Zeeman effect:

Hydrogen-like atom in external magnetic field

$$H_B = -(\vec{\mu}_\ell + \vec{\mu}_s)\cdot \vec{B}_{ext}$$

now field acts not only electron itself (on μ_s) as in spin-orbital interaction but also on magnetic dipole moment made by the electron motion along the orbit μ_e

We already derived in SI units

$$\vec{\mu}_s = -\frac{g}{2}\frac{e}{m_e}\vec{S}$$

Similarly

$$\vec{\mu}_\ell = -\frac{e}{2m_e}\vec{L}$$

note the absense of g (gyro magnetic factor) in $\vec{\mu}_e$. $g \simeq 2$

$\Longrightarrow$

$$\boxed{\begin{gathered}\mu_s = -\frac{g}{2}\frac{e}{m_e c}\vec{S} \\ \mu_e = -\frac{1}{2}\frac{e}{m_e c}\vec{L} \\ \text{CGS units} \\ \text{or Gaussian units}\end{gathered}}$$

as in the text book eq 11.04

$$\boxed{\begin{aligned}\hat{H}_B &= \frac{e}{2mc}\left(\vec{L} + 2\vec{S}\right)\cdot\vec{B} \\ \vec{J} &= \vec{L} + \vec{S}\end{aligned}}$$

here we put $g = 2$

Our goal is to find energy correction due to $\hat{H}_B$ For weak field (see what is weak below)

Yet again we need to worry about degeneracy of $|n\,j\,\ell\,s\,m_j\rangle$ states since so far the relativistic correction and spin orbital correction lift degeneracy in j and ℓ but not in m_j

Luckily if we direct $\vec{B}$ along z-axis $\vec{B} = B\cdot\hat{k}$ then $[\hat{H}_B, \hat{J}_z] = 0 \iff |n, j, \ell, m_j\rangle$ are good and $[\hat{H}_B, \hat{L}^2] = 0$ state where degeneracy matrix V is diagonal

notize ! no s !

So we can work in "non-degenerate" formalism

$$\boxed{E^{(1)}_{nj\ell m_j} = \langle n, j, \ell, m_j | \hat{H}_B | n, j, \ell, m_j \rangle}$$

$$E^{(1)} = \frac{eB\hat{k}}{2mc} \langle n, j, \ell, m_j \,|\, \hat{L} + 2\hat{S} \,|\, n, j, \ell, m_j \rangle$$

← direction, not an operator

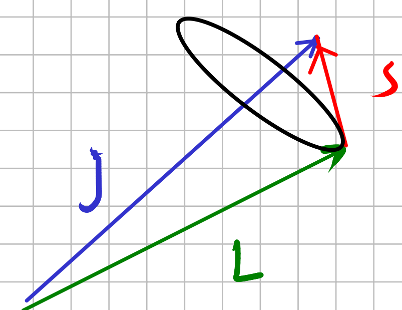
$$= \frac{e}{2mc} \langle \hat{L} + 2\hat{S} \rangle$$

short hand notation

recall $\hat{J} = \hat{L} + \hat{S} \implies \hat{L} + 2\hat{S} = \hat{J} + \hat{S}$

$$\langle \hat{L} + 2\hat{S} \rangle = \langle \hat{J} + \hat{S} \rangle$$

since $\hat{S}$ precess around conserved $\hat{J}$



$\hat{J}$ conserves but $\hat{L}$ and $\hat{S}$ precess around $\hat{J}$

we can say $\langle \hat{J} + \hat{S} \rangle = \langle \hat{J} + \hat{S}_{\parallel J} \rangle$

time averaged $\vec{S}_{ave} = \vec{S}_{\parallel J}$ along J

$$\vec{S}_{\parallel} = \frac{\vec{J} \cdot \vec{S}}{J} \cdot \frac{\vec{J}}{J} = \frac{\hat{J} \cdot \hat{S}}{J^2} \cdot \hat{J}$$

just a number

unit vector along J

component (projection) along $\vec{J}$

The rest is easy

$$\hat{L} = \hat{J} - \hat{S} \Rightarrow \hat{L}^2 = \hat{J}^2 + \hat{S}^2 - 2\hat{J}\hat{S}$$

$$\hat{J}\hat{S} = (\hat{J}^2 - \hat{L}^2 + \hat{S}^2) \cdot \frac{1}{2}$$

$$\Rightarrow \langle L + 2S \rangle = \langle \hat{J} + \hat{S}_{\parallel} \rangle = \langle \hat{J} + \frac{\hat{J}\cdot\hat{S}}{J^2} \cdot \hat{J} \rangle =$$

$$= \langle \left(1 + \frac{\hat{J}\cdot\hat{S}}{\underbrace{J^2}_{=j(j+1)}}\right) \hat{J} \rangle = \underbrace{\left[1 + \frac{1}{2} \frac{j(j+1) - \ell(\ell+1) + s(s+1)}{j(j+1)} \right]}_{\text{Landé g-factor}} \langle \underbrace{\hat{J}}_{\hbar m_j} \rangle$$

along $\hat{k}$ (z axis)

$$\Rightarrow \boxed{\begin{array}{l} E^{(1)}_{\text{Zeeman}} = \mu_B \, g_J \, B_{ext} \, m_j \\ \underbrace{\mu_B}_{\text{Bohr magneton}} = \dfrac{e\hbar}{2mc} = 5.788 \cdot 10^{-5} \dfrac{eV}{T} \end{array}}$$

We were a bit sloppy and mix classical vectors and operators averages

But the end result is precise. Alternative treatment in chapter 11.7

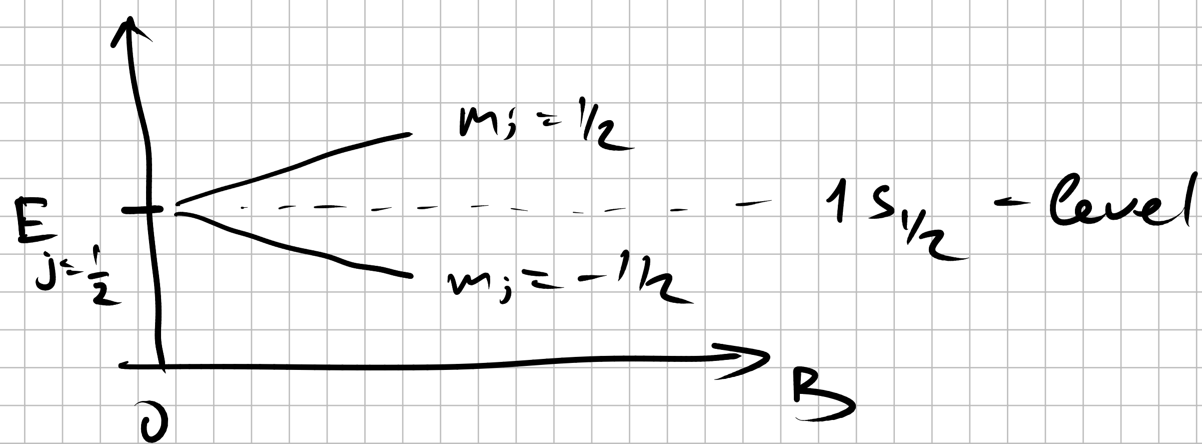
in SI system

$$\mu_B = \frac{e\hbar}{2m} \leftarrow \text{no speed of light}$$

The energy level splitting which is proportional to $\vec{B}$ and m_j is

the base of atomic magnetometers

and we are using it in my lab to measure pT level of $B \simeq 6 \cdot 10^{-17}\,eV$



So far we did perturbation treatment

i.e we assumed $(\mu_e + \mu_s) B_{ext}$ to be small.

What does it mean small? Compare to what?

Essentially we are saying $B_{ext} \ll B_{inner}$

same as in spin orbital coupling

$$\boxed{B_{inner}} = \frac{\overset{=1 \text{ for Hydrogen}}{Z}}{4\pi\varepsilon_0}\,\frac{e\cdot \overset{\sim\hbar}{L}}{m c^2 \underset{\sim a_0}{r}^3}$$

SI units

see our derivation of spin-orbital coupling

$$= 9\cdot10^{9}\frac{[Jm]}{[C^2]}\,\frac{1.6\cdot10^{-19}[C]\;1.05\cdot10^{-34}[J\cdot s]}{9.1\cdot10^{-31}\cdot[kg]\cdot\left(3\cdot10^{8}[\frac{m}{s}]\right)^2\cdot\left(5.29\cdot10^{-11}[m]\right)^3}$$

$$\simeq \frac{1.5\cdot10^{-43}}{1.2\cdot10^{-44}} \simeq \boxed{12.4\;[T]}$$

Tesla

So small B approximation is fine

for comparison: $B_{earth} \approx 50\mu T$, $B_{DC\ human\ made} \approx 17\ T$

Strong field Zeeman effect

$B_{ext} \gg B_{int}$

In this case S-O coupling is a perturbation on top of it and we focus on $\hat{H}_0$ states.

The good states (V is diagonal) would be $|n, \ell, m_\ell, s, m_s\rangle$

and $E^{(1)} = \langle \frac{e}{2mc} B_{ext} (L_z + 2S_z)\rangle$

we directed B along z

$$\Rightarrow \boxed{E^{(1)} = \frac{e}{2mc} B_{ext} (m_\ell + 2m_s) = \mu_B \cdot B_{ext} (m_\ell + 2m_s)}$$

Gaussian units

Intermediate B case is hard

see Griffiths ch. 7.4.3