Homework 05 Name: ______________________

Problem 1 (10 points)
In class we showed that identical particle can be either in symmetric (+) or anti-symmetric state (−). Thus for the case of two identical particles

$$\Psi(\vec{r}_1, \vec{r}_2)_\pm = A[\Psi_a(\vec{r}_1)\Psi_b(\vec{r}_2) \pm \Psi_a(\vec{r}_2)\Psi_b(\vec{r}_1)]$$

(a) Assuming that Ψ_a and Ψ_b form orthonormal set, find normalization constant A for the case $a \neq b$.
(b) What is the normalization constant A if $a = b$, i.e. particles are in the same state? Does it make sense to consider anti symmetric case for this sub problem? Why so?

Problem 2 (30 points, each item 5 points)
Disregard the spin coordinate in this problem.
Suppose we have N identical bosons of mass m in a one dimensional box of length $L = Na$ (the allowed x is between 0 and L) with the Hamiltonian

$$\hat{\mathbf{H}} = \sum_{i=1}^{N} \hat{\mathbf{p}}_i^2/(2m) + \lambda/2 \sum_{i \neq j} \delta(x_i - x_j)$$

(a) Set $\lambda = 0$. How should the ground state wave function look? Explicitly show it for $N = 3$. **Hint:** Google about Slater determinant (for fermions) and permanents (for bosons), I suggest to read about Slater determinant first.
(b) How should it look for N particles, how many summed products should it have? Do not express it explicitly this time, just call it $\Psi(x_1, x_2, ..., x_N)$.
(c) Now assume that λ is not zero but small. Find the ground state energy. Hint: recall perturbation methods.
(d) Replace all bosons with fermions. Assume that $\lambda = 0$. Explicitly, show the ground state wave function for $N = 3$.
(e) What is the ground state energy for $\lambda = 0$ and $N \gg 1$?
(f) Allow $\lambda \neq 0$ and $N \gg 1$, prove that 1st order correction is independent of λ. Hint: what should be the value of the fermions wave function if two arbitrary coordinates are the same?

Problem 3 (10 points)
In class we discussed white dwarfs supported by electrons degeneracy pressure. Once gravitational pull becomes larger than electron degeneracy pressure, the electrons are "pressed" into a proton, and together they form a neutron (which is also a fermion). Assume that all electrons and protons replaced with neutron. Also, assume that initially the star was made of heavy elements for which the number of neutrons equal to number of protons.
(a) Find the maximum mass of this neutron star which can be supported by the neutron degeneracy pressure. Express it in the solar mass ($M_\odot$) units. Important note: we need to assume that a neutron cannot be smaller than a certain size ($r_n = 0.8$ fm $= 0.8 \times 10^{-15}$ m). Beyond this we cannot treat them as neutrons.
(b) What is the maximum radius of such neutron star? Compare to the size of Williamsburg.

Problem 4 (10 points)
Numerical problem.
Let's revisit variational principle and hydrogen molecule ion. In class we derived the expression for ground energy upper estimate as a function of R/a (distance between hydrogen nuclei normalized to the Bohr radius).
(a) Make a plot of the upper bound for the ground level energy estimate vs R/a in the range of 0.1 – 10. Plot the energy in units of E_g (hydrogen ground energy). Use expression provided in the posted notes (in case if there were typos made in class). Do it for symmetric and anti-symmetric case.
(b) Calculate R/a which corresponds to the minimum of energy estimate with 3 digit precision.
(c) Calculate the best ground level energy estimate with 3 digit precision in the units of the hydrogen ground energy.