Operational amplifiers (continued). Useful circuits with Op-Amp.

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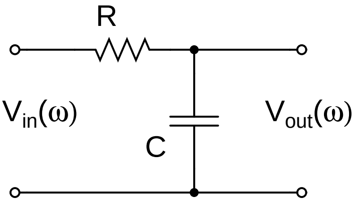
Week 10

Integrator



$$V_{out}(t) = V_c(t) = \frac{Q(t)}{C} = \int \frac{I(t)}{C} dt = \int \frac{V_{in}(t) - V_c(t)}{RC} dt$$

for $V_c \approx 0$

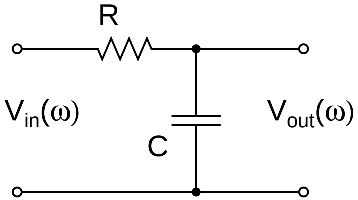
$$V_{out}(t) \approx \frac{1}{RC} \int V_{in}(t) dt$$

Integral representation in Fourier space

$$\begin{aligned}
F(t) &= \int_{-\infty}^{t} f(t')dt' = \int_{-\infty}^{t} dt' \int_{-\infty}^{+\infty} f(\omega)e^{i\omega t'} d\omega \\
&= \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{t} f(\omega)e^{i\omega t'} dt' \\
&= \int_{-\infty}^{+\infty} d\omega \left[\frac{f(\omega)}{i\omega} e^{i\omega t'} \Big|_{-\infty}^{t} \right] \\
&= \int_{-\infty}^{+\infty} d\omega \frac{f(\omega)}{i\omega} e^{i\omega t} = \int_{-\infty}^{+\infty} d\omega F(\omega) e^{i\omega t}
\end{aligned}$$

$$\begin{aligned}
F(t) &= \int_{-\infty}^{t} f(t')dt' \\
F(\omega) &= \frac{f(\omega)}{i\omega}
\end{aligned}$$

Integrator



$$V_{out}(\omega) = G(\omega) V_{in}(\omega) = \frac{Z_c}{R + Z_c} V_{in}(\omega) = \frac{1}{1 + i\omega RC} V_{in}(\omega)$$

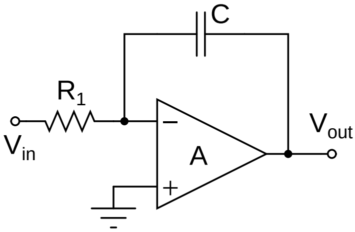
for $\omega \gg \omega_{3dB}$

$$V_{out}(\omega) \approx \frac{1}{RC} \frac{V_{in}(\omega)}{i\omega}$$

True Integrator / low-pass filter

We need to keep

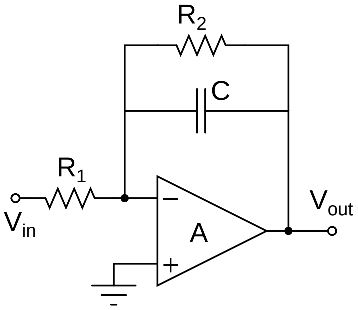
$$I = \frac{V_{in}}{R}$$



$$G(\omega) = -\frac{Z_c}{R_1} = -\frac{1}{i\omega R_1 C}$$

The only one problem remains: if any DC voltage is applied at input, output will reach a rail at power supply voltage.
This can be though as a lack of feedback since at DC capacitor blocks everything.

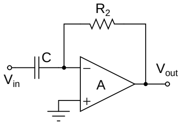
Low-pass filter / Integrator improved



$$G(\omega) = -\frac{Z_c \| R_2}{R_1} = -\frac{R_2}{R_1}\frac{1}{1 + i\omega R_2 C}$$

The only one problem remains: if any DC voltage is applied at input, output will reach a rail at power supply voltage.

Differentiator / high-pass filter



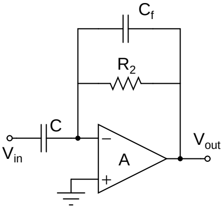
$$V_{in} = \frac{Q}{C} = \frac{1}{C}\int I dt \rightarrow I = C\frac{dV_{in}}{dt}$$
$$V_{out} = -IR_2$$

$$V_{out} = -R_2 C\frac{dV_{in}}{dt}$$

Fourier space

$$V_{out}(\omega) = -\frac{Z_{R_2}}{Z_c} = -i\omega R_2 C V_{in}(\omega) = \omega R_2 C V_{in}(\omega) e^{-i\frac{\pi}{2}}$$

Differentiator compensated



$$V_{out}(\omega) = -\frac{Z_{R_2} \| Z_f}{Z_c} V_{in}(\omega) = -\frac{i\omega R_2 C}{1 + i\omega R_2 C_f} V_{in}(\omega)$$

$\omega \ll \frac{1}{R_2 C_f}$

$$V_{out}(\omega) = -i\omega R_2 C V_{in}(\omega)$$

$\omega \gg \frac{1}{R_2 C_f}$

$$V_{out}(\omega) = -\frac{C}{C_f} V_{in}(\omega)$$

Thermistor linearization

$$R_{th}(T) = R_0 e^{-\gamma(T-T_0)}$$

Where $\gamma = 0.04$, $T_0 = 20$ °C, $R_0 = 10$ kOhm. Below circuit linearizes the output voltage vs temperature ($R = R_0$ as an example).