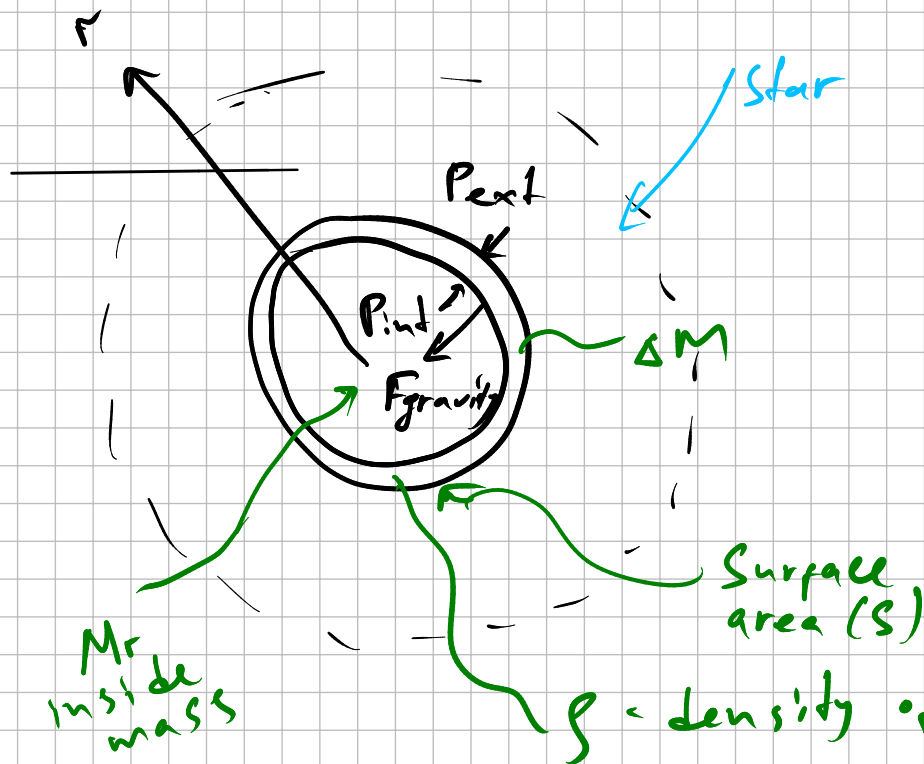


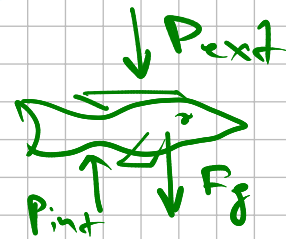
White dwarf stars

While a star sustains nuclear reaction it stays hot, so gravitational pull is counteracted by the light pressure and thermal pressure.

Once star cools the above mechanism is no good and we are left with the degeneracy pressure.



Hydrostatic Equilibrium condition
 ↑ same as for fish



$$\Delta M a = (P_{int} - P_{ext}) \cdot S - G \frac{\Delta M \cdot M_r}{r^2} = 0$$

$$0 = -\Delta P \cdot S - G \frac{\rho \cdot \Delta V}{r^2} M_r$$

ρ = density of star

$$-\Delta P \cdot \delta = G \rho \cdot \frac{\delta \cdot \delta r}{r^2} M_r \quad \leftarrow \frac{4\pi r^3}{3} \rho \quad \text{assuming } \rho = \text{const}$$

$$\frac{\Delta P}{\Delta r} \rightarrow \frac{dP}{dr} = -G \rho \frac{M_r}{r^2}$$

$$\frac{dP}{dr} = -G \rho^2 \frac{4\pi}{3} r \Rightarrow dP = -G \rho^2 \frac{4\pi}{3} r dr$$

center of
the star

$$P_c = P(r=0) = \int_R^0 dP = - \int_0^R G \rho^2 \frac{4\pi}{3} r dr + P(R) = 0$$

outside of
star radius R

$$P_c = G \rho^2 \frac{2\pi}{3} R^2$$

$$\rho = \frac{M_s}{V_s} = \frac{M_s}{\frac{4\pi}{3} R^3} \quad \leftarrow \text{mass of the star}$$

$$P_c = G \cdot \frac{M_s^2}{\left(\frac{4\pi}{3} R^3\right)^2} \cdot \frac{2\pi}{3} R^2 = G \frac{3 M_s^2}{8\pi}$$

$$P_c = \frac{3}{8\pi} G \frac{M_s^2}{R^4}$$

For star to be stable (non collapsing)

$$P_c \leq P_f = \frac{2}{3} \frac{\hbar^2 (3\pi^2 N)^{5/3}}{10\pi^2 m_f} V^{-5/3}$$

$$= \frac{2}{3} \frac{\hbar^2}{10\pi^2} \left(3\pi^2 \frac{N}{V} \right)^{5/3} \frac{1}{m_f} =$$

$$= \left(N = \frac{1}{2} \frac{M_s}{m_{\text{proton}}} \right) \frac{1}{m_{\text{electron}}} =$$

number of electrons
 $\frac{1}{2}$ - for every proton
 there is a neutron

$$= \frac{2}{3} \frac{\hbar^2}{10\pi^2} (3\pi^2)^{5/3} \left(\frac{1}{2} \frac{M_s}{m_p} \frac{1}{\frac{4\pi}{3} R^3} \right)^{5/3} \frac{1}{m_{\text{electron}}} \geq P_c = \frac{3}{8\pi} G \frac{M_s^2}{R^4}$$

$$\frac{2}{3} \frac{\hbar^2}{10\pi^2} (3\pi^2)^{5/3} \left(\frac{3}{8\pi} \frac{M_s}{m_p} \right)^{5/3} \frac{1}{R^5} \frac{1}{m_{\text{electron}}} \geq \frac{3}{8\pi} G \frac{M_s^2}{R^4}$$

$$R \leq \frac{16}{9} \frac{1}{10\pi} \frac{\hbar^2}{G} \frac{1}{M_s^{1/3}} (3\pi^2)^{5/3} \left(\frac{3}{8\pi} \frac{1}{m_p} \right)^{5/3} \frac{1}{m_e}$$

let's estimate maximum radius for a sun like star:

$$M_s = M_\odot = 2 \cdot 10^{30} \text{ kg}$$

$$m_p = 1.67 \cdot 10^{-27} \text{ kg}, \quad m_e = 9.1 \cdot 10^{-31} \text{ kg}$$

$$\hbar = 1.05 \cdot 10^{-34} \text{ J}\cdot\text{s}$$

$$G = 6.674 \cdot 10^{-11} \frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$$

$$R = 2.84 \cdot 10^6 \text{ m}$$

, compare to Earth radius

$$R_e = 6.4 \cdot 10^6 \text{ m}$$

white
dwarf
radius

electron degeneracy
supported radius

Note if star mass is larger \Rightarrow R is smaller but pressure is higher

at some high pressure

electrons are "pressed into" proton $\Rightarrow p + e = n$

and become neutron \Rightarrow neutron star with very different $R_n < R_e$