So far we ignored the spin of electron and magnetic dipole moment ($\vec{\mu}$) assosiated with it.
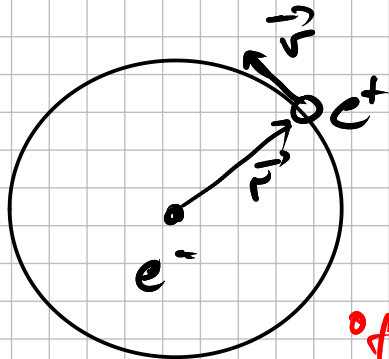
If we have suprounding magnetic field $\vec{B}$ we would have energy term of such interaction

$$H = -\vec{\mu} \cdot \vec{B} \quad \text{simple classical physics.}$$

↑do not confuse with reduced mass (μ)

If $\vec{B}$ external, we have Zeeman effect
(we will cover it soon)

But there is also internal $\vec{B}$ field generated by apparent motion of proton in the electron rest frame.

Biot-Savart law (Phys 102)

$$|\vec{B}| = \frac{\mu_0 I}{2r} = \frac{\mu_0 q \frac{e}{T}}{2r} = \frac{\mu_0 e Z}{2r} \frac{v}{2\pi r} \cdot \frac{r}{r} \cdot \frac{m_e}{m_e}$$

permitivity of free space (yet another μ ¨)

$$B = \frac{\mu_0 Z e}{4\pi r^3 m_e} \cdot \boxed{m_e r v}$$

$$= L = \text{angular momentum}$$
$$\vec{L} = m_e \, \vec{r} \times \vec{v}$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 Z e}{4\pi m_e} \frac{1}{r^3} \vec{L}$$

recall that $\sqrt{\varepsilon_0 \mu_0} = 1/c$

$$\vec{B} = \frac{1}{4\pi\varepsilon_0} \frac{Z e}{m_e c^2} \frac{1}{r^3} \vec{L} \quad \Rightarrow \quad \boxed{\vec{B} = \frac{1}{4\pi\varepsilon_0} \frac{Z e}{m_e c^2} \frac{1}{r^3} \vec{L}}$$

what about magnetic dipole momentum of electron?

$$\vec{M} = A \cdot I \sim \boxed{\pi r^2} \cdot \left(\frac{-e}{t}\right) \sim \pi r^2 \cdot (-e) \cdot \boxed{\frac{v}{2\pi r} \cdot \frac{m_e}{m_e} \cdot \frac{\sim}{r}} \sim S$$

area

radius of electron!

area of electron is tricky

actually $M = g \cdot \frac{e}{2m_e} \vec{S}$
where $g = 2.002\cdots$

$$\vec{M} \sim \frac{-e}{2m} \vec{S}$$

classical estimate $\Rightarrow$ the real value $\boxed{\vec{M} = - \frac{e}{m_e} \cdot \vec{S}}$

Combining all together :

$$\boxed{\hat{H}_1 = -\vec{\mu}\cdot\vec{B} = +\frac{1}{2}\frac{Ze^2}{m_e^2 c^2}\frac{1}{r^3}\hat{S}\,\hat{L}}$$

note $\frac{1}{4\pi\epsilon_0}$ is gone! We moved to CGS units
$\frac{e^2}{4\pi\epsilon_0} \rightarrow e^2$ Gaussian units

Our perturbation due to

Spin-Orbit coupling

Thomas precession since electron is accelerating (to move along orbit) and we cannot use it as inertial frame

So if we used only classical approach we would get the almost correct expression.

$$\frac{g}{2} = 1.001...$$

Time to think about states of electron.
so far we used $|n, \ell, m\rangle$
but we should include spin into consideration

$$|n, \ell, m\rangle \otimes |s, m_s\rangle$$

analogous to ℓ in

$\hat{H}_0$ has no clue about spin i.e. comute this way we ignored it.

We need to find good basis which
states also eigen states of $\hat{M}_1$ or we have
to deal with degenerate $\tilde{V}$ matrix.
If we do it right $\tilde{V}$ will be diagonal and
our life is "easy".

Let's look at total momentum
operator $\hat{J} = \hat{L} + \hat{S}$

its eigen function are constructed from

$|\ell, m\rangle \otimes |s, m_s\rangle \rightarrow |j, \ell, s, m_j\rangle$,

so they are eigen function of $\hat{M}_0, \hat{S}^2, \hat{L}^2, \hat{J}^2, \hat{J}_z$

Old $|s, m_s\rangle$
$|\ell, m_\ell\rangle$

are bad basis
because $\hat{S}, \hat{L}$
do not conserve
$[\hat{M}_1, \hat{L}] \neq 0$
$[\hat{M}_1, \hat{S}] \neq 0$

$j = |\ell - s| \dots |\ell + s|$
$m_j = -J, \dots 0, \dots, J$

Note

$$J^2 = (\hat{L} + \hat{S})^2 = (L^2 + S^2 + 2\hat{L}\hat{S})$$

$$\hat{L}\hat{S} = \frac{1}{2}(\hat{J}^2 - \hat{L}^2 - \hat{S}^2)$$

$$\hat{L}\hat{S}|j, \ell, s, m_j\rangle = \frac{1}{2}\hbar^2(j(j+1) - \ell(\ell+1) - s(s+1))$$

$\frac{1}{2}$ for electron

Reminder about angular momentum / spin addition

$$\hat{J} = + \left\{ \begin{array}{l} \hat{L} \; ; \; \hat{L}_z \to m_\ell \\ \hat{S} \; : \; \hat{S}_z \to m_z \end{array} \right\} + \implies \begin{array}{l} \text{maximum} \quad m = \ell + s \\ \text{minimum} \quad m = |\ell - s| \end{array}$$
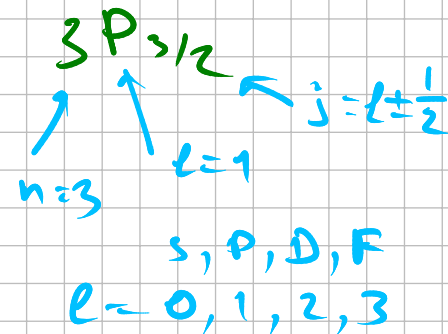
Think about vector addition

Generally speaking, there are multiple way to get the same m_j out of different m_ℓ, m_s so we expect such $|j, m_j\rangle$ to be super-positions of same ℓ, s but different $m_\ell + m_s = m_j$

Wonderful properties

$$\hat{J}^2 |j, \ell, s, m_j\rangle = \hbar^2 j(j+1) |j, \ell, s, m_j\rangle$$
$$\hat{L}^2 |j, \ell, s, m_j\rangle = \hbar^2 \ell(\ell+1) |j, \ell, s, m_j\rangle$$
$$\hat{S}^2 |\text{---} \qquad \rangle = \hbar^2 s(s+1) |j, \ell, s, m_j\rangle$$
$$\hat{J}_z |\text{----} \qquad \rangle = \hbar m_z |j, \ell, s, m_j\rangle$$

spectroscopic level notation

$3P_{3/2}$

$n=3$ $\ell=1$ $j=\ell \pm \frac{1}{2}$

S, P, D, F
$\ell = 0, 1, 2, 3$

$$\Rightarrow \hat{L}\hat{S}|j,\ell,s,m_j\rangle = \tfrac{1}{2}\hbar^2\left(\underset{\hat{J}^2}{j(j+1)} - \underset{\hat{L}^2}{\ell(\ell+1)} - \underset{\hat{S}^2}{\tfrac{3}{4}}\right)|j,\ell,s,m_j\rangle$$

$$j = \ell \pm \tfrac{1}{2}$$

electron spin

$$\langle n,j,\ell,s,m_j|\frac{\hat{L}\hat{S}}{r^3}|n,j,\ell,s,m_j\rangle =$$

note ↗ same as $|n,\ell,m\rangle = |R_{n\ell}\rangle|Y_{\ell m}\rangle$

$$= \langle n,j,\ell,s,m_j|\frac{1}{r^3}|n,j,\ell,s,m_j\rangle \cdot \tfrac{1}{2}\hbar^2\left(j(j+1) - \ell(\ell+1) - \tfrac{3}{4}\right)$$

this one is tricky

$$= \frac{Z^3}{\ell(\ell+\tfrac{1}{2})(\ell+1)\,n^3\,a_0^3}$$

$$\begin{cases} (\ell+\tfrac{1}{2})(\ell+\tfrac{1}{2}+1) - \ell(\ell+1) - \tfrac{3}{4} \\ (\ell-\tfrac{1}{2})(\ell-\tfrac{1}{2}+1) - \ell(\ell+1) - \tfrac{3}{4} \end{cases}$$

$$\begin{cases} \ell^2+\ell+\tfrac{1}{4}+\ell+\tfrac{1}{2} -\ell^2-\ell - \tfrac{3}{4} \\ \ell^2-\ell+\tfrac{1}{4}+\ell-\tfrac{1}{2} -\ell^2-\ell - \tfrac{3}{4} \end{cases}$$

$$\begin{cases} \ell & , \; j = \ell+\tfrac{1}{2} \\ -(\ell+1) & , \; j = \ell-\tfrac{1}{2} \end{cases}$$

all together

$$E_{s-o}^{(1)} = \frac{m_e c^2 Z^4 \alpha^4}{4 n^3 \ell(\ell+\frac{1}{2})(\ell+1)} \times \begin{cases} \ell & , \quad j = \ell + \frac{1}{2} \\ -(\ell+1) & , \quad j = \ell - \frac{1}{2} \end{cases}$$

spin orbit
correction

Note $E_{s-o}^{(1)} = 0$ for $\ell = 0$! since $\vec{L} = 0$

Remember sodium dublet in physics 251?

$3P_{3/2}$

$3P_{1/2}$

$\lambda = 5890 \text{Å}$ $\lambda = 5896 \text{Å}$

$3S_{1/2}$

n

ℓ j
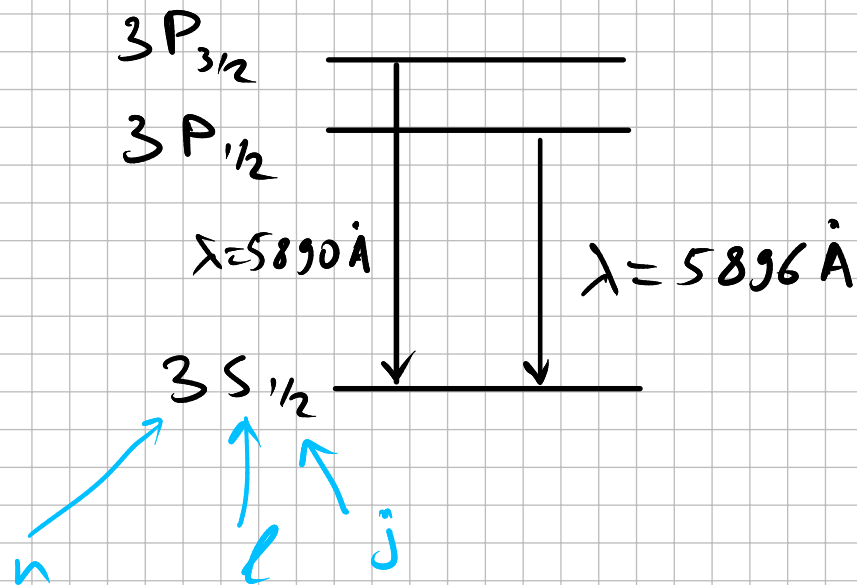
It is interesting that $E_{s-o}^{(1)} \sim m_e c^2 \alpha^4$

HW: same as relativistic correction
so we need to add them together