Harmonic oscillator

$$ma = F \Rightarrow m\ddot{x} = -kx$$

$$\ddot{x} = -\frac{k}{m}x \qquad \frac{k}{m} = \omega^2$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

$$\boxed{\ddot{x} = -\omega^2 x}$$

solution for

$$\boxed{x(t) = A \cdot \cos(\omega t + \varphi)}$$

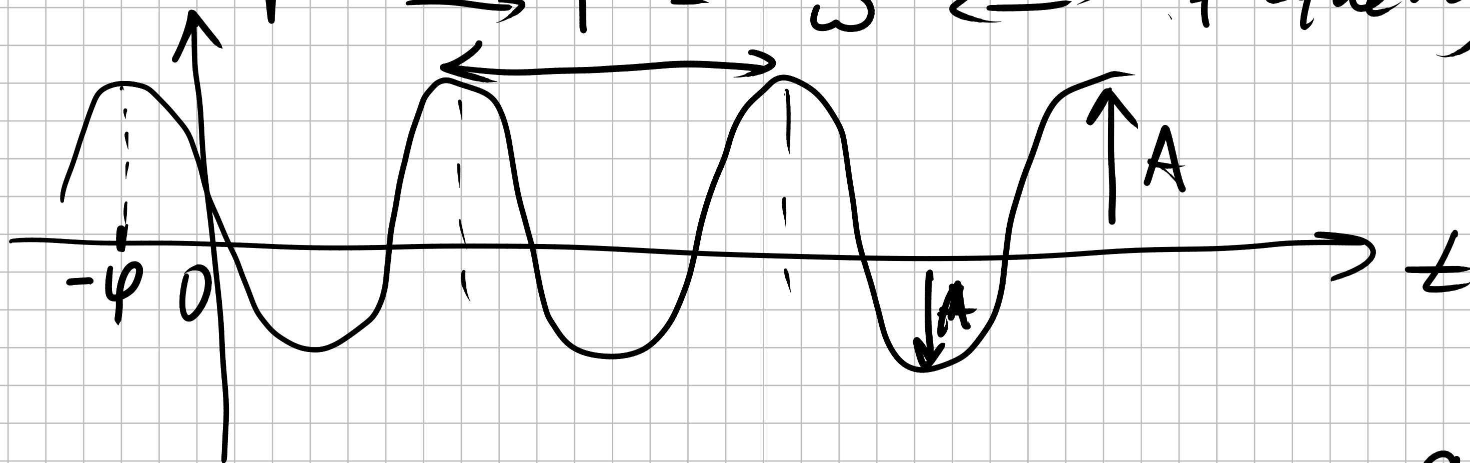
$$\dot{x}(t) = \frac{dx}{dt} = v(t) = A \cdot (-)\sin(\omega t + \varphi)\frac{d(\omega t + \varphi)}{dt}$$

$$= -A\sin(\omega t + \varphi) \cdot \omega$$

$$\ddot{x}(t) = \frac{d^2x}{dt^2} = \frac{dv(t)}{dt} = -\underbrace{A \cdot \cos(\omega t + \varphi)}_{x} \cdot \omega^2 =$$

$$= -x \cdot \omega^2$$

period $\rightarrow T = \frac{2\pi}{\omega} \iff$ frequency $f = \frac{1}{T}$



$\omega = 2\pi \cdot f$

angular frequency

If we observed $x(0) = x_0$

$v(0) = v_0$

ω — is given

φ - ?

A - ?

$$x(t) = A\cos(\omega t + \varphi)$$

$$v(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \varphi)$$

$$x(t=0) = x_0 = A\cos(\omega \cdot 0 + \varphi) = A\cos\varphi$$

$$v(t=0) = v_0 = -\omega A \sin(\omega \cdot 0 + \varphi) = -\omega A \sin\varphi$$

$$\frac{v_0}{x_0} = -\omega \cdot \frac{\sin\varphi}{\cos\varphi} = -\omega \tan\varphi \Rightarrow \varphi$$

$$(\omega x_0)^2 + v_0^2 = (\omega A)^2 \cos^2\varphi + (\omega A)^2 \sin^2\varphi$$

$$= (\omega A)^2$$

$$\Rightarrow \quad A = \sqrt{\frac{(\omega x_0)^2 + v_0^2}{\omega^2}}$$

New set up

if $\varphi = 0$ $\Rightarrow$ $x = A\cos(\omega t)$

$v = -\omega A \sin(\omega t)$

t when v is maximum

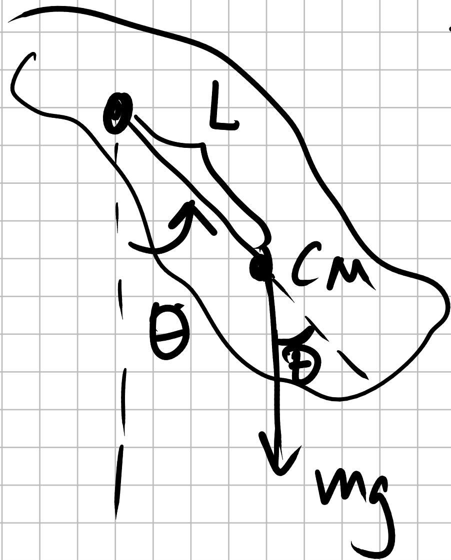
$$\Rightarrow \sin(\omega t) = 1$$

$$\omega t = \frac{\pi}{2} + 2\pi n, \quad n = \ldots -2, -1, 0, 1, 2 \ldots$$

$$t = \frac{\pi}{2\omega} + \frac{2\pi n}{\omega} = \frac{T}{4} + nT$$

$\omega = \frac{2\pi}{T}$

Physical pendulum



$$I\cdot\alpha = \tau_{net} = \tau_g =$$

$$= -L\cdot mg\sin\theta$$

$$I\cdot\alpha = I\cdot\ddot{\theta} = -Lmg\sin\theta$$

$$\ddot{\theta} = -\underbrace{\frac{Lmg}{I}}_{=\omega^2}\sin\theta = -\omega^2\underbrace{\sin\theta}_{\approx\theta}$$

When $\theta \ll 1$

do not confuse with angular velocity

in is **angular frequency**

We have to make approximation

$\theta << 1$ (small angle)

$\Rightarrow \sin\theta \approx \theta$

$$\boxed{\ddot{\theta} = -\omega^2 \theta}$$

compare to

$$\boxed{\ddot{x} = -\omega^2 x}$$

harmonic oscillator equation

$$\boxed{\omega = \sqrt{\frac{Lmg}{I}}}$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

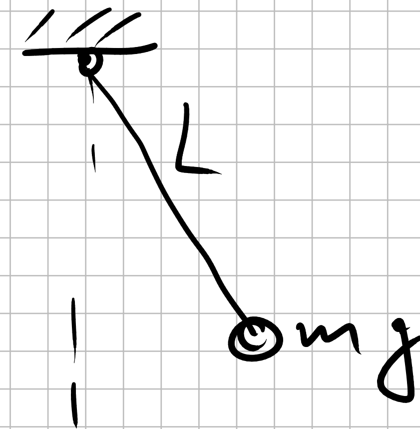
$$\theta(t) = A \cdot \cos(\omega t + \varphi)$$

θ_A – maximum deviation from equilibrium

Simple pendulum

$$\boxed{\omega = \sqrt{\frac{Lmg}{I}}}$$

$$I = mL^2$$



$$= \sqrt{\frac{Lmg}{mL^2}} = \boxed{\sqrt{\frac{g}{L}} = \omega}$$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}}$$