Electronics 1.
Kirchhoff’s laws,
Thévenin’s and Norton’s theorems

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Week 2
Kirchhoff’s Current Law

The algebraic sum of currents entering and exiting a node equals zero.

Convention (quite arbitrary): currents going into the nodes are positive, the ones which go out of the node are negative.

Kirchhoff’s Voltage Law

The algebraic sum of all voltage changes (aka voltage drops) in a loop equals zero.

Notes:

- Chose a direction along which you travel a network. If you go over a resistor and current runs the same way then voltage change is negative, otherwise it's positive.
- If you go over a voltage source from negative terminal to positive the voltage change is positive, otherwise negative.
Example

our goal is to find $I_1$, $I_2$, and $I_3$

We chose $V_A = 0$

For node $A$:

$$I_1 - I_2 - I_3 = 0 \quad (1)$$

We need 2 more independent equations.

For this we will go over 2 small loops as indicated by arrows.

$$V_{DC} + V_{CA} + V_{AD} = 0 \quad (2)$$

$$V_{AB} + V_{BC} + V_{CA} = 0 \quad (3)$$

Notice:

$V_{AB} = +E_1$, $V_{BC} = -R_2 \times I_2$, $V_{CA} = +R_3 \times I_3$,

$V_{DC} = +R_1 \times I_1$, $V_{AD} = -E_2$. 
Example (continued)

\[ I_1 - I_2 - I_3 = 0 \]

\[ V_{DC} + V_{CA} + V_{AD} = 0 \]

\[ V_{AB} + V_{BC} + V_{CA} = 0 \]

\[ R_1 \times I_1 + R_3 \times I_3 - E_2 = 0 \]

\[ E_1 - R_2 \times I_2 + R_3 \times I_3 = 0 \]
Maple as the math aid

\begin{align*}
\text{solve} & \{ I_1 - I_2 - I_3 = 0, \ E_1 - R_2 \cdot I_2 + R_3 \cdot I_3 = 0, \ R_1 \cdot I_1 + R_3 \cdot I_3 - E_2 = 0 \}, \ [I_1, I_2, I_3]\} \\
\begin{bmatrix}
I_1 &= \frac{\frac{R_3 \ E_1 + R_3 \ E_2 + R_2 \ E_2}{R_3 \ R_1 + R_1 \ R_2 + R_3 \ R_2}}{I_2 &= \frac{\frac{R_3 \ E_1 + R_3 \ E_2 + R_1 \ E_1}{R_3 \ R_1 + R_1 \ R_2 + R_3 \ R_2}} \ I_3 &= -\frac{\frac{R_1 \ E_1 - R_2 \ E_2}{R_3 \ R_1 + R_1 \ R_2 + R_3 \ R_2}}
\end{bmatrix}
\end{align*}
\[ V_{out} := \frac{V_{in} \cdot R_2}{(R_1 + R_2)} \cdot \frac{R_L}{R_L + \frac{R_1 \cdot R_2}{R_1 + R_2}} \]

\[ V_{in} := 10; R_1 := 10; R_2 := 1; \]

\[ plot(V_{out}, R_L = .1 .. 10) \]
Thévenin’s and Norton’s equivalent circuit theorems

Any combination of voltage sources, current sources and resistors with two terminals is electrically equivalent

Thévenin’s theorem
to a single voltage source $V_{TH}$ and a single series resistor $R_{TH}$ connected in series.

Norton’s theorem
to a single current source $I_{N}$ and a single series resistor $R_{N}$ connected in parallel.

Note above circuits are equivalent to each other when

$$R_{TH} = R_{N} \text{ and } I_{N} = \frac{V_{TH}}{R_{TH}}$$