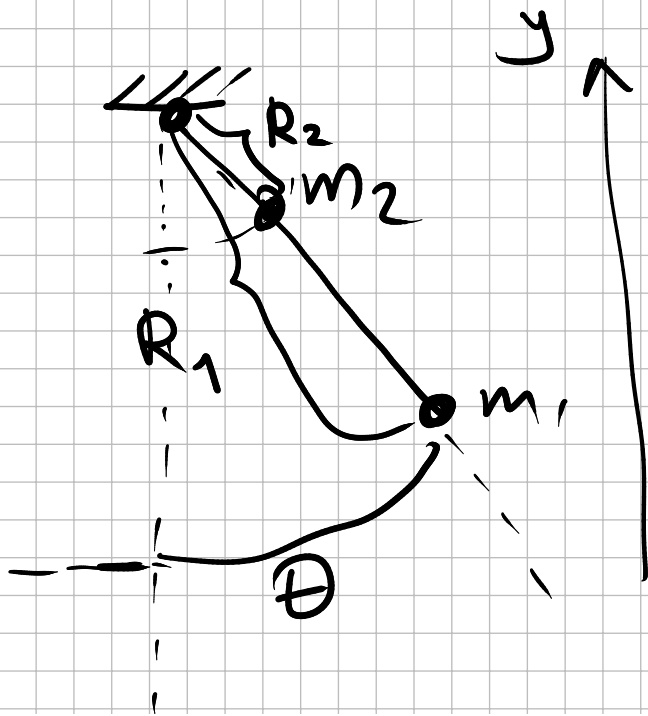


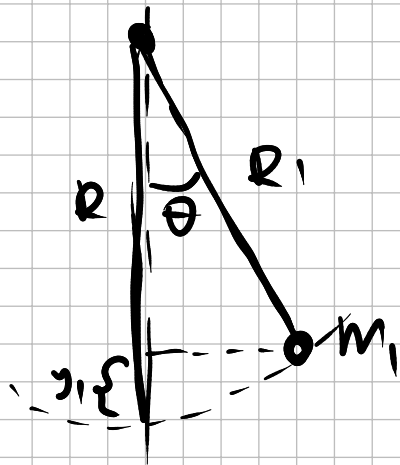
For free moving ^{rigid} objects

$$E = \frac{M_{\text{tot}} v^2}{2} + \frac{I_{\text{cm}} \cdot \omega^2}{2} + M_{\text{tot}} \cdot g \cdot y_{\text{cm}}$$

Fixed axis of rotation



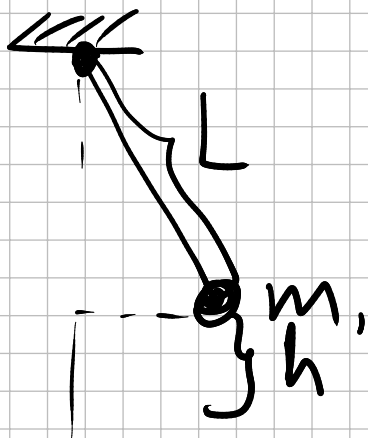
$$\begin{aligned} E &= \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} + m_1 g y_1 + m_2 g y_2 \\ &= \frac{m_1 (R_1 \cdot \omega)^2}{2} + \frac{m_2 (R_2 \cdot \omega)^2}{2} + \\ &\quad + m_1 g R_1 (1 - \cos \theta) + \\ &\quad + m_2 g R_2 (1 - \cos \theta) \end{aligned}$$



$$y_1 = R_1 - R_1 \cos \theta$$

$$E = \underbrace{(m_1 R_1^2 + m_2 R_2^2)}_{I_{A, \text{tot}}} \frac{\omega^2}{2} + \frac{m_1 \cdot y_1 + m_2 \cdot y_2}{m_1 + m_2} \cdot \underbrace{(m_1 + m_2)}_{M_{\text{tot}}} g$$

$$E = I_{A, \text{tot}} \cdot \frac{\omega^2}{2} + M_{\text{tot}} \cdot g \cdot y_{\text{cm}}$$



$$E = \frac{I_A \omega^2}{2} + mgh \quad : \text{ Before}$$

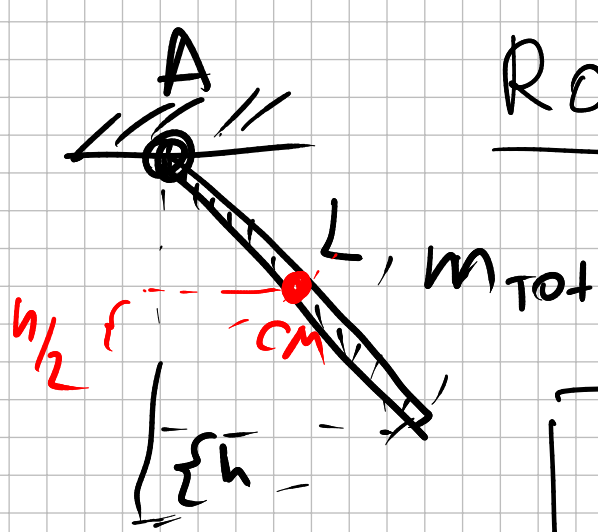
$$= \frac{I_A \omega_f^2}{2} + mg \cdot 0 \quad : \text{ After}$$

$$mgh = \frac{I_A \omega^2}{2}$$

$$\omega^2 = \frac{2mgh}{I_A} = \frac{2mgh}{mL^2} = \frac{2gh}{L^2}$$

$$v^2 = (\omega \cdot L)^2 = 2gh$$

distance
from the
axis



Rod

$$\omega^2 = \frac{2 M_{\text{tot}} g \cdot h/2}{I_{\text{tot}A}} = \frac{2 M_{\text{tot}} \cdot g \cdot h/2}{\cancel{M_{\text{tot}} \frac{L^2}{3}}}$$

$$\omega^2 = \cancel{6} \frac{g h}{L^2} \rightarrow 3$$

Moments of Inertia

Hoop about cylinder axis

$$I = MR^2$$

Annular cylinder (or ring) about cylinder axis

$$I = \frac{M}{2} (R_1^2 + R_2^2)$$

Solid cylinder (or disk) about cylinder axis

$$I = \frac{MR^2}{2}$$

Solid cylinder (or disk) about central diameter

$$I = \frac{MR^2}{4} + \frac{ML^2}{12}$$

Thin rod about axis through center \perp to length

$$I = \frac{ML^2}{12}$$

Thin rod about axis through one end \perp to length

$$I = \frac{ML^2}{3}$$

Solid sphere about any diameter

$$I = \frac{2MR^2}{5}$$

Thin spherical shell about any diameter

$$I = \frac{2MR^2}{3}$$

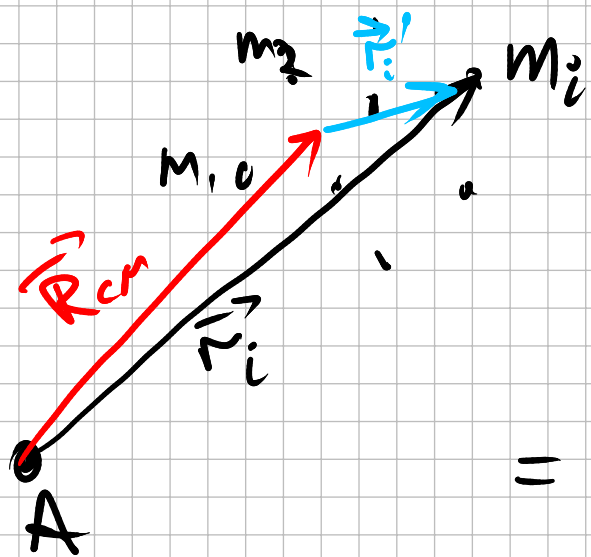
Hoop about any diameter

$$I = \frac{MR^2}{2}$$

Slab about \perp axis through center

$$I = \frac{M(a^2 + b^2)}{12}$$

Central axis theorem



$$I_A = \sum_i m_i \cdot r_i^2 =$$

$$= \sum_i m_i \left(\vec{R}_{CM} + \vec{r}'_i \right)^2 =$$

$$= \sum_i m_i \left(R_{CM}^2 + r_i'^2 \right) +$$

$$+ \sum_i m_i 2 \left(\vec{R}_{CM} \cdot \vec{r}'_i \right) =$$

$$= \left(\sum_i m_i \right) R_{CM}^2$$

$$+ \left(\sum m_i r_i'^2 \right) + I_{CM}$$

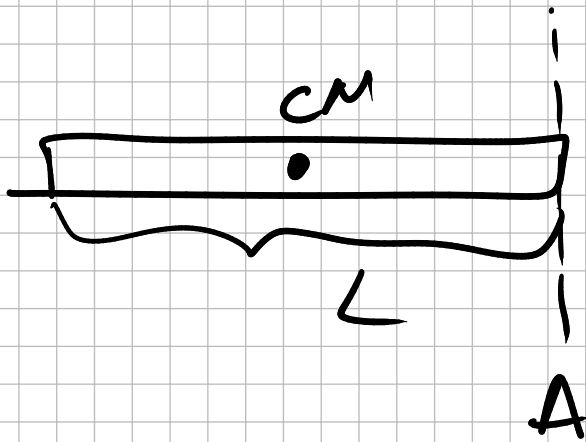
$$+ 2 \cdot \vec{R}_{CM} \cdot \left(\frac{\sum m_i \vec{r}'_i}{M_{tot}} \right) \cdot M_{tot}$$

$\vec{R}_{CM} = \vec{0}$

$$I_A = M_{\text{Tot}} \cdot R_{\text{cm}}^2 + I_{\text{cm}}$$

displacement of CM.
from axis of rotation

Example



$$I_{\text{cm}} = \frac{ML^2}{12}$$

$$I_A = M \cdot \left(\frac{L}{2}\right)^2 + \frac{ML^2}{12}$$

$$= M \cdot L^2 \left(\frac{3 \cdot 1}{3 \cdot 4} + \frac{1}{12} \right) =$$

$$= ML^2 \cdot \frac{4}{12} = \frac{ML^2}{3}$$