Homework 01 Name: _____

This homework has a theme: 1D or 2D square wells with perturbations.

Problem 1 (10 points)

Find exact solutions (eigenstates and eigenvalues) of the one dimensional Schrodinger equation for a particle with mass (m) moving in the infinite square well potential $U_0(x)$

$$U_0(x) = \begin{cases} 0 & \text{if} |x| \leq a; \\ \infty & \text{if} |x| > a \end{cases}$$

Make sure that you normalize wave functions to 1.

Problem 2 (10 points)

The potential energy from the problem 1 got a perturbation term

$$U_1(x) = \begin{cases} \lambda V_o & \text{if} |x| \leq a; \\ 0 & \text{if} |x| > a \end{cases}$$

- Find the first order corrections for every energy state.
- Express condition for λ which still satisfies the small perturbation approximation.

Problem 3 (10 points)

Solve problem 2 precisely (it can be done without the perturbation approximation) and show that energy values match to the 1st order of λ.

Problem 4 (10 points)

Consider a particle with mass (m) moving in one-dimensional potential

$$U(x) = m\omega^2 x^2/2 + \lambda x^4 \tag{1}$$

Calculate the energy of the ground state to the first order in λ.

Hint: review the harmonic oscillator problem from QM I. You do not need to derive 0th order energy states expressions, use them right away.

Problem 5 (10 points)

A particle with mass (m) moves in two-dimensional infinite square well potential described as

$$U_0(x) = \begin{cases} 0 & \text{if} |x| \leq a \text{ and } |y| \leq a; \\ \infty & \text{otherwise} \end{cases}$$

It is perturbed by

$$U_1(x) = \begin{cases} \lambda V_o xy/a^2 & \text{if} |x| \leq a \text{ and } |y| \leq a; \\ 0 & \text{otherwise} \end{cases}$$

Find expression for the ground state energy and its non vanishing perturbative correction.