Blackbody Radiation

Experiment objectives: explore radiation from objects at certain temperatures, commonly known as "blackbody radiation"; make measurements testing the Stefan-Boltzmann law; measure the inverse-square law for thermal radiation.

Theory

A familiar observation to us is that dark-colored objects absorb more thermal radiation (from the sun, for example) than light-colored objects. You may have also observed that a good absorber of radiation is also a good emitter (like dark-colored seats in an automobile). Although we observe thermal radiation ("heat") mostly through our sense of touch, the range of energies at which the radiation is emitted can span the visible spectrum (thus we speak of high-temperature objects being "red hot" or "white hot"). For temperatures below about $600°C$, however, the radiation is emitted in the infrared, and we cannot see it with our eyes, although there are special detectors (like the one you will use in this lab) that can measure it.

An object which absorbs all radiation incident on it is known as an "ideal blackbody". In 1879 Josef Stefan found an empirical relationship between the power per unit area radiated by a blackbody and the temperature, which Ludwig Boltzmann derived theoretically a few years later. This relationship is the **Stefan-Boltzmann law:**

$$S = \sigma T^4 \tag{1}$$

where S is the radiated power per unit area (W/m^2), T is the temperature (in Kelvins), and $\sigma = 5.6703 \times 10^{-8} W/m^2K^4$ is the Stefan's constant.

Most hot, opaque objects can be approximated as blackbody emitters, but the most ideal blackbody is a closed volume (a cavity) with a very small hole in it. Any radiation entering the cavity is absorbed by the walls, and then is re-emitted out. Physicists first tried to calculate the spectral distribution of the radiation emitted from the ideal blackbody using *classical thermodynamics*. This method involved finding the number of modes of oscillation of the electromagnetic field in the cavity, with the energy per mode of oscillation given by kT. The classical theory gives the **Rayleigh-Jeans law:**

$$u(\lambda, T) = \frac{8\pi kT}{\lambda^4} \tag{2}$$

where $u(\lambda)(J/m^4)$ is the spectral radiance – energy radiated per unit area at a single wavelength λ. This law agrees with the experiment for radiation at long wavelengths (infrared), but predicts that $u(\lambda)$ should increase infinitely at short wavelengths. This is not observed experimentally (Thank heaven, or we would all be constantly bathed in ultraviolet light - a true ultraviolet catastrophe!). In reality, the peak of radiation distribution as a function of its wavelength depends on the blackbody temperature as described by **Wien's law:**

$$\lambda_{max}T = 2.898 \times 10^{-3} m \cdot K \tag{3}$$

and the spectral radiance approaches zero for short wavelengths.

The breakthrough came when Planck assumed that the energy of the oscillation modes can only take on discrete values rather than a continuous distribution of values, as in classical physics. With this assumption, Planck's law was derived:

$$u(\lambda, T) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1} \tag{4}$$

where c is the speed of light and $h = 6.626076 \times 10^{-34} J \cdot s$ is the Planck's constant. This proved to be the correct description.

Radiation sensor operation principle

Imagine a metal wire connected to a cold reservoir at one end and a hot reservoir at the other. Heat will flow between the ends of the wire, carried by the electrons in the conductor, which will tend to diffuse from the hot end to the cold end. Vibrations in the conductor's atomic lattice can also aid this process. This diffusion causes a potential difference between the two ends of the wire. The size of the potential difference depends on the temperature gradient and on details of the conductive material, but is typically in the few 10s of $\mu V/K$. A thermocouple, shown on the left, consists of two different conductive materials joined together at one end and connected to a voltmeter at the other end. The potential is, of course, the same at the joint, but the difference in material properties causes $\Delta V = V_1 - V_2 \neq 0$ between the separated ends. This ΔV is measured by the voltmeter and is proportional to ΔT. Your radiation sensor is a thermopile, simply a "pile" of thermocouples connected in series, as shown at the right. This is done to make the potential difference generated by the temperature gradient easier to detect.

Important: When using the thermal radiation sensor, make each reading quickly to keep the sensor from heating up. Use sheets of white isolating foam (with the silvered surface facing the lamp) to block the sensor between measurements.

Sensor calibration: To convert the radiation sensor readings V_S to the detected thermal radiation intensity S_{det} (power per unit area), you need to use the voltage-to-power conversion factor 22 V/W, and the area of the sensor $2mm \times 2mm$:

$$S_{det}[W/m^2] = \frac{V_S[V]}{22[V/W]} \cdot \frac{1}{4 \cdot 10^{-6}[m^2]}$$

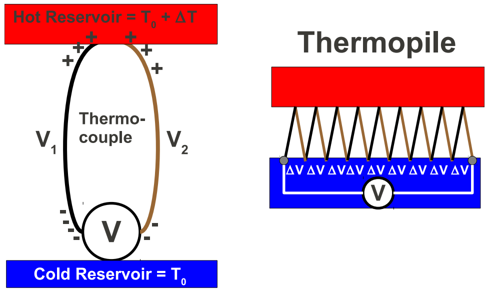

Figure 1: *Left*: thermocouple construction; *right*: thermopile - an array of thermocouples connected in series.

Test of the Stefan-Boltzmann Law

Equipment needed: Radiation sensor, multimeters, Stefan-Boltzmann Lamp, Power supply.

Before starting actual experiment take some time to have fun with the thermal radiation sensor. Can you detect your lab partner? What about people across the room? Point the sensor in different directions and see what objects affect the readings. **These exercises are fun, but you will also gain important intuition about various factors which may affect the accuracy of the measurements!**

1. **Before turning on the lamp**, measure the resistance of the filament of the Stefan-Boltzmann lamp at room temperature. Record the room temperature, visible on the wall thermostat.

2. To indirectly measure the temperature of the filament, we will use the known dependence of its resistance on the temperature, given in table shown in Table. 1. To ensure the accurate measurement, we will again use the four-point probe method (review the video on the course web site, if you need a refresher) by measuring the voltage drop across the lamp. VERY IMPORTANT: make all connections to the lamp when the power is off, and ask the instructor to check your connections before proceeding.

3. Place the thermal sensor at the same height as the filament, with the front face of the sensor approximately 5 cm away from the filament and fix their relative position. Make sure no other objects are viewed by the sensor other than the lamp.

4. Turn on the lamp power supply. Set the voltage, V, in steps of 1-2 volt from 1-6 volts. At each V, record the current running through the lamp and the voltage from the radiation sensor. Calculate the resistance of the lamp using Ohm's Law and determine the temperature T of the lamp from the table shown in Table 1. Don't forget to use Kelvin scale for the temperatures (conversion equation is $T[K] = T[^oC] + 273$).

5. Calculate the values of T^4 - these are going to be the x-values for the graph. Are they more or less equally distributed? If not (which is probably the case), estimate the big gaps, and measure additional points to fill them in.

R/R_{300K}	Temp K	Resistivity $\mu\Omega$cm	R/R_{300K}	Temp K	Resistivity $\mu\Omega$cm	R/R_{300K}	Temp K	Resistivity $\mu\Omega$cm
1.0	300	5.65	5.48	1200	30.98	16.29	3000	92.04
1.43	400	8.06	6.03	1300	34.08	16.95	3100	95.76
1.87	500	10.56	6.58	1400	37.19	17.62	3200	99.54
2.34	600	13.23	7.14	1500	40.36	18.28	3300	103.3
2.85	700	16.09	7.71	1600	43.55	18.97	3400	107.2
3.36	800	19.00	8.28	1700	46.78	19.66	3500	111.1
3.88	900	21.94	8.86	1800	50.05	26.35	3600	115.0
4.41	1000	24.93	9.44	1900	53.35			
4.95	1100	27.94	10.03	2000	56.67			

Table 1: Table of tungsten's resistance and resisitivity as a function of temperature. Here, R_{300K} is the resistance of tungsten at the temperature of 300 K. This dependence can be approximated by the following relationship between the filament temperature T (in Kelvin)s and the relative resistivity R/R_{300K}: $T = 292 \cdot (R/R_{300K})^{5/6}$.

In the lab report plot the reading from the radiation sensor (convert to W/m^2) (on the y axis) versus the temperature T^4 on the x axis. According to the Stefan-Boltzmann Law, the data should show a linear dependence, since according to Eq.(1) $S \propto T^4$. Fit the experimental data using a linear fit and its uncertainty. For an ideal blackbody we expect the slope to be equal to the Stephen constant $\sigma = 5.6703 \times 10^{-8} W/m^2K^4$. However, there exists no ideal black bodies. For real objects the Eq.(1) is modified, and written as:

$$S = \epsilon\sigma T^4, \tag{5}$$

where the coefficient ϵ is called *emissivity* and is defined as the ratio of the energy radiated from a material's surface to that radiated by a perfect blackbody at the same temperature. The values of ϵ vary from 0 to 1, with one corresponding to an ideal blackbody. All real materials have $\epsilon < 1$, although some come quite close to the ideal (for example, carbon black has $\epsilon = 0.95$). The emissivity of a tungsten wire varies from $\epsilon = 0.032$ (at $30°C$) to $\epsilon = 0.35$ (at $3300°C$).

Unfortunately, it is impossible to measure the exact value of emissivity from the experimental data, as the Stephan-Boltzman law describes the amount of radiation *emitted* by the object per unit area. To relate S to the amount of *detected* radiation S_{det} one needs to know the surface area of the filament - something we cannot measure without breaking the bulb (please don't!). All we can say is that the emitted and detected radiation intensity are proportional to one another. As a result, in this lab we are going to only verify the validity of functional dependence described by Eq. (5) by testing the linear dependence of the detected radiation on the filament temperature T^4.

To do that, fit the experimental data using the linear fit, find the proportionality coefficient and its uncertainty. To examine the quality of the fit more carefully, make a separate plot of the *residual* - the difference between the experimental points and the fit values. For a proper fit function, we expect the residuals to be randomly distributed around zero within the experimental measurement uncertainties. Analyze your results. Do the points seem to systematically differ from the fit line in a particular region? Can you think of a reason why that would be?

Test of the inverse-square law

Equipment needed: Radiation sensor, Stefan-Boltzmann lamp, multimeter, power supply, meter stick. A point source of radiation emits that radiation according to an inverse square

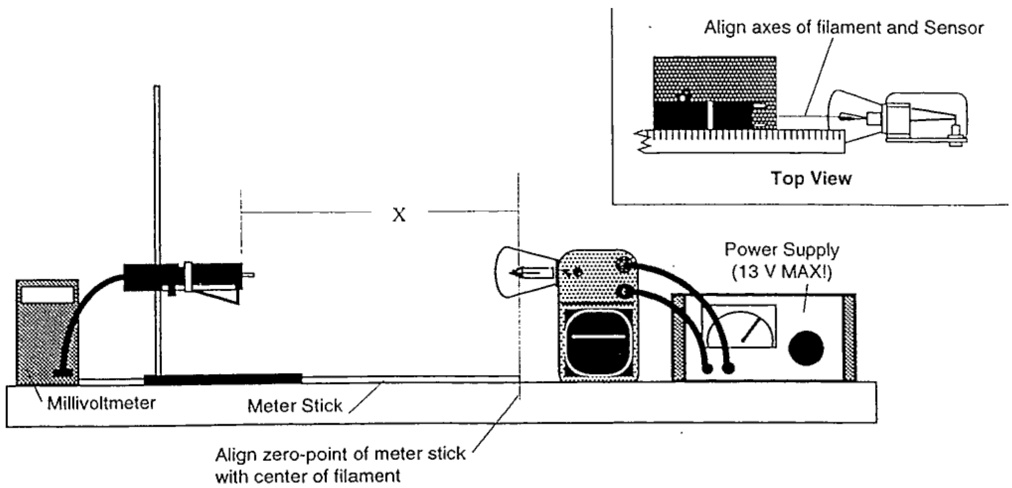

Figure 2: Inverse square law setup

law: that is, the intensity of the radiation in (W/m^2) is proportional to the inverse square of the distance from that source. This way, the intensity at the certain distance from the blackbody integrated over surface of the sphere of such radius is always constant. Mathematically, we expect the relationship between the detected intensity S_{det}, the total power of the radiation source P_0, and the distance to the point source r to be:

$$S_{det}(r) = \frac{P_0}{2\pi r^2} \tag{6}$$

1. Set up the equipment as shown in Fig. 2. Tape the meter stick to the table. Place the Stefan-Boltzmann lamp at one end, and the radiation sensor in direct line on the other side. The zero-point of the meter stick should align with the lamp filament (or, should it?). Adjust the height of the radiation sensor so it is equal to the height of the lamp. Align the system so that when you slide the sensor along the meter stick the sensor still aligns with the axis of the lamp. Connect the multimeter (reading millivolts) to the sensor and the lamp to the power supply.

2. With the **lamp off**, slide the sensor along the meter stick. Record the reading of the voltmeter at 10 cm intervals. Average these values to determine the ambient

level of thermal radiation. You will need to subtract this average value from your measurements with the lamp on.

3. Turn on the power supply to the lamp. Set the voltage to approximately 5-7 V. **Do not exceed 13 V!** Adjust the distance between the sensor and lamp from 2.5-100 cm and record the sensor reading. **Before the actual experiment think carefully about at what distances you want to take the measurements. Is taking them at constant intervals the optimal approach? At what distances would you expect the sensor reading change more rapidly?**

4. Make a plot of the corrected radiation measured from the lamp versus the distance from the lamp to the sensor x. Fit the data to

$$S_{det} = S_0 + \frac{C}{(x - x_0)^2}. \tag{7}$$

5. What are the values of S_0, C and x_0 (and, of course, their uncertainties)?

6. Compare Eqs.(6) and (7). What are the physical meanings of the parameters S_0, C and x_0. Do their values make sense, considering your experimental arrangements and measurements?

7. Can the lamp be considered a point source? If not, how could this affect your measurements?

Universal thermometer

Blackbody radiation gives us an ability to measure the temperature of remote objects. Have you ever asked yourself how do astronomer know the temperature of stars or other objects many light years away? The answer - by measuring the light they emit and analyzing its spectrum composition using the expressions for the blackbody radiation spectrum. Wein's law Eq.(3) links the wavelength at which the most radiation is emitted to the inverse of the object's temperature, thus the colder stars emit predominantly in red (hence the name "red giants"), while emission pick for hot young stars is shifted to the blue, making them emit in all visible spectrum.

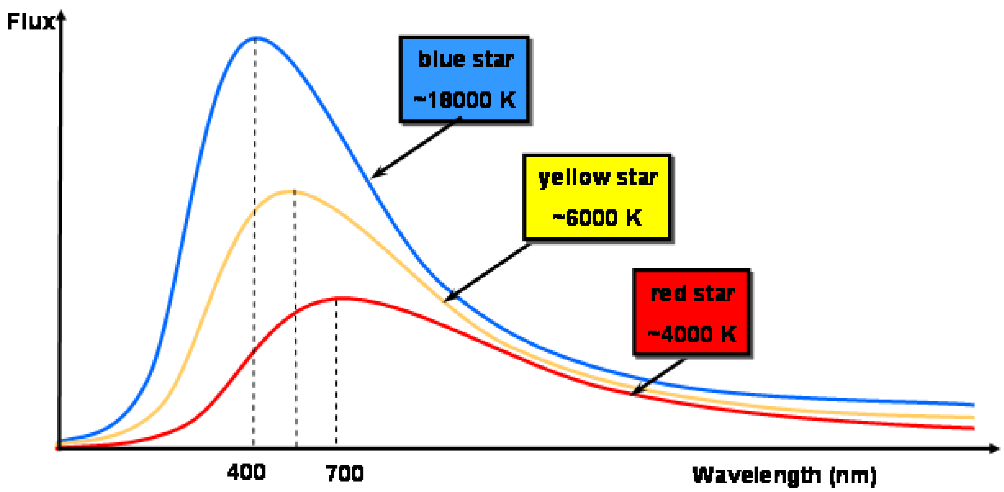

Figure 3: Black body radiation spectrum for objects with different temperatures.

The human bodies, of course, are much cooler than stars and emit in infrared range. This radiation is invisible to a human eye, but using proper detection methods it is possible to create thermal maps of the surroundings with accuracy better than 1/10th of a degree. Forward-looking infrared (FLIR) cameras have wide range of applications, from surveillance and military operations to building inspection and repairs, night-time navigation and hunting. As I write this in Fall 2020, in the middle of COVID19 pandemic, more and more locations use such infrared sensors to measure visitors' temperature at the building entrances or the check points in airports.