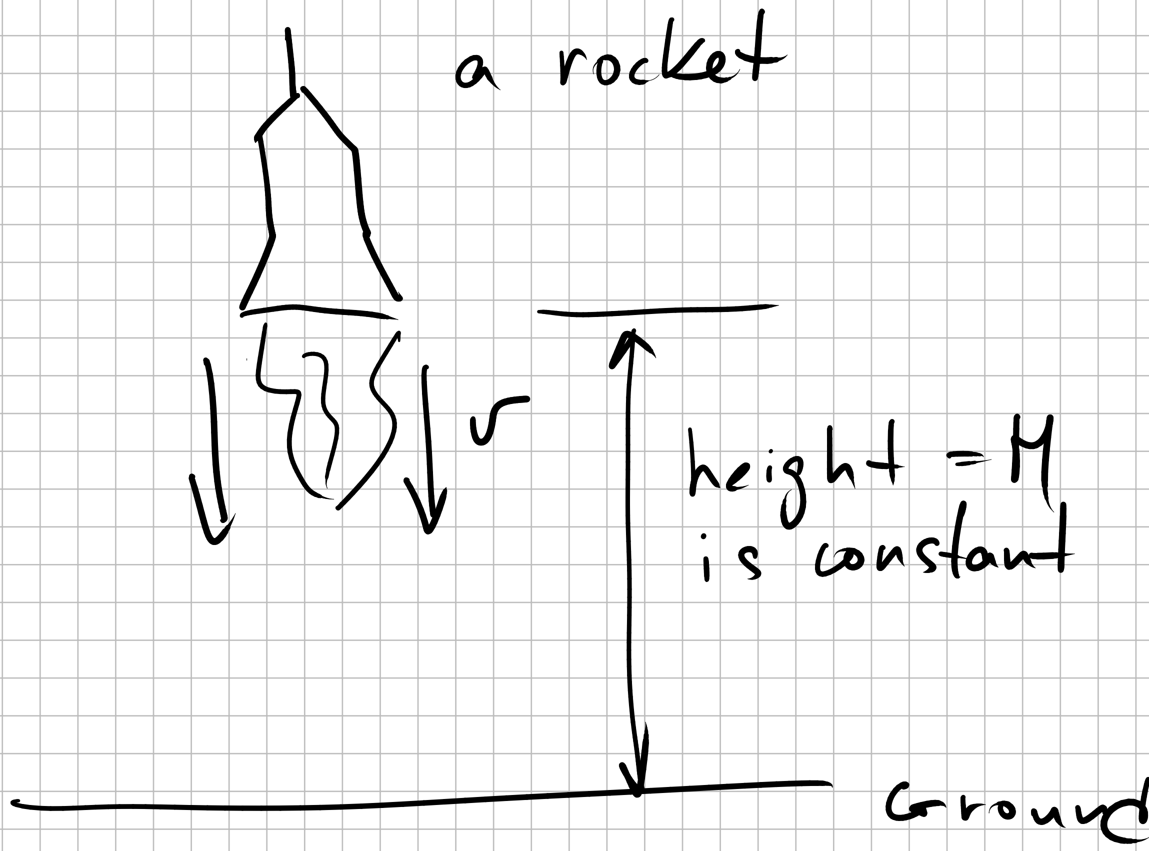
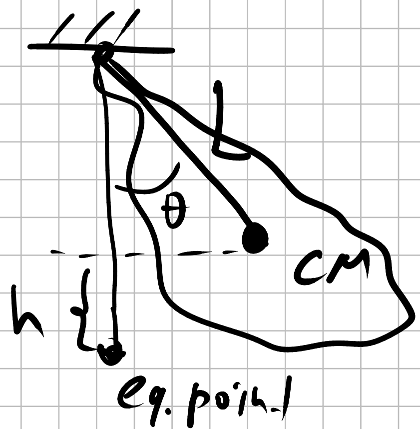
Extra problem – 10 points bonus



Where is the energy goes if H is constant.

Resolve the momentum conserv. paradox.

Physical Pendulum

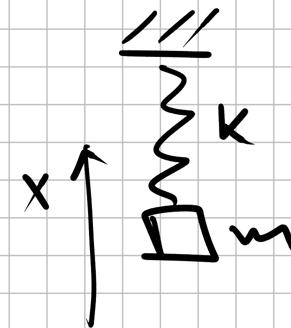


$$\omega = \sqrt{\frac{mgL}{I}}$$

$$\ddot{\theta} = -\omega^2 \theta$$

$$\theta(t) = \theta_A \cdot \cos(\omega t + \varphi)$$

Weight on a spring



$$\omega = \sqrt{\frac{k}{m}}$$

$$\ddot{x} = -\omega^2 x$$

$$x(t) = A \cdot \cos(\omega t + \varphi)$$

Energy conservation

$$K + U = E$$

angular velocity

$$\frac{I \cdot (\dot{\theta})^2}{2} + mgh =$$

$$E_{Tot} = \frac{I(\dot{\theta})^2}{2} + mg(L - L\cos\theta)$$

$$\frac{mv^2}{2} + \frac{kx^2}{2} = E_{Total}$$

$$\frac{mv^2}{2} + \frac{kx^2}{2} = E_{Tot}$$

$$L - L\cos\theta = L(1-\cos\theta) = L\left(1-\sqrt{1-\sin^2\theta}\right)$$

$$\theta \ll 1 \Rightarrow \sin\theta \approx \theta$$

$$L\left(1-\sqrt{1-\theta^2}\right) \approx L\left(1-\left(1-\frac{\theta^2}{2}\right)\right) = L\frac{\theta^2}{2}$$

$$y=\theta^2$$

$$\sqrt{1-y} \approx 1-\frac{y}{2}$$

$$y \ll 1$$

$$E_{tot} = \underbrace{\frac{I(\dot\theta)^2}{2}}_{K} + \underbrace{mgL\,\frac{\theta^2}{2}}_{U}$$

$$\theta = \theta_A \cos(\omega t+\varphi)$$

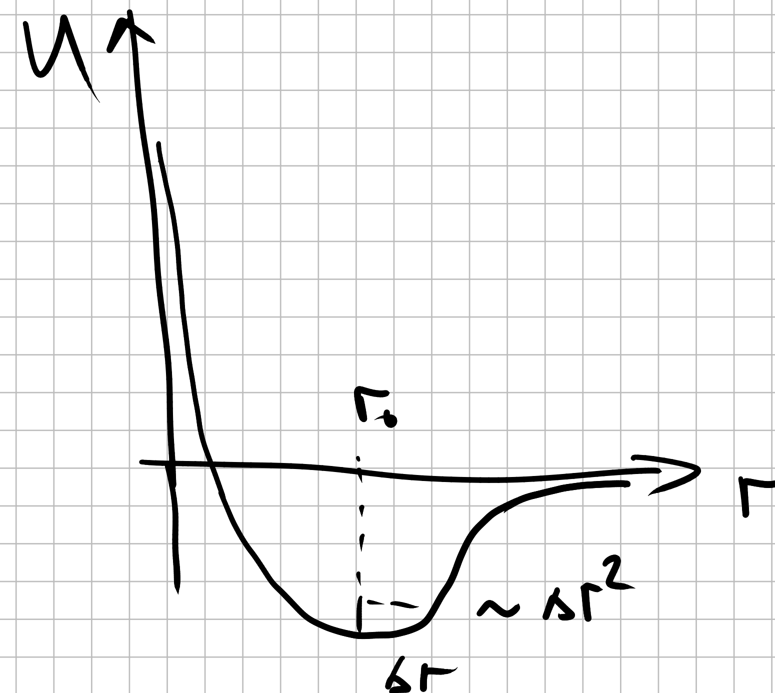
$$\dot\theta = \theta_A(-)\sin(\omega t+\varphi)\cdot\omega$$

$$E_{tot} = \frac{mv^2}{2} + \frac{kx^2}{2}$$

$$= \underbrace{\frac{m(\dot x)^2}{2}}_{K} + \underbrace{\frac{kx^2}{2}}_{U}$$

$$\dot x = A(-\omega)\sin(\omega t+\varphi)\omega$$

Diatomic moleculer



$$E = \frac{I\,\theta_A^2\,\omega^2}{2}\sin^2(\omega t+\varphi) + \frac{mgL}{2}\theta_A^2\cos^2(\omega t+\varphi)$$

$$\omega^2 = \frac{mgL}{I}$$

$$= \frac{\theta_A^2}{2} mgL \underbrace{\left(\sin^2(\ldots)+\cos^2(\ldots)\right)}_{1}$$

$$= \theta_A^2 \frac{mgL}{2} = E_{tot}$$

$$E_{tot} = \frac{KA^2}{2}$$

Finding 'g'

$$\omega = \sqrt{\frac{g}{L}}$$

$$f = \frac{\omega}{2\pi}$$

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$$

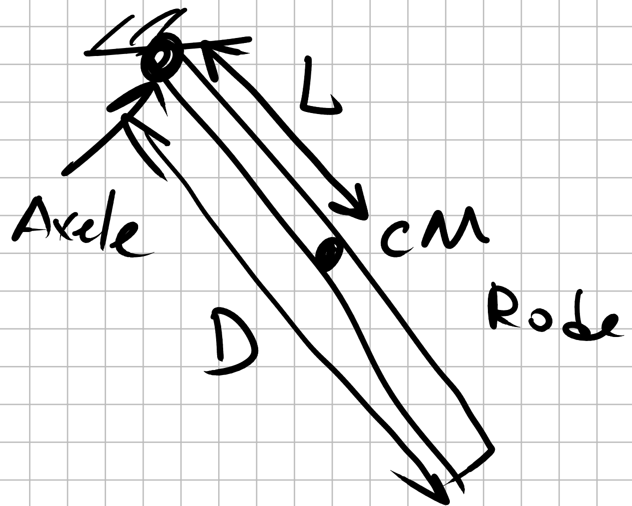
$$g = \frac{(2\pi)^2 \cdot L}{T^2} =$$

$L \simeq 0.52\,m$

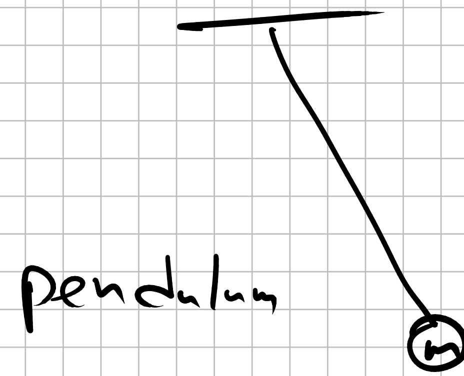
$10T = 14.2\,s$

$T = 1.42\,s$

$$= \frac{(6.28)^2 \cdot 0.52}{1.42^2} \simeq 10.8\,\frac{m}{s^2}$$



vs



$$\omega = \frac{mgL}{I_A} = \frac{mg\frac{D}{2}}{I_A}$$

$$D = 2L$$

$$I_A = I_{cm} + mL^2 = \frac{mD^2}{12} + mL^2 = \frac{m(2L)^2}{12} + mL^2$$

$$= mL^2\left(\frac{4}{12} + 1\right) =$$

$$= mL^2\left(\frac{1}{3} + 1\right) = mL^2\frac{4}{3} = \frac{m(L \cdot 2)^2}{3}$$

$$= \frac{mD^2}{3}$$

$$\omega = \frac{mg\,\frac{D}{2}}{mD^2/3} = \frac{g}{D \cdot \frac{2}{3}}$$