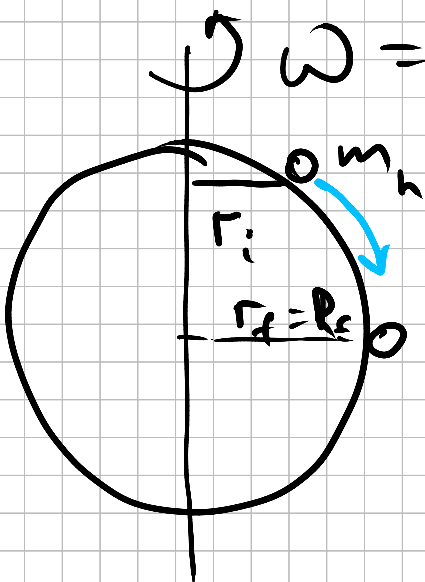
Angular momentum conservation

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{\text{all external forces}}$$



$$\omega = \frac{2\pi}{T}$$

$$\frac{d\vec{L}}{dt} = 0$$

$$L = I_i\,\omega_i = I_f \cdot \omega_f$$

$$(I_E + \underbrace{I_{h_i}}_{m_h \cdot r_i^2})\,\omega_i = (I_E + \underbrace{I_{h_f}}_{m_i \cdot r_f^2})\,\omega_f$$

$\frac{2}{5} M R_E^2$

$$I_E = \frac{2}{5} M R_E^2 = \frac{2}{5} \cdot (5.9 \cdot 10^{24} kg) \cdot (6.378 \cdot 10^6 m)^2$$

$$r_i = 0$$

$$r_f = R_E = 6.378 \cdot 10^6 m$$

$$\frac{\omega_f}{\omega_i} = \frac{I_E + \overbrace{I_{h_i}}^{=0 \Leftarrow r_i = 0}}{I_E + I_{h_f}} =$$

$$\overbrace{\frac{2\pi}{T_i} \Big/ \left(2\pi / T_f\right)}^{} = \frac{\omega_i}{\omega_f} = \frac{I_E + I_{h_f}}{I_E + I_{h_i}} = \frac{T_f}{T_i} = \frac{T_i + \Delta T}{T_i}$$

$$\frac{\cancel{I_E}\left(1 + \frac{I_{h_f}}{I_E}\right) \Leftarrow 1}{\cancel{I_E}\left(1 + \frac{I_{h_i}}{I_E}\right) \Leftarrow 1} = \frac{\cancel{T_i}}{\cancel{T_i}}\left(1 + \frac{\Delta T}{T_i}\right)$$

$$\frac{1}{1+x} \text{ (wrere } x \ll 1\text{)} \approx 1 - x$$

$$\left(1+\frac{I_{hf}}{I_E}\right)\left(1-\frac{I_{hi}}{I_E}\right) = 1 + \frac{\Delta T}{T_i}$$

$$\cancel{1} + \frac{I_{hf}}{I_E} - \frac{I_{hi}}{I_E} - \underbrace{\cancel{\frac{I_{hf}\, I_{hi}}{I_E \cdot I_E}}}_{\ll 1} = \cancel{1} + \frac{\Delta T}{T_i}$$

$$\Delta T = T_i \cdot \left(\frac{I_{hf} - \overset{0}{\overset{\shortparallel}{I_{hi}}}}{I_E}\right)$$

$$= 24 \cdot 3600\,\text{s} \cdot \frac{4.06 \cdot 10^{15} - 0}{9.6 \cdot 10^{37}} =$$

$$= 3.6 \cdot 10^{-18}\,\text{s}$$

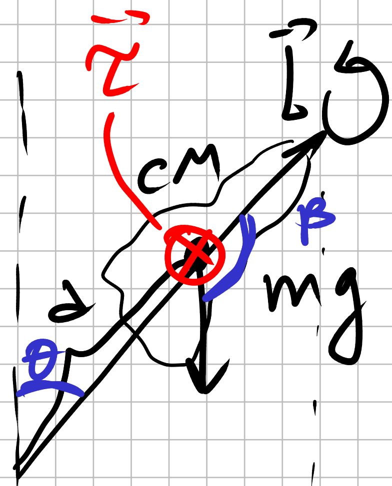
Gyroscope precession

$$\tau = \vec{r} \times \vec{F}$$

$$L = I\omega$$

$$\frac{dL}{dt} = \tau = d \cdot mg \cdot \sin\beta$$

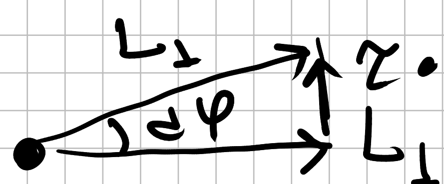
$\sin\beta \, // \, \sin\theta$



ω

$$I \cdot \omega = L$$

Top view



$$\tau \cdot dt = dL$$

$$d\varphi = \frac{dL}{L_\perp} = \frac{\tau \cdot dt}{L_\perp} = \frac{d \cdot mg \sin\theta}{L_0 \sin\theta} dt$$

$\uparrow L \cdot \sin\theta$

Precession speed

$$\frac{d\varphi}{dt} = \boxed{\omega_{\text{Precession}}} = \frac{d \cdot mg}{L} = \boxed{\frac{d \cdot mg}{I \cdot \omega}}$$