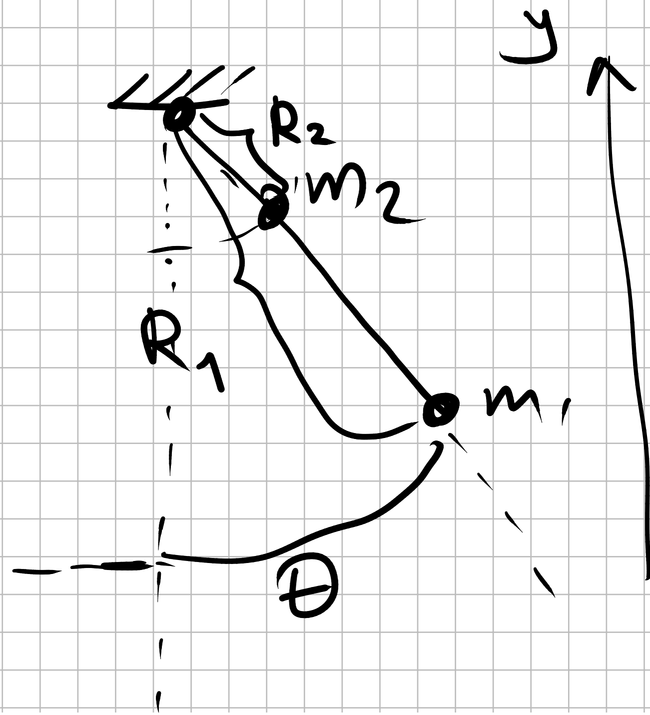
For free moving rigid objects

$$\boxed{E = \frac{M_{Tot}\, v^2}{2} + \frac{I_{\cancel{x}\,cm} \cdot \omega^2}{2} + M_{Tot} \cdot g \cdot y_{cm}}$$

Fixed axis of rotation

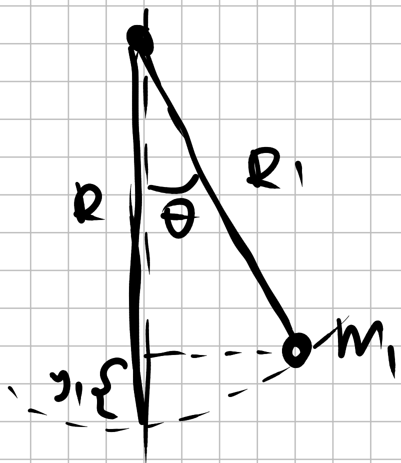


$$E = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} + m_1 g y_1 + m_2 g y_2$$

$$= \frac{m_1 (R_1 \cdot \omega)^2}{2} + \frac{m_2 (R_2 \cdot \omega)^2}{2} +$$

$$+ m_1 g R_1 (1 - \cos\theta) +$$

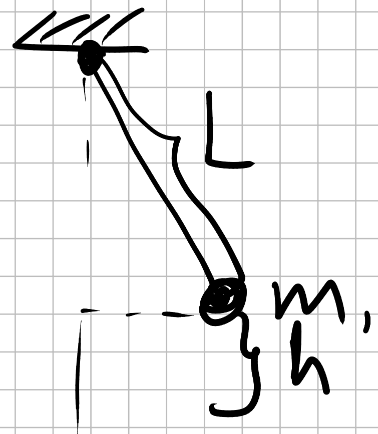
$$+ m_2 g R_2 (1 - \cos\theta)$$



$$y_1 = R_1 - R_1 \cos\theta$$

$$E = \underbrace{\left(m_1 R_1^2 + m_2 R_2^2\right)}_{I_{A_{Tot}}} \frac{\omega^2}{2} + \underbrace{\frac{m_1 \cdot y_1 + m_2 \cdot y_2}{m_1 + m_2}}_{y_{cm}} \cdot \underbrace{(m_1 + m_2)}_{M_{Tot}} g$$

$$\boxed{E = I_{tot_A} \cdot \frac{\omega^2}{2} + M_{tot} \cdot g \cdot y_{cm}}$$



$$E = \frac{I_A \overset{0}{\cancel{\omega^2}}}{2} + mgh \quad : \text{Before}$$

$$= \frac{I_A \omega_f^2}{2} + mg\cdot 0 \quad : \text{After}$$

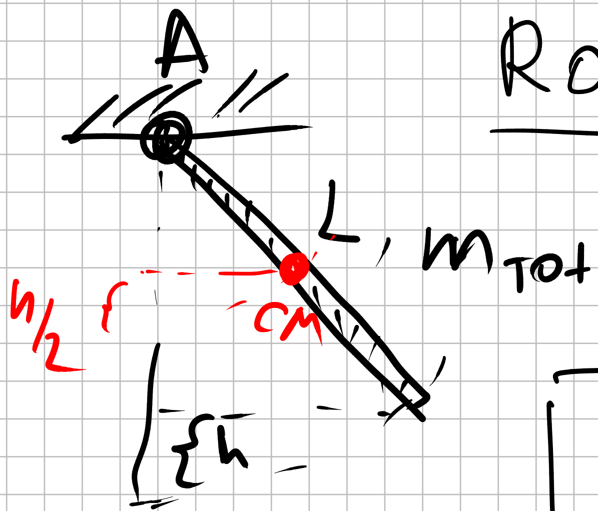
$$mgh = \frac{I_A \omega^2}{2}$$

$$\boxed{\omega^2 = \frac{2mgh}{I_A} = \frac{2mgh}{mL^2} = \frac{2gh}{L^2}}$$

$$v^2 = (\omega \cdot L)^2 = 2gh$$

distance from the axis

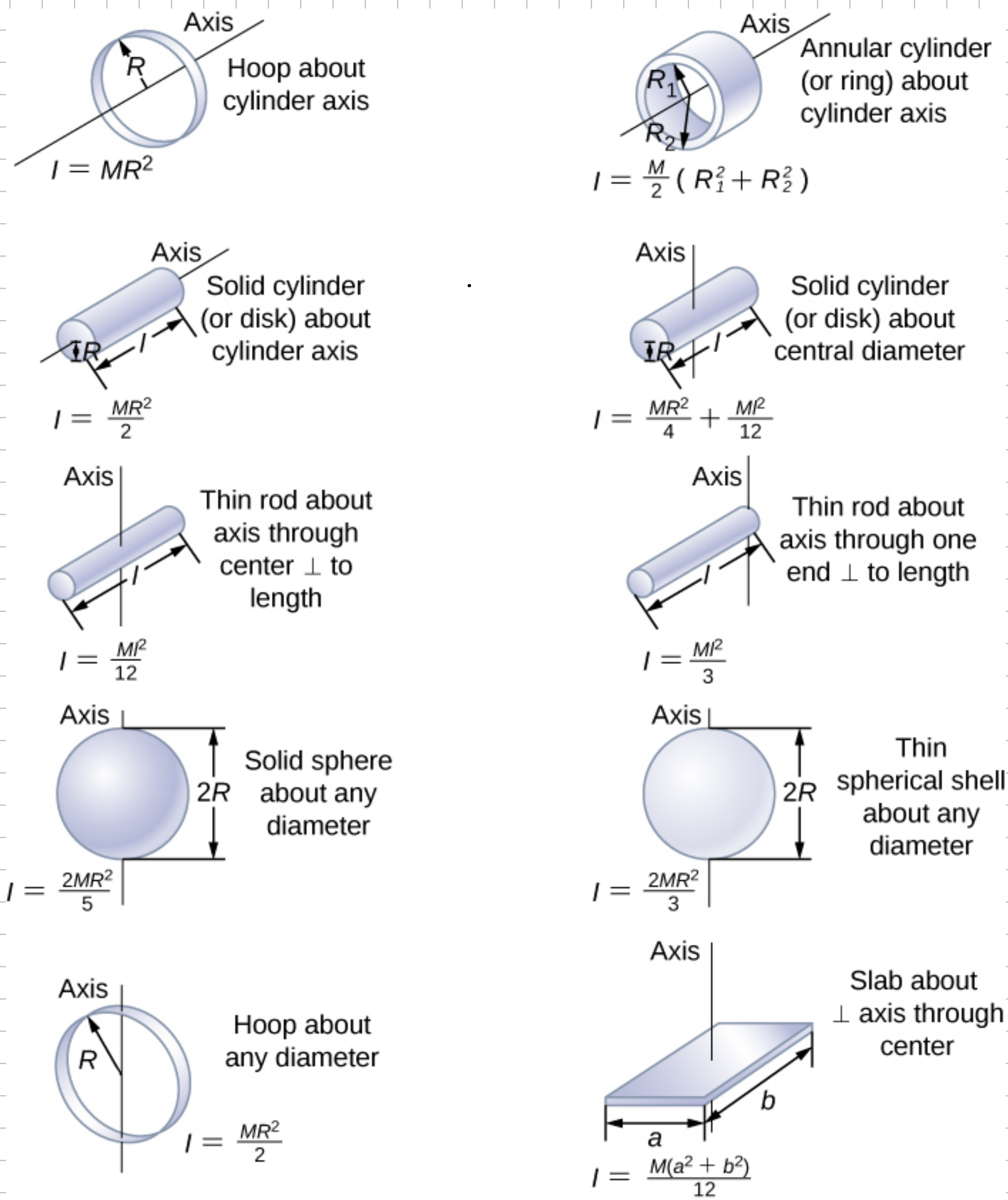
Rod



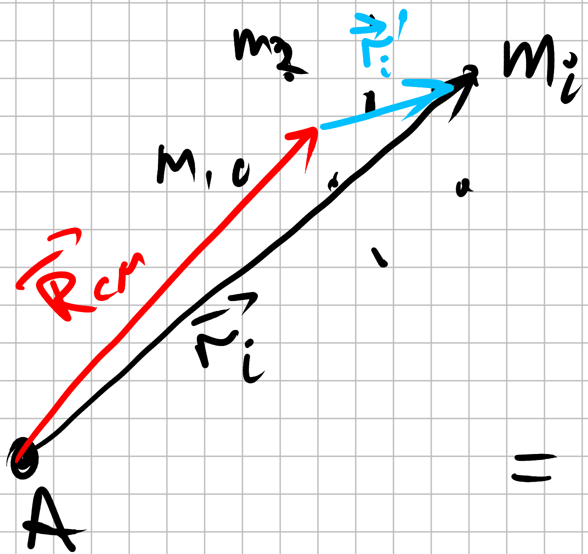
$$\omega^2 = \frac{2 M_{Tot}\, g \cdot h/2}{I_{Tot_A}} = \frac{2 \cancel{M_{Tot}} \cdot g\, h/2}{\cancel{M_{Tot}} \frac{L^2}{3}}$$

$$\boxed{\omega^2 = \frac{\cancel{6}^{\,3}\, g h}{L^2}}$$

Moments of Inertia



Central axis theorem



$$I_A = \sum_i m_i \cdot \vec{r}_i^{\,2} =$$

$$= \sum_i m_i \left(\vec{R}_{cm} + \vec{r}_i\,' \right)^2 =$$

$$= \sum_i m_i \left(R_{cm}^2 + r_i'^2 \right) +$$

$$+ \sum_i m_i 2 \left(\vec{R}_{cm} \cdot \vec{r}_i\,' \right) =$$

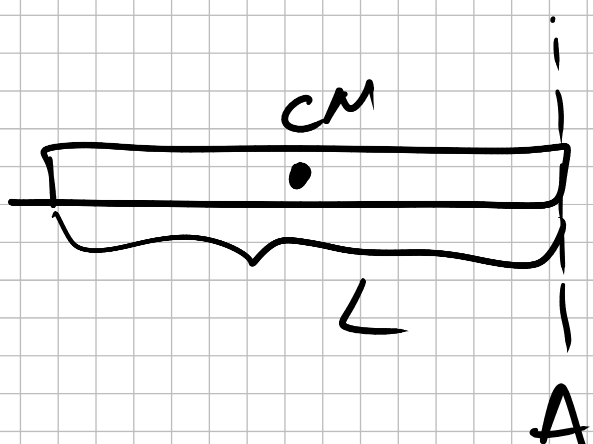
$$= \underbrace{\left(\sum_i m_i \right)}_{M_{tot}} R_{cm}^2 + \underbrace{\left(\sum m_i r_i'^2 \right)}_{I_{cm}} +$$

$$+ 2 \cdot \vec{R}_{cm} \cancel{\underbrace{\frac{\left(\sum_i m_i \vec{r}_i\,' \right)}{M_{tot}}}_{= \vec{R}_{cm}' = \vec{0}}} \cdot M_{tot}$$

$$\boxed{I_A = M_{Tot} \cdot R_{CM}^2 + I_{CM}}$$

displacement of C.M. from axis of rotation

Example



$$I_{CM} = \frac{ML^2}{12}$$

$$I_A = M \cdot \left(\frac{L}{2}\right)^2 + \frac{ML^2}{12}$$

$$= M \cdot L^2 \left(\frac{3 \cdot 1}{3 \cdot 4} + \frac{1}{12}\right) =$$

$$= ML^2 \cdot \frac{4}{12} = \frac{ML^2}{3}$$