

## Homework 03

Name: \_\_\_\_\_

### Problem 1 (10 points)

In class we discussed the importance of the dimensionless fine-structure constant

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

which was used to calculate Hydrogen-like atom relativistic corrections to energy (see chapter 11.5).

Plug the fundamental constants in SI units and calculate  $\alpha$  for 3 extra digits.

Do you see a problem? What is missing in the definition? Why is it missing?

### Problem 2 (10 points)

Prove Hellmann–Feynman theorem, which states: if Hamiltonian depends on parameter  $\lambda$  and its eigen energy and state depend on  $\lambda$  as well, i.e. we have  $\hat{\mathbf{H}}|\psi_\lambda\rangle = E_\lambda|\psi_\lambda\rangle$  then

$$\frac{dE}{d\lambda} = \langle \psi_\lambda | \frac{d\hat{\mathbf{H}}}{d\lambda} | \psi_\lambda \rangle$$

Hint:

$$\frac{dE}{d\lambda} = \frac{d}{d\lambda} \langle \psi_\lambda | \hat{\mathbf{H}} | \psi_\lambda \rangle$$

### Problem 3 (10 points)

Using Hellmann–Feynman theorem, calculate

$$\left\langle n, l, m \left| \frac{1}{r} \right| n, l, m \right\rangle$$

for Hydrogen-like atom.

Hint:

$$\begin{aligned} \hat{\mathbf{H}}_r &= -\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{\hbar^2 l(l+1)}{2\mu r^2} - Z\frac{e^2}{r} \\ \hat{\mathbf{H}}_r |R_{nl}\rangle &= -\frac{\mu e^4 Z^2}{2\hbar^2 (N+l)^2} |R_{nl}\rangle \end{aligned} \quad (1)$$

### Problem 4 (10 points)

Calculate the first-order correction to the ground and first excited states of a one dimensional harmonic oscillator due to the relativistic correction to its kinetic energy. The mass of the oscillator is  $m$ , and its natural frequency is  $\omega$ .

What would be an analog of “fine structure constant” in this system?