Quantum Mechanics foundation recap.
(QM 1 in two pages)

Physical system described by its state ψ

we learn about it by making measurement described by operator

$\hat{O}$: $\hat{O}|\psi\rangle = \sum_n O_n C_n |O_n\rangle$

probability amplitude

projection i.e. measured value

eigen state

of operator $\hat{O}$

important properties of operator and state set

1. $\langle O_n | O_m \rangle = \delta_{nm}$ ← orthonormal set

2. $\sum_n |O_n\rangle\langle O_n| = \hat{1}$ ← completnes

if there is another operator $\hat{Q}$, with eigen state $|q_m\rangle$ it can be decompose in super position of $|O_n\rangle$

3. $|q_m\rangle = \sum_n C_{nm} |O_n\rangle$, $C_{nm} = \langle O_n | q_m \rangle$

Evolution of the system in time described by Schrodinger eq.

$$\boxed{i\hbar \frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle}$$

$\hat{H}$ - Hamiltonian of the system represent its Energy

$$\hat{H}|E_n\rangle = E|E_n\rangle$$

if $\hat{H}$ does not depend on time

$$\boxed{|E_n(t)\rangle = e^{-\frac{iE_n}{\hbar}t}|E_n(0)\rangle}$$

i.e. it is trivial

we still need $|E_n(0)\rangle$!

to make it useful

$$\boxed{|\psi(0)\rangle = C_n|E_n\rangle \Rightarrow |\psi(t)\rangle = \sum c_n e^{-\frac{iE_n}{\hbar}}|E_n(0)\rangle}$$