No cloning theorem

Suppose we have a "magical" operator ($\hat{c}$) which takes any state to be "erased" $|e\rangle$ and convert it to a given clone of the input:

(*) $$|\varphi\rangle|e\rangle \xrightarrow{\hat{c}} |\varphi\rangle|\varphi\rangle$$

template ↑ ↑ clone

(**) $$|\psi\rangle|e\rangle \xrightarrow{\hat{c}} |\psi\rangle|\psi\rangle$$

Note that we need to keep input or template

Now lets calculate

$$\alpha(*) + \beta(**) \xrightarrow{\hat{c}} \alpha|\varphi\rangle|\varphi\rangle + \beta|\psi\rangle|\psi\rangle \quad (1)$$

|| from other hand we do

$$(\alpha|\varphi\rangle + \beta|\psi\rangle)|e\rangle \xrightarrow{\hat{c}} (\alpha|\varphi\rangle + \beta|\psi\rangle)(\alpha|\varphi\rangle + \beta|\psi\rangle)$$

$$= \alpha^2|\varphi\rangle|\varphi\rangle + \beta^2|\psi\rangle|\psi\rangle + \alpha\cdot\beta(|\varphi\rangle|\psi\rangle + |\psi\rangle|\varphi\rangle) \quad (2)$$

Clearly RHS of eq. 1 is not the same as RHS of eq. 2, so we arrive to the contradiction !

$\Longrightarrow$ Cloning is impossible in a <u>general</u> case.

This has an important implication:

<u>security</u> of <u>quantum</u> <u>communication</u>, since you cannot evesdrop / remove states from a channel, clone, and do measurements on clones undetected.

Note that teleporting is possible and was demonstrated with photons:

$$|\varphi\rangle|e\rangle \xrightarrow{\hat{T}} |0\rangle|\varphi\rangle$$

undetermined