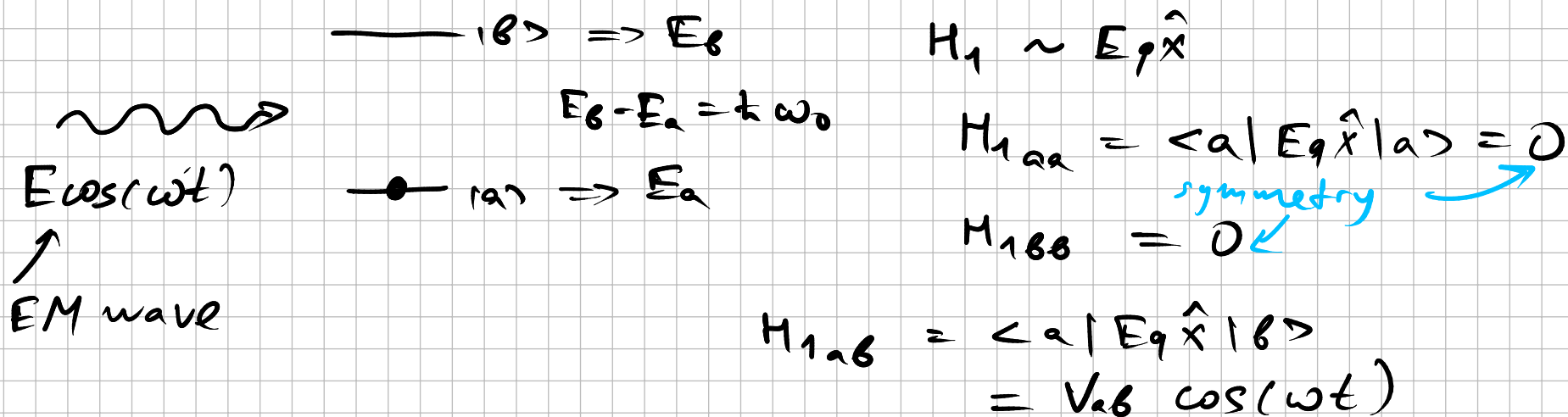


# Two level atom - with periodic excitation

"There are no two level atoms and Rubidium is not one of them"

Bill Phillips (Noble prize winner)

Nevertheless it is useful approximation



$|b\rangle \Rightarrow E_b$   
 $E_b - E_a = \hbar \omega_0$   
 $|a\rangle \Rightarrow E_a$

$H_1 \sim E_0 \hat{x}$   
 $H_{1aa} = \langle a | E_0 \hat{x} | a \rangle = 0$   
 $H_{1bb} = 0$   
 $H_{1ab} = \langle a | E_0 \hat{x} | b \rangle = V_{ab} \cos(\omega t)$

$E \cos(\omega t)$   
EM wave

*symmetry*

Last time we derived

$$c_f(t) = \delta_{fi} - \frac{i}{\hbar} \int_0^t e^{i(E_f - E_i)t'/\hbar} \langle f | H_1 | i \rangle dt'$$

$|c_a(0)|^2 = 1$  i.e. atoms initially in 'a' state

$$|c_b(0)|^2 = 0$$

$$c_b(t) = -\frac{i}{\hbar} \int_0^t e^{i\omega_0 t'} V_{ab} \cos(\omega t') dt'$$

$$= -\frac{i V_{ab}}{\hbar} \int_0^t e^{i\omega_0 t'} \frac{(e^{i\omega t'} + e^{-i\omega t'})}{2} dt'$$

$$= -\frac{i V_{ab}}{2\hbar} \left[ \frac{e^{i(\omega_0 + \omega)t'}}{\omega_0 + \omega} + \frac{e^{i(\omega_0 - \omega)t'}}{\omega_0 - \omega} \right]_0^t$$

$$= -\frac{i V_{ab}}{2\hbar} \left[ \frac{e^{i(\omega_0 + \omega)t} - 1}{\omega_0 + \omega} + \frac{e^{i(\omega_0 - \omega)t} - 1}{\omega_0 - \omega} \right]$$

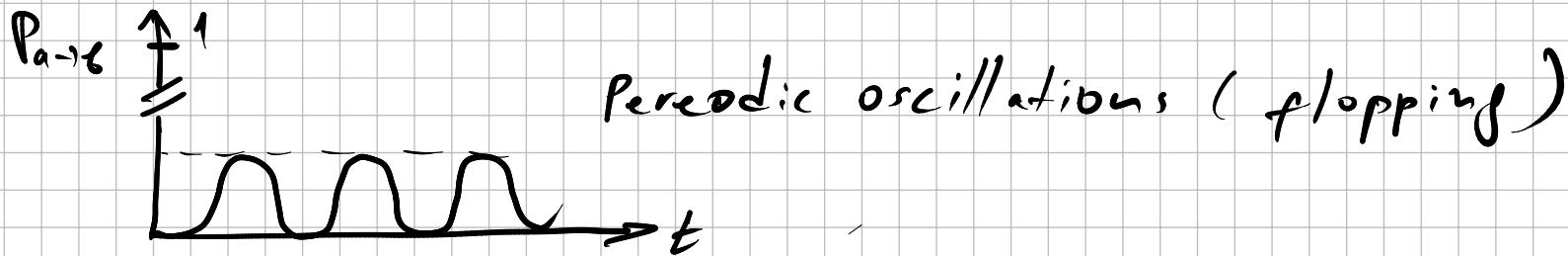
for optical transitions  $\omega_0 \approx 10^{14}$  Hz, RF  $\omega_0 \approx 10^{10}$  Hz

$$c_b(t) = -\frac{i V_{ab}}{2\hbar} \frac{e^{i(\omega_0 - \omega)t} - 1}{\omega_0 - \omega} \approx -\frac{i V_{ab}}{\hbar} (i) \frac{e^{i\frac{(\omega_0 - \omega)t}{2}} - e^{-i\frac{(\omega_0 - \omega)t}{2}}}{2(\omega_0 - \omega)} e^{+i\frac{(\omega_0 + \omega)t}{2}}$$

$$C_b(t) = \frac{V_{ab}}{\hbar} \frac{\sin\left(\frac{\omega_0 - \omega}{2}t\right)}{\omega_0 - \omega} e^{i\left(\frac{\omega_0 - \omega}{2}\right)t}$$

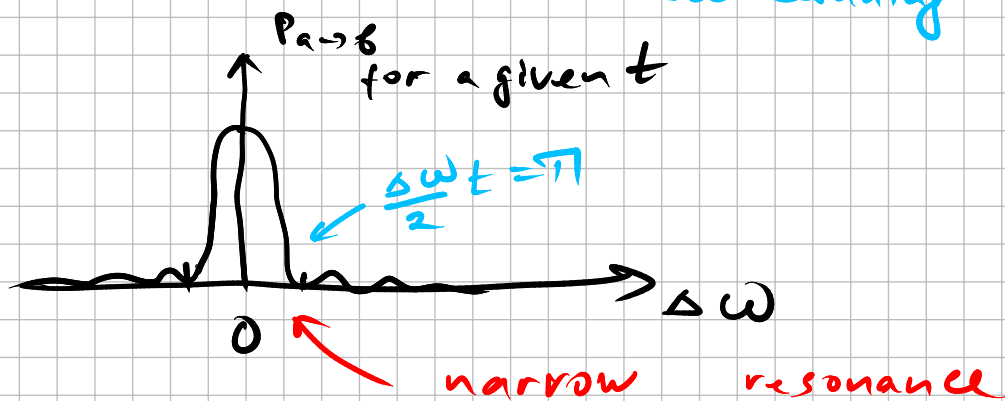
$$\Rightarrow P_{a \rightarrow b}(t) = |C_b(t)|^2 = \frac{|V_{ab}|^2}{\hbar^2} \frac{\sin^2\left(\frac{\omega_0 - \omega}{2}t\right)}{(\omega_0 - \omega)^2}$$

probability to move from level 'a' to 'b'



$$P_{a \rightarrow b \text{ max}} = \left| \frac{V_{ab}}{\hbar (\omega_0 - \omega)} \right|^2 = \left| \frac{V_{ab}}{\hbar \Delta\omega} \right|^2 \quad \text{at } \frac{\omega_0 - \omega}{2} t = \frac{\pi}{2} + n\pi$$

$\Delta\omega$  detuning



one issue

$$P_{a \rightarrow b \text{ max}} \rightarrow \infty \quad \Delta\omega \rightarrow 0$$

1st it cannot be  $> 1$   
 $\Rightarrow$  2nd we out of perturbation regime!

## Two level atom exact (almost) solution

Recall that during time perturbation derivation we got the exact expressions for  $C_n(t)$

$$\frac{d}{dt} C_f(t) = -\frac{i}{\hbar} \sum_n C_n(t) e^{i \frac{(E_f^{(0)} - E_n^{(0)})t}{\hbar}} \langle \psi_f | H_1(t) | \psi_n \rangle$$

for the two level case and  $H_1$  which has only off diagonal elements

$$H_{1ab} = V_{ab} \cos(\omega t)$$

we have

$$\begin{cases} \frac{d}{dt} C_a(t) = -\frac{i}{\hbar} C_b(t) e^{-i\omega_0 t} V_{ab} \cos(\omega t) \\ \frac{d}{dt} C_b(t) = -\frac{i}{\hbar} C_a(t) e^{+i\omega_0 t} V_{ab}^* \cos(\omega t) \end{cases}$$

$$\left\{ \begin{aligned} \dot{C}_a(t) &= -\frac{i}{\hbar} C_b(t) e^{-i\omega_0 t} \frac{e^{i\omega t} + e^{-i\omega t}}{2} V_{ab} \\ \dot{C}_b(t) &= -\frac{i}{\hbar} C_a(t) e^{+i\omega_0 t} \frac{e^{-i\omega t} + e^{i\omega t}}{2} V_{ab}^* \end{aligned} \right.$$

$e^{-i(\omega_0 + \omega)t}$  fast oscillations  
 $e^{+i(\omega_0 + \omega)t}$  we drop them

$$H_1 = V_{ab} \cos(\omega t) \rightarrow V_{ab} e^{-i\omega t}$$

Rotating wave approximation

Why do we drop them?

they would force  $C_a, C_b$  oscillate with frequency  $(\omega_0 + \omega) \sim 10^{14}$  which is HUGE

there are no detectors which are that fast!  
 So, who cares about such terms.

$$\left\{ \begin{aligned} \dot{C}_a(t) &= -\frac{i}{2\hbar} C_b(t) e^{-i(\omega_0 - \omega)t} V_{ab} \\ \dot{C}_b(t) &= -\frac{i}{2\hbar} C_a(t) e^{i(\omega_0 - \omega)t} V_{ab}^* \end{aligned} \right.$$

There is a general solution  
 but we will do the case of zero detuning  
 $\Delta\omega = 0 \iff \omega_0 = \omega$

$$\begin{cases} \dot{c}_a = -\frac{i}{2\hbar} c_b V_{ab} \\ \dot{c}_b = -\frac{i}{2\hbar} c_a V_{ba} \end{cases}$$

$$\begin{cases} \ddot{c}_a = -\frac{i}{2\hbar} \dot{c}_b V_{ab} = -\frac{V_{ab} \cdot V_{ba}}{\hbar^2} c_a = -\frac{|V_{ab}|^2}{\hbar^2} c_a \\ \ddot{c}_b = -\frac{|V_{ab}|^2}{\hbar^2} c_b \end{cases}$$

well we know the solution  $C = A \cos \Omega t + B \sin \Omega t$

where

$$\Omega = \frac{|V_{ab}|}{2\hbar}, \text{ Rabi frequency}$$

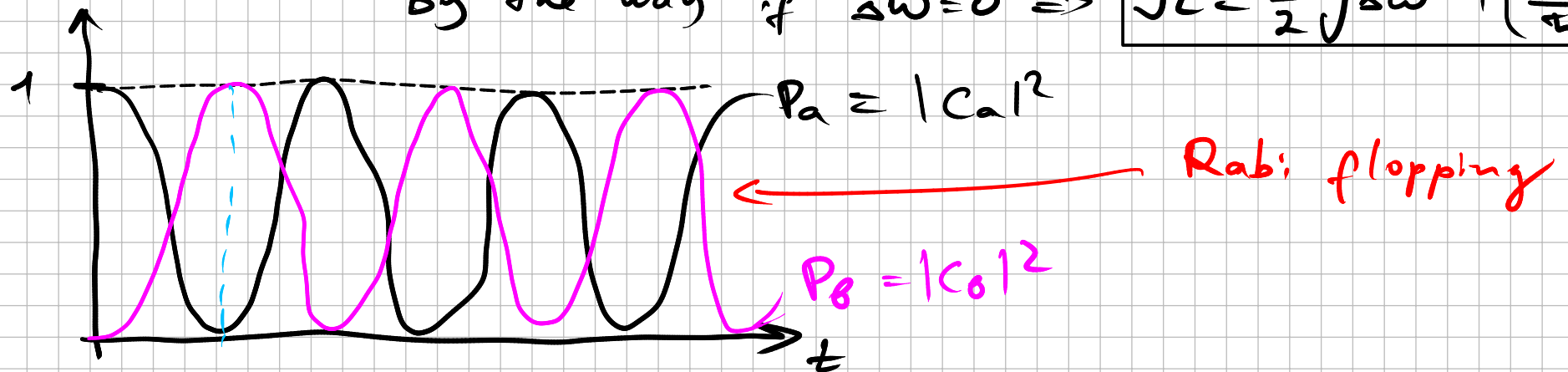
let's use initial condition

$$\begin{aligned} c_a(0) &= 1 \leftarrow \text{everything} \\ c_b(0) &= 0 \quad \text{at ground} \\ & \quad \text{level} \end{aligned}$$

$$C_a = \cos(\Omega t), \quad C_b = \sin(\Omega t)$$

by the way if  $\Delta\omega = 0 \Rightarrow$

$$\Omega = \frac{1}{2} \sqrt{\Delta\omega^2 + \left(\frac{|V_{ab}|}{\hbar}\right)^2}$$

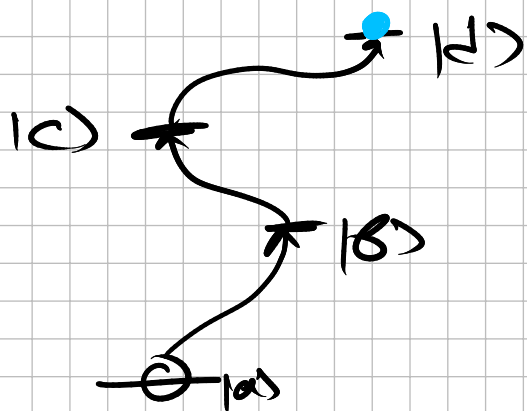


$$\pi: \Omega t = \pi$$

so called  $\pi$  pulse time

which brings all population from 'a' to 'b'

Now we can do quantum control



i.e. populate any level even  
if we cannot generate  
 $\omega = \omega_b - \omega_a$

HW: show that for  
 $\Delta\omega = 0$  and  $t \rightarrow 0$   
perturbation theory gives the same  
answer