

Zeeeman effect:

Hydrogen-like atom in external magnetic field

$$M_B = - (\vec{\mu}_L + \vec{\mu}_S) \cdot \vec{B}_{\text{ext}}$$

now field acts not only electron itself (on μ_S) as in spin-orbital interaction but also on magnetic dipole moment made by the electron motion along the orbit μ_L

We already derived in SI units

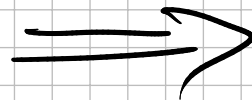
$$\vec{\mu}_S = -\frac{g e}{2 m_e} \vec{S}$$

Similarly

$$\vec{\mu}_L = -\frac{e}{2 m_e} \vec{L}$$

note the absence of g (gyro magnetic factor)

in $\vec{\mu}_L$. $g \approx 2$



$$\mu_S = -\frac{g e}{2 m_e c} \vec{S}$$

$$\mu_L = -\frac{1}{2} \frac{e}{m_e c} \vec{L}$$

CGS units
or Gaussian units

as in the text book
eq 11.04

$$\hat{H}_B = \frac{e}{2mc} (\vec{L} + 2\vec{S}) \cdot \vec{B}$$

$$\vec{J} = \vec{L} + \vec{S}$$

here we put $g=2$

Our goal is to find energy correction due to \hat{H}_B

For weak field (see what is weak below)

Yet again we need to worry about degeneracy of $|n, j, l, s, m_j\rangle$ states since so far the relativistic correction and spin orbital correction lift degeneracy in j and l but not in m_j

Luckily if we direct \vec{B} along z -axis $\vec{B} = B \cdot \hat{k}$ then $[\hat{H}_B, \hat{J}_z] = 0 \iff |n, j, l, m_j\rangle$ are good and $[\hat{H}_B, \hat{L}^2] = 0$ state where degeneracy matrix V is diagonal

notice!
no s !

So we can work in "non-degenerate" formalism

$$E_{n, j, l, m_j}^{(1)} = \langle n, j, l, m_j | \hat{H}_B | n, j, l, m_j \rangle$$

$$E^{(1)} = \frac{eB\hat{k}}{2mc} \langle n, j, l, m_j | \hat{L} + 2\hat{S} | n, j, l, m_j \rangle$$

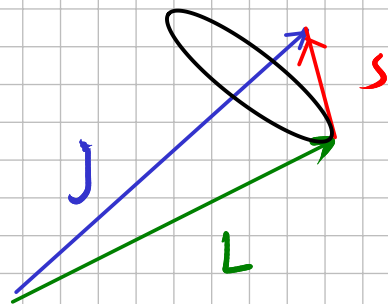
← direction, not an operator

$$= \frac{e}{2mc} \langle \hat{L} + 2\hat{S} \rangle \quad \text{short hand notation}$$

recall $\hat{J} = \hat{L} + \hat{S} \Rightarrow \hat{L} + 2\hat{S} = \hat{J} + \hat{S}$

$$\langle \hat{L} + 2\hat{S} \rangle = \langle \hat{J} + \hat{S} \rangle$$

since \hat{S} precesses around conserved \hat{J}



\hat{J} conserves but \hat{L} and \hat{S} precess around \hat{J}

we can say $\langle \hat{J} + \hat{S} \rangle = \langle \hat{J} + \hat{S}_{||J} \rangle$ time averaged $\hat{S}_{ave} = \hat{S}_{||J}$
 along J

$$\hat{S}_{||} = \left(\frac{\hat{J} \cdot \hat{S}}{|\hat{J}|} \right) \cdot \left(\frac{\hat{J}}{|\hat{J}|} \right) = \left(\frac{\hat{J} \cdot \hat{S}}{|\hat{J}|^2} \right) \cdot \hat{J}$$

unit vector along J

component (projection) along J

just a number

The rest is easy

$$\hat{L} = \hat{J} - \hat{S} \Rightarrow \hat{L}^2 = \hat{J}^2 + \hat{S}^2 - 2\hat{J}\hat{S}$$

$$\hat{J}\hat{S} = (\hat{J}^2 - \hat{L}^2 + \hat{S}^2) \cdot \frac{1}{2}$$

$$\begin{aligned} \Rightarrow \langle L + 2S \rangle &= \langle \hat{J} + \hat{S} \rangle = \langle \hat{J} + \frac{\hat{J} \cdot \hat{S}}{J^2} \cdot \hat{J} \rangle = \\ &= \langle \left(1 + \frac{\hat{J} \cdot \hat{S}}{J^2} \right) \hat{J} \rangle = \underbrace{\left[1 + \frac{1}{2} \frac{j(j+1) - l(l+1) + s(s+1)}{j(j+1)} \right]}_{\text{Landé } g\text{-factor}} \langle \hat{J} \rangle \end{aligned}$$

+ m_j
along \hat{K} (z axis)

$$\Rightarrow E_{\text{zeeman}}^{(1)} = \mu_B g_J B_{\text{ext}} m_j$$

$$\mu_B = \frac{e\hbar}{2mc} = 5.788 \cdot 10^{-5} \frac{\text{eV}}{\text{T}}$$

Bohr magneton

We were a bit sloppy and mix classical vectors and operators averages

in SI system

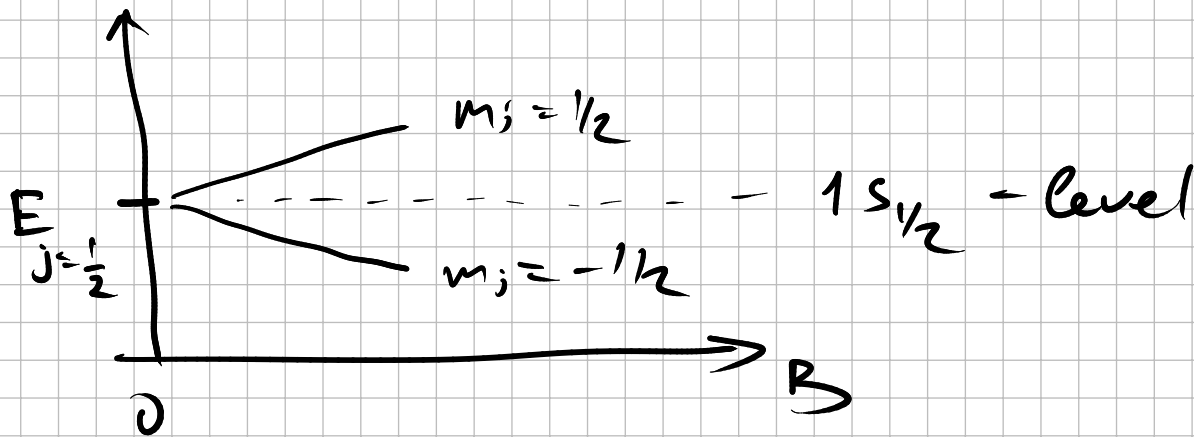
$$\mu_B = \frac{e\hbar}{2m} \leftarrow \text{no speed of light}$$

But the end result is precise. Alternative treatment in chapter 11.7

The energy level splitting which is proportional to \vec{B} and m_j is

the base of atomic magnetometers

and we are using it in my lab to measure PT level of $\vec{B} \approx 6 \cdot 10^{-14}$ eV



So far we did perturbation treatment
 i.e. we assumed $(m_e + m_s) B_{ext}$ to be small.

What does it mean small? Compare to what?

Essentially we are saying $B_{ext} \ll B_{inner}$

same as in
 spin orbital
 coupling

$$B_{inner} = \frac{Z}{4\pi\epsilon_0} \frac{e \cdot L}{m c r^3} \quad \begin{matrix} = 1 \text{ for Hydrogen} \\ \sim \hbar \end{matrix}$$

see our derivation
 of spin-orbital coupling

SI units

$$= 9 \cdot 10^9 \frac{[Nm]}{[C^2]} \frac{1.6 \cdot 10^{-19} [C] \cdot 1.05 \cdot 10^{-34} [Js]}{9.1 \cdot 10^{-31} [kg] \cdot (3 \cdot 10^8 \frac{[m]}{s})^2 \cdot (5.29 \cdot 10^{-11} [m])^3}$$

$$\approx \frac{1.5 \cdot 10^{-43}}{1.2 \cdot 10^{-44}} = 12.4 [T]$$

Tesla

So small
 B approximation
 is fine

for comparison: $B_{earth} = 50 \mu T$, $B_{DC \text{ human made}} = 17 T$

Strong field Zeeman effect $B_{ext} \gg B_{int}$

In this case S-O coupling is a perturbation on top of \hat{H}_0 and we focus on \hat{H}_0 states.

The good states (V is diagonal)

would be $|n, \ell, m_\ell, s, m_s\rangle$

$$\text{and } E^{(1)} = \left\langle \frac{e}{2mc} B_{ext} (L_z + 2S_z) \right\rangle$$

we directed
B along z

$$\Rightarrow E^{(1)} = \frac{e}{2mc} B_{ext} (m_\ell + 2m_s) = \mu_B \cdot B_{ext} (m_\ell + 2m_s)$$

↑
Gaussian
units

Intermediate B case is hard

see Griffiths ch. 7.4.3