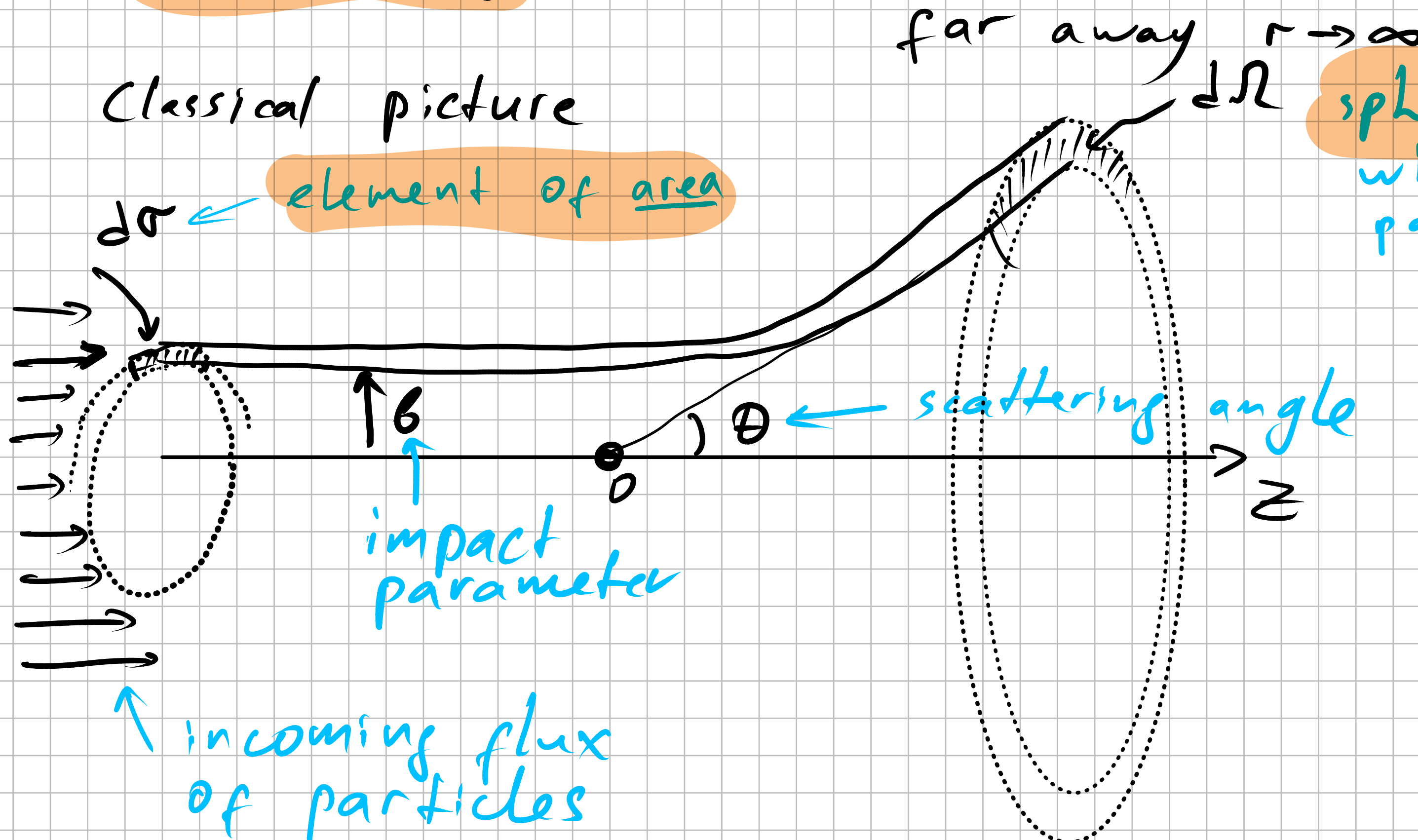
Scattering

Goal: find size and shape of the object

Classical picture



We assume no dependance on φ !

$d\sigma$ and $d\Omega$ seem to be linked, i.e. proportional

$$d\sigma = D(\theta)\, d\Omega$$

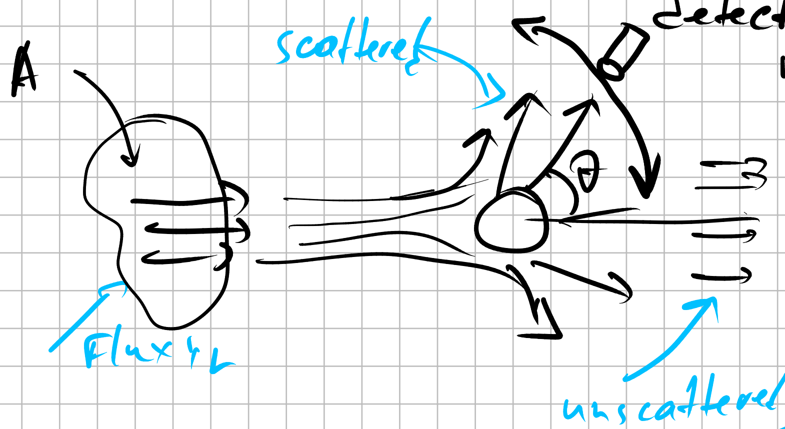
Canonical name: Differential cross-section often labeled as $\frac{d\sigma}{d\Omega}$

$$d\sigma = \frac{d\sigma}{d\Omega} d\Omega$$

good luck to understand this form of equation

Experimentally we usually have access to

$D(\theta)\, d\Omega$



detector of fixed size $d\Omega$
moving along variable θ

\# of "clicks" in detector
proportional to

$$\text{Flux} \cdot d\Omega = \text{Flux} \cdot D(\theta)\, d\Omega = dN_{\substack{\text{detected}\\ \text{scattered}}}$$

$$\text{Flux} = \frac{\#\ \text{particle}}{\text{unit area} \cdot \text{unit time}}$$

$$\int d\sigma = \int D(\theta)\, d\Omega$$

$$\sigma = \int D(\theta)\, d\Omega = \int \frac{dN_{\text{scattered}}}{\text{Flux}} = \frac{N_{\text{scattered}}}{\text{Flux}}$$

$$N_{in} = A \cdot \text{Flux}$$

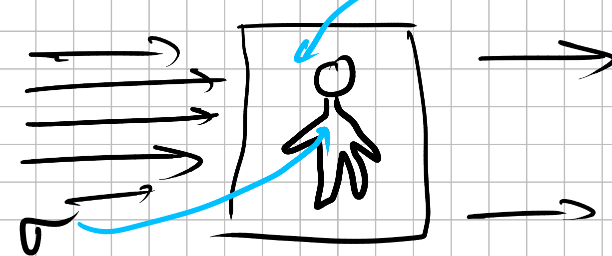
↑ # particles which we send in

$$\boxed{\frac{N_{sc}}{N_{in}} = \frac{\sigma}{A}}$$

size of the "shadow"

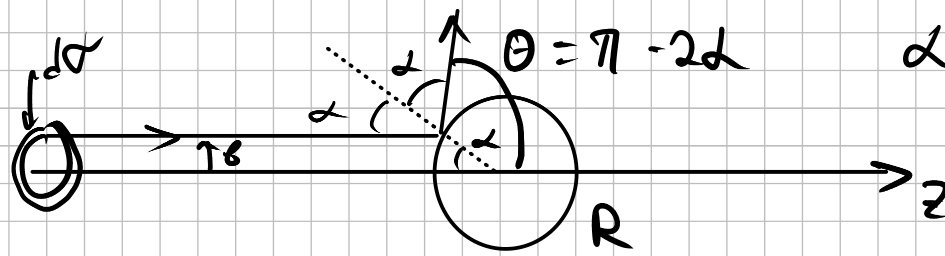
missing particles from $\theta = 0$

A



$D(\theta)$ carry information about object and interaction forces

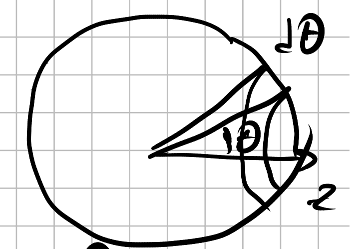
Example: scattering on a sphere



$\theta = \pi - 2\alpha$

if $b > R \Rightarrow \theta = 0$

$\alpha = \arcsin \frac{b}{R}$



$$d\sigma = 2\pi b \, db = D(\theta) \cdot d\Omega = D(\theta) \cdot \underbrace{\frac{2\pi r \cdot r \sin\theta}{r^2} d\theta}_{d\Omega}$$

$$D(\theta) = \frac{d\sigma}{d\Omega} = \frac{2\pi b \, db}{2\pi \sin\theta \, d\theta} = \frac{b}{\sin\theta} \cdot \frac{db}{d\theta}$$

differentiate LHS and RHS

$$d\left(\theta = \pi - 2\alpha = \pi - 2 \arcsin \frac{b}{R} \right)$$

$$d\theta = -2 \frac{1}{\sqrt{1 - \underbrace{\left(\frac{b}{R}\right)}_{\sin\alpha}^2}} \cdot \frac{db}{R} = -2 \left| \frac{1}{\cos \underbrace{\alpha}_{\frac{\pi}{2} - \frac{\theta}{2}}} \right| \frac{db}{R} = -2 \left| \frac{1}{\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right)} \right| \frac{db}{R}$$

$$= -2 \left| \frac{1}{\sin \frac{\theta}{2}} \right| \frac{db}{R}$$

$$\frac{db}{d\theta} = -\frac{R}{2}\left|\sin\frac{\theta}{2}\right| \Rightarrow$$

$R \cdot \sin\alpha = R\cos\theta/2$

$$D(\theta) = \frac{b}{\sin\theta}\cdot\frac{db}{d\theta} = -\frac{b}{\sin\theta}\,\frac{R}{2}\left|\sin(\theta/2)\right| = -R^2\frac{\overbrace{\cos(\tfrac{\theta}{2})\sin(\tfrac{\theta}{2})}^{\frac{1}{2}\sin\theta}}{2\sin\theta}$$

$$\boxed{D(\theta) = -\frac{R^2}{4}}$$

Finaly recall that

$$\sigma = \int D(\theta)\,d\Omega = -\int_0^{\pi}\frac{R^2}{4}\,2\pi\sin\theta\,d\theta = \boxed{\pi R^2 = \sigma}$$

as expected
this "shadow" area
of a sphere

Important fact: Rezerford scattering of a charged particle on a coulomb potential $\sim \frac{Qq}{r}$ gives $\sigma = \infty$

$$D(\theta) \sim \frac{1}{\sin^4(\theta/2)}$$

It was experimentally confirmed with exact $D(\theta)$

Usually $\theta_{scattering}$ grows as impact parameter b decreases.

Think about $b \to \infty$ far away from collision $\theta \to 0$

$b \to 0$ we guarantee to have a hit

$\Rightarrow$ θ grows