Homework 02 Name: ____________________

Problem 1 (10 points)

A particle with mass (m) moves in two-dimensional infinite square potential, i.e.

$$U_0(x, y) = \begin{cases} 0 & \text{if} |x| \le a \text{ and } |y| \le a; \\ \infty & \text{otherwise} \end{cases}$$

This potential is perturbed with $U_1(x, y) = u_1 \delta(x)\delta(y)$. Find corrections to the **4th** lowest energy state. Note that system is degenerate for any but the ground state.

Is degeneracy of the 4th level fully lifter? Can you provide some arguments why it might be the case without even doing full set of calculations?

Also find the new set of eigen states (wave functions) of this previously degenerate level.

Problem 2 (10 points)

Prove the following property of the parity operator

$$\hat{\mathbf{\Pi}} |n, l, m\rangle = (-1)^l |n, l, m\rangle$$

Here, $|n, l, m\rangle$ labeled states of hydrogen like atom, i.e. an electron moving in the central potential ($\sim 1/r$).

Reminder: parity operator (or inversion operator) act as $\hat{\mathbf{\Pi}} |\psi(\vec{\mathbf{r}})\rangle = |\psi(-\vec{\mathbf{r}})\rangle$

Hint: express $|n, l, m\rangle$ via radial and angular functions $R_{nl}(r)Y_{lm}(\theta, \phi)$. Observe that $\vec{\mathbf{r}} \to -\vec{\mathbf{r}}$ corresponds to $(\theta, \phi) \to (\pi - \theta, \phi + \pi)$. Do it for several n, l, m to spot the pattern. Also, note that Y_{lm} consist of polynomials of $\cos(\theta)$ and $\sin(\theta)$.

Problem 3 (10 points)

In class, we saw that the Stark effect 1st order correction to the hydrogen ground state is 0. **Estimate** the 2nd order correction to the ground state.

Hint 1: It will involve calculation sum like $\sum |\langle 1| \hat{\mathbf{H}}_1 |n\rangle|^2/(E_1 - E_n)$. To estimate the upper bound for correction, replace the all denominators in the sum with the smallest possible $E_1 - E_m$ and pull out of the sum.

Hint 2: $\langle n| \hat{\mathbf{o}} \sum_\psi |\psi\rangle\langle\psi| \hat{\mathbf{o}} |n\rangle = \langle n| \hat{\mathbf{o}}\hat{\mathbf{I}}\hat{\mathbf{o}} |n\rangle$, where $\{\psi\}$ is the complete set of states.

Problem 4 (10 points)

We will use result of this problem during derivation of non relativistic corrections to Hydrogen like atoms.

Let's say we have a degenerate state with energy E and a corresponding set of eigen functions (states) $\{|\psi_a\rangle, |\psi_b\rangle, |\psi_c\rangle, |\psi_d\rangle, \cdots\}$. Prove that if perturbation Hamiltonian $\hat{\mathbf{H}}_1$ is diagonal in this set basis, i.e. $\langle\psi_\alpha| \hat{\mathbf{H}}_1 |\psi_\beta\rangle \sim \delta_{\alpha\beta}$, then the 1st order correction to energy level is given in similar to non degenerate case. I.e. by expression like,

$$E^{(1)} = \langle\psi_\alpha| \hat{\mathbf{H}}_1 |\psi_\alpha\rangle$$

where $\alpha = \{a, b, c, d, \cdots\}$.

Hint: express H_1 matrix, and see how the characteristic equation looks.