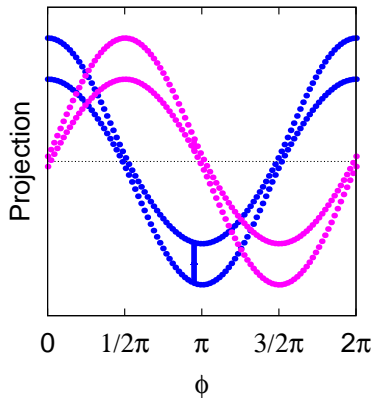
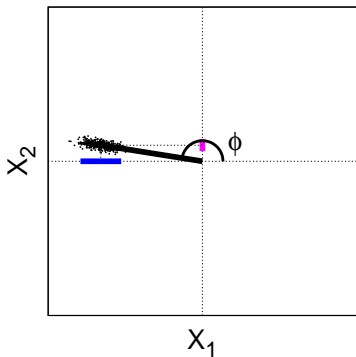
Phase squeezed states

$$\Delta X_1 \Delta X_2 = 1/4$$

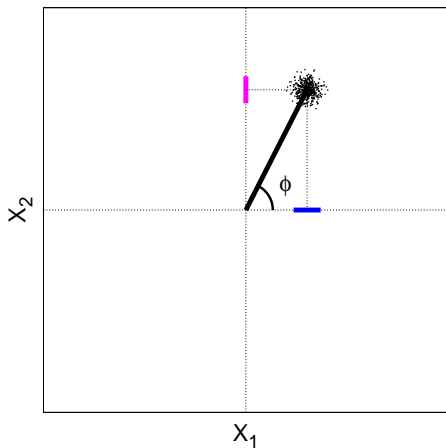


Phase squeezed states

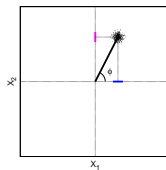
$$\Delta X_1 \Delta X_2 = 1/4$$



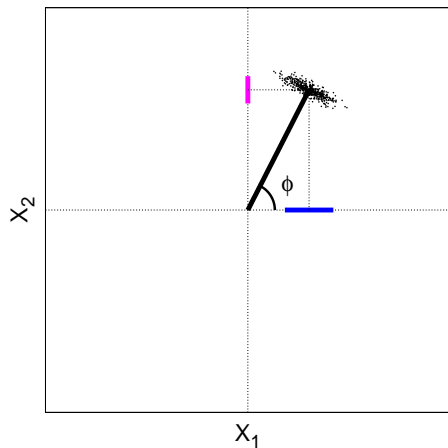
Squeezed quantum states zoo



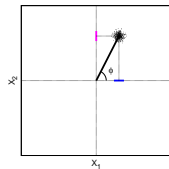
Unsqueezed
coherent



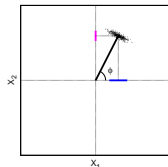
Squeezed quantum states zoo



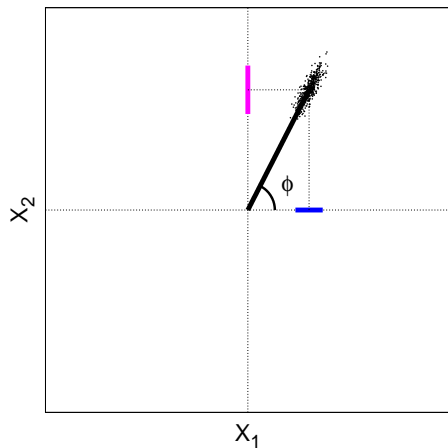
Unsqueezed
coherent



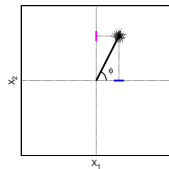
Amplitude
squeezed



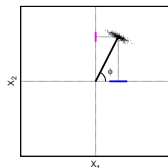
Squeezed quantum states zoo



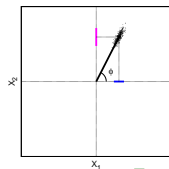
Unsqueezed
coherent



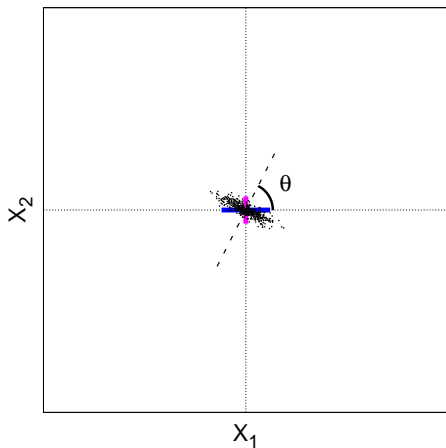
Amplitude
squeezed



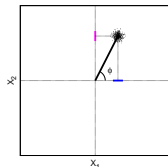
Phase
squeezed



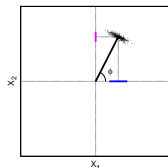
Squeezed quantum states zoo



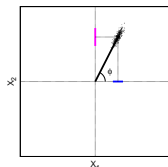
Unsqueezed
coherent



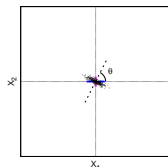
Amplitude
squeezed



Phase
squeezed

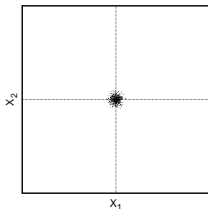


Vacuum
squeezed



Squeezed field generation recipe

Take a vacuum
state $|0\rangle$

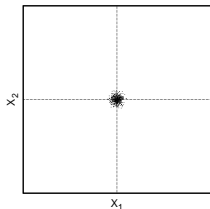


$$H = \frac{1}{2}$$

Squeezed field generation recipe

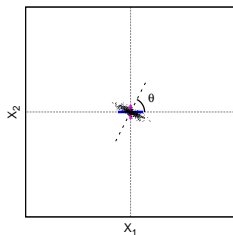
Take a vacuum state $|0\rangle$

Apply squeezing operator $|\xi\rangle = \hat{S}(\xi)|0\rangle$



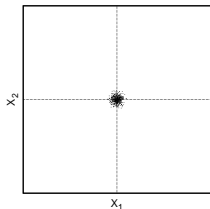
$$H = \frac{1}{2}$$

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$



Squeezed field generation recipe

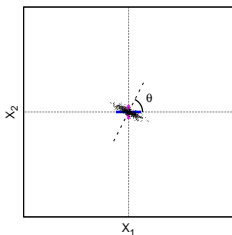
Take a vacuum state $|0\rangle$



$$H = \frac{1}{2}$$

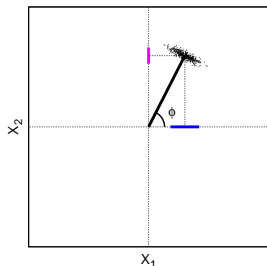
Apply squeezing operator $|\xi\rangle = \hat{S}(\xi)|0\rangle$

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$



Apply displacement operator $|\alpha, \xi\rangle = \hat{D}(\alpha)|\xi\rangle$

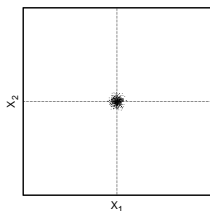
$$\hat{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$



$$\begin{aligned}\langle \alpha, \xi | X_1 | \alpha, \xi \rangle &= \text{Re}(\alpha), \\ \langle \alpha, \xi | X_2 | \alpha, \xi \rangle &= \text{Im}(\alpha)\end{aligned}$$

Squeezed field generation recipe

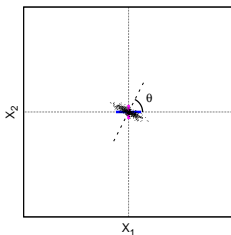
Take a vacuum state $|0\rangle$



$$H = \frac{1}{2}$$

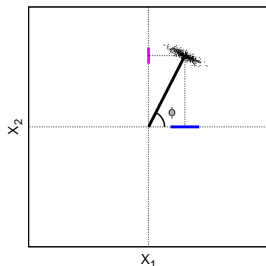
Apply squeezing operator $|\xi\rangle = \hat{S}(\xi)|0\rangle$

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$



Apply displacement operator $|\alpha, \xi\rangle = \hat{D}(\alpha)|\xi\rangle$

$$\hat{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$

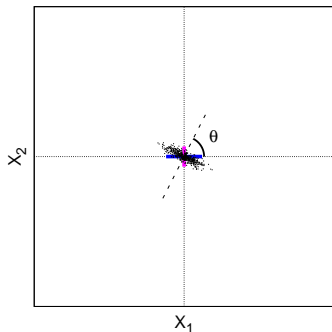


$$\langle \alpha, \xi | X_1 | \alpha, \xi \rangle = \text{Re}(\alpha),$$

$$\langle \alpha, \xi | X_2 | \alpha, \xi \rangle = \text{Im}(\alpha)$$

Notice $\Delta X_1 \Delta X_2 = \frac{1}{4}$

Squeezed state $|\xi\rangle = \hat{S}(\xi)|0\rangle$ properties



$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}, \xi = r e^{i\theta}$$

If $\theta = 0$

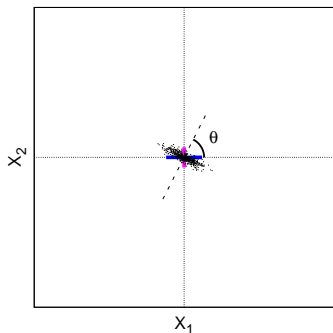
$$\langle \xi | (\Delta X_1)^2 | \xi \rangle = \frac{1}{4} e^{-2r}$$

$$\langle \xi | (\Delta X_2)^2 | \xi \rangle = \frac{1}{4} e^{2r}$$

$$\langle \xi | (\Delta X_1)^2 | \xi \rangle = \frac{1}{4} (\cosh^2 r + \sinh^2 r - 2 \sinh r \cosh r \cos \theta)$$

$$\langle \xi | (\Delta X_2)^2 | \xi \rangle = \frac{1}{4} (\cosh^2 r + \sinh^2 r + 2 \sinh r \cosh r \cos \theta)$$

Photon number of squeezed state $|\xi\rangle$



Probability to detect given number of photons $C = \langle n | \xi \rangle$ for squeezed vacuum

- even

$$C_{2m} = (-1)^m \frac{\sqrt{(2m)!}}{2^m m!} \frac{(e^{i\theta} \tanh r)^m}{\sqrt{\cosh r}}$$

- odd

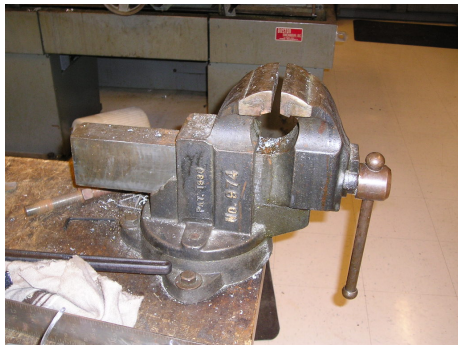
$$C_{2m+1} = 0$$

Average number of photons in general squeezed state

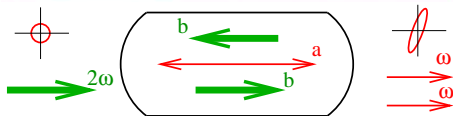
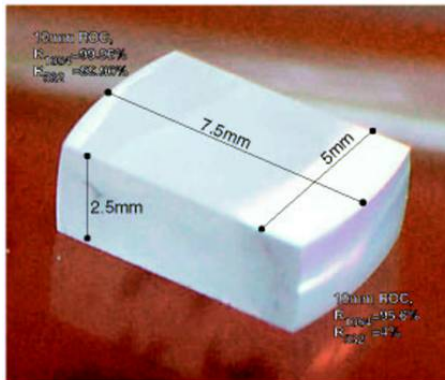
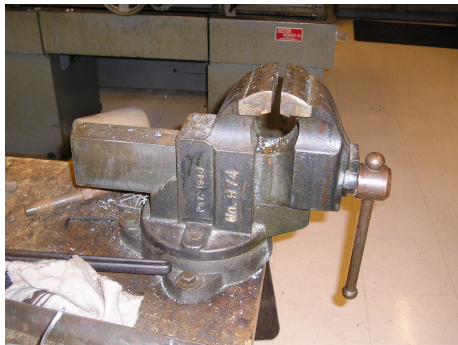
$$\langle \alpha, \xi | a^\dagger a | \alpha, \xi \rangle = \alpha + \sinh^2 r$$

Tools for squeezing

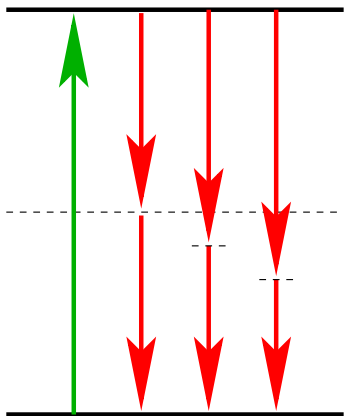
Tools for squeezing



Tools for squeezing



Two photon squeezing picture

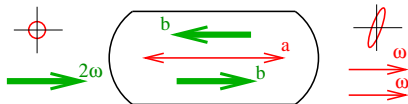


Squeezing operator

$$\hat{S}(\xi) = e^{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}}$$

Parametric down-conversion in crystal

$$\hat{H} = i\hbar\chi^{(2)}(a^2 b^\dagger - a^{\dagger 2} b)$$



Squeezing

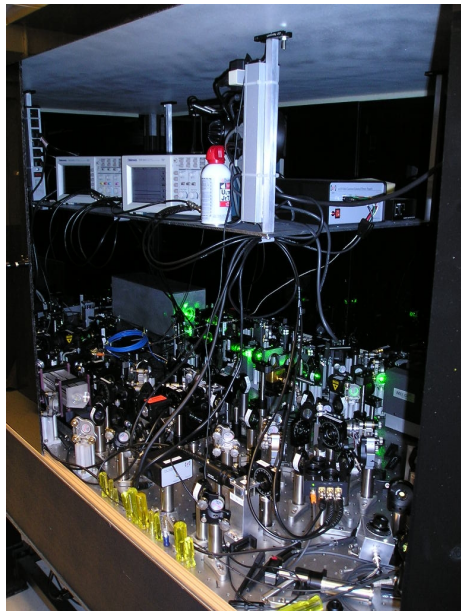
result of correlation of upper and lower sidebands

Squeezer appearance

Squeezer appearance



Squeezer appearance



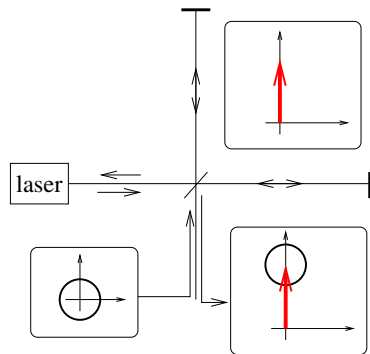
Possible squeezing applications

- improvements any shot noise limited optical sensors
- noise less signal amplification
- secure communications (you would notice eavesdropper)
- photon pair generation, entanglement, true single photon sources
- quantum memory probe and information carrier
- interferometers sensitivity boost (for example gravitational wave antennas)
- light free measurements

Squeezing and interferometer

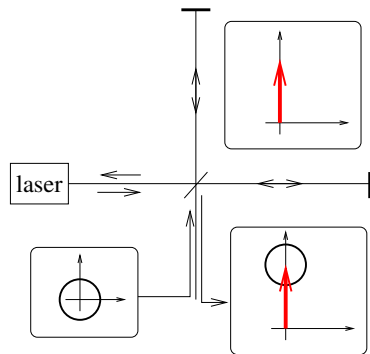
Squeezing and interferometer

Vacuum input

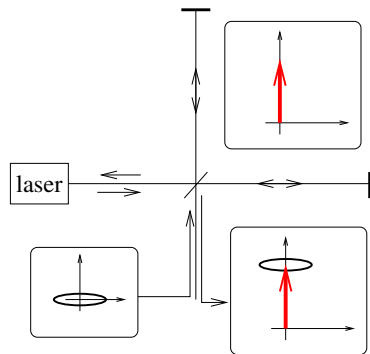


Squeezing and interferometer

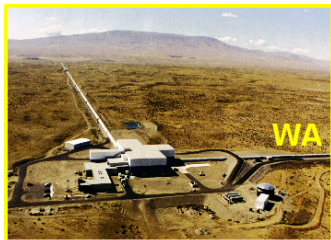
Vacuum input



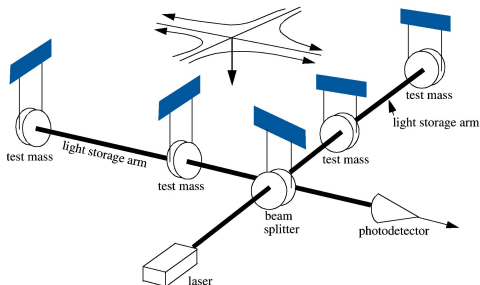
Squeezed input



Laser Interferometer Gravitational-wave Observatory



- $L = 4 \text{ km}$
- $h \sim 10^{-21}$
- $\Delta L \sim 10^{-18} \text{ m}$
- $\Delta \phi \sim 10^{-10} \text{ rad}$

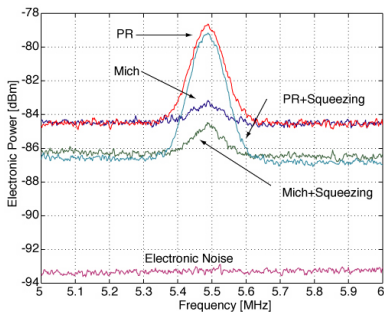


Interferometer sensitivity improvement with squeezing

Table top demonstration

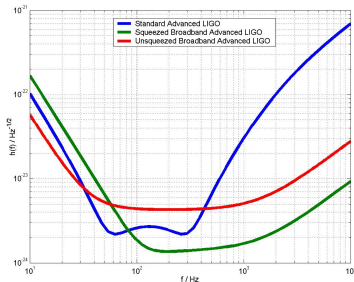
Pioneered by M. Xiao, L. Wu,
and H. J. Kimble

Phys. Rev. Lett. 59, 278-281 (1987)



Kirk McKenzie *et. al.*
Phys. Rev. Lett. 88, 231102 (2002)
The Australian National University

Projected
advanced LIGO sensitivity

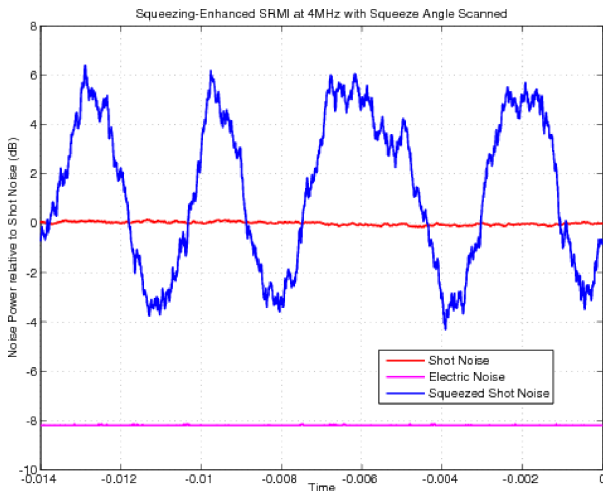


T. Corbitt *et.al.*
Phys. Rev. D 70, 022002
(2004) MIT

Requirements to squeezer

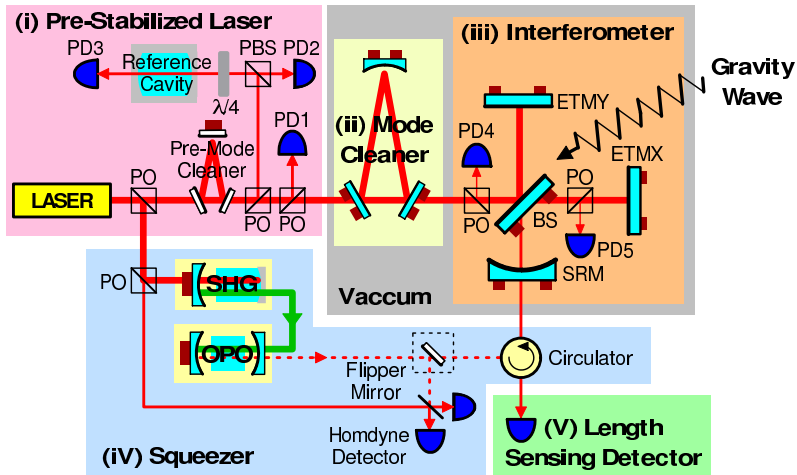
- squeezing at low frequency (10Hz - 10kHz)
- frequency dependent quadrature (angle) of squeezing
- stability (long time of operation)

Squeezing level vs time (unlocked)

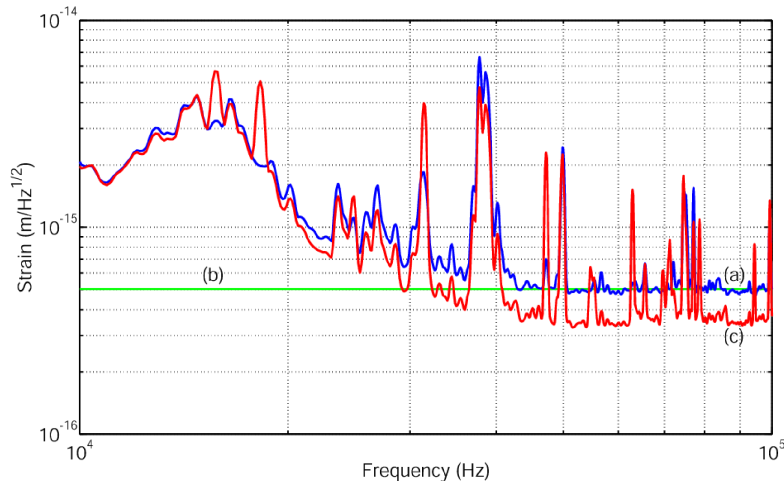


“A quantum-enhanced prototype gravitational-wave detector”,
Nature Physics, **4**, 472-476, (2008).

GW 40m detector and squeezer

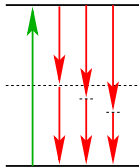


GW 40m detector with 4dB of squeezed vacuum

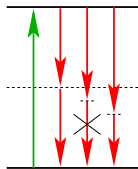
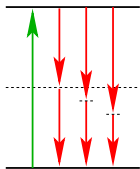


Signal to noise improvement by factor of 1.43

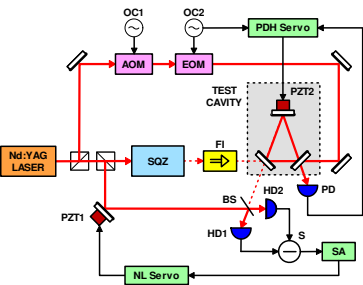
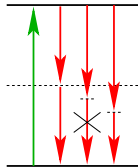
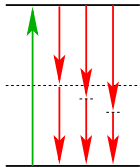
Cavity parameters with squeezing



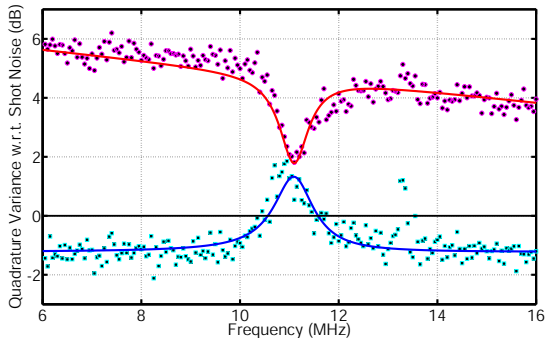
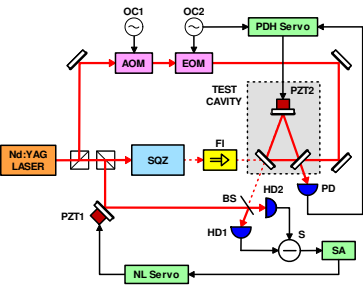
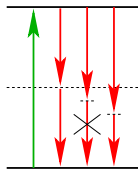
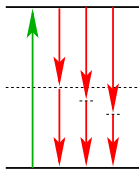
Cavity parameters with squeezing



Cavity parameters with squeezing

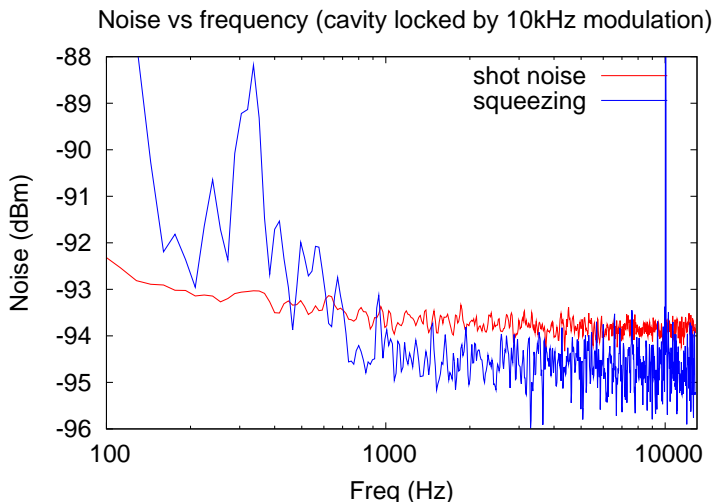


Cavity parameters with squeezing



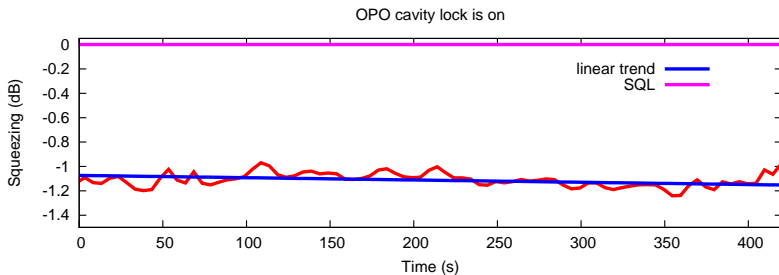
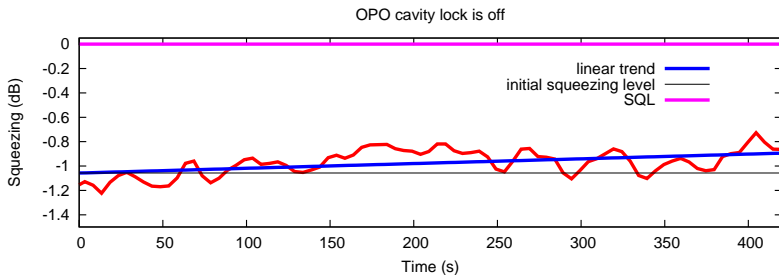
“Noninvasive measurements of cavity parameters by use of squeezed vacuum”, *Physical Review A*, **74**, 033817, (2006).

Low frequency squeezing with light free noise lock



“Quantum noise locking”,
J. Opt. B: Quantum Semiclass. Opt., 7, S421, (2005).

Squeezing level vs time (homodyne angle lock is on)



Summary for crystal squeezing

Pros

- mainstream: many different nonlinear crystals available
- so far the best squeezers
 - maximum squeezing value detected **11.5 dB at 1064 nm**
 - Moritz Mehmet, Henning Vahlbruch, Nico Lastzka, Karsten Danzmann¹, and Roman Schnabel, "Observation of squeezed states with strong photon-number oscillations", Phys. Rev. A **81**, 013814 (2010)
- well understood

Cons

- crystals have limited transparency window
- thus squeezing is hard to generate at visible wavelength
 - at 795 nm only 4-6 dB squeezing is reported
- this limits such squeezer for spectroscopy applications

Summary

- Squeezing is exiting
- many measurements benefit from squeezing
- there is still a lot to do