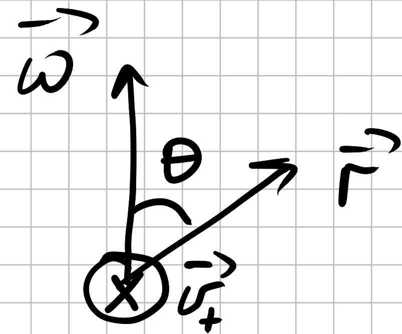
Polar coordinates

$\vec{v}_t = \vec{\omega} \times \vec{r}$

$\vec{v}_t \perp \vec{\omega}$

$\vec{v}_t \perp \vec{r}$



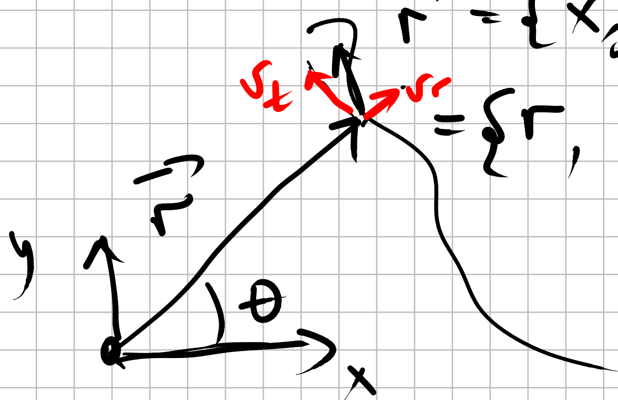
cross product

$|\vec{v}_t| = |\vec{\omega} \times \vec{r}| =$

$= |\vec{\omega}| \cdot |\vec{r}| \cdot \sin(\theta)$

normal multiplication

$\Rightarrow$ if $\vec{\omega} \parallel \vec{r} \Rightarrow \vec{v}_t = \vec{0}$



$\vec{r} = \{x, y\}$

$= \{r, \theta\}$

$\vec{r} \rightarrow \vec{v}$

$\{v_x, v_y\} \Rightarrow \{\vec{v}_r, \vec{v}_t\}$

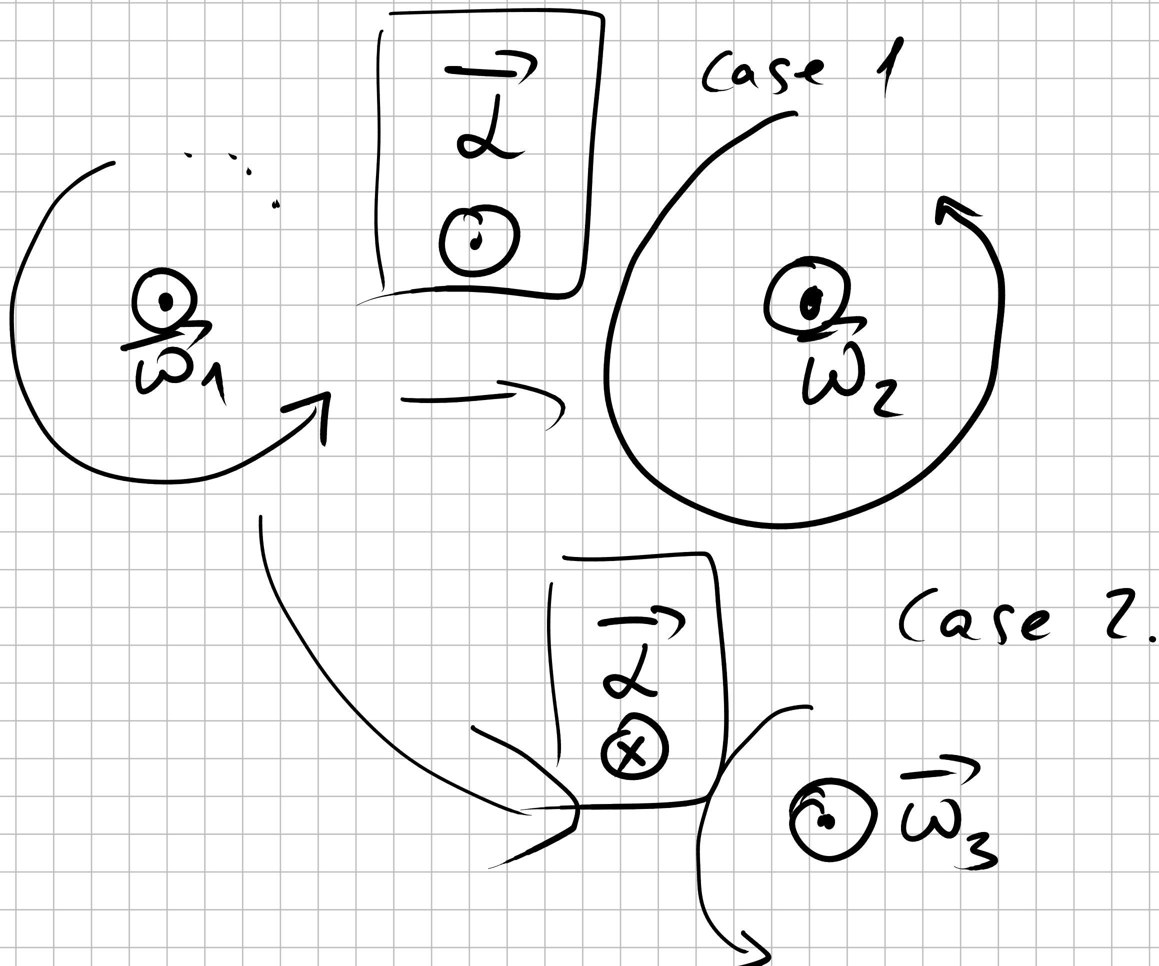
$\vec{v}_t = \vec{\omega} \times \vec{r}$

$\vec{\omega} \times \vec{r} = \dot{\vec{\theta}}$

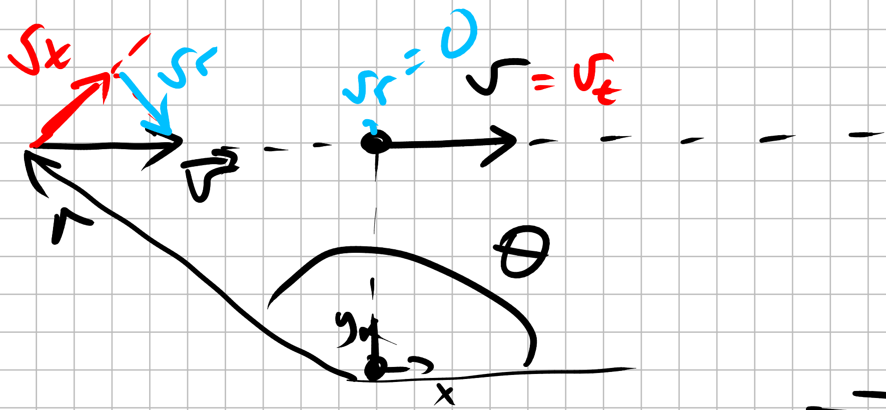
$\dot{\vec{\theta}} = \frac{d\vec{\theta}}{dt} = \vec{\omega}$

angular velocity

$$\frac{d}{dt}\vec{\omega} = \dot{\vec{\omega}} = \vec{\alpha}$$ angular acceleration



Example

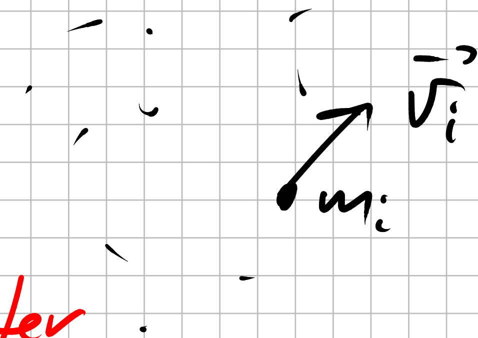


$$\vec{v} = \{ v_r, v_t \}$$

$\Rightarrow$ Polar coordinates are not inertial reference frames

Kinetic energy

$$K = \sum \frac{m_i}{2} \vec{v}_i^{\,2} =$$

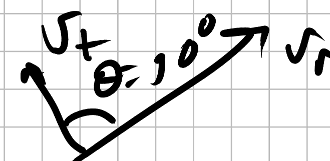


with respect to center of mass

$$= \sum_i \frac{m_i}{2} \left(\vec{v}_{cm} + \vec{v}_i{}' \right)^2 =$$

$$= \sum_i \frac{m_i}{2} \left(\vec{v}_{cm} + \vec{v}_r{}' + \vec{v}_t{}' \right)^2 =$$

$$= \Big/ \text{assume } \vec{v}_{cm} = \vec{0} \Big/ =$$



$v_r \cdot v_t \cdot \cos\theta$

$$= \sum_i \frac{m_i}{2} \left(\vec{v}_r + \vec{v}_t \right)^2 =$$

$$= \sum_i \frac{m_i}{2} \left(\vec{v}_r^{\,2} + \vec{v}_t^{\,2} + 2 \underbrace{\vec{v}_r \cdot \vec{v}_t}_{=0} \right)$$

$$= \sum_i \frac{m_i}{2}\left(\underbrace{v_r^2}_{0 \text{ for rigid objects}} + v_t^2\right) = \sum_i \frac{m_i}{2} v_{t_i}^2$$

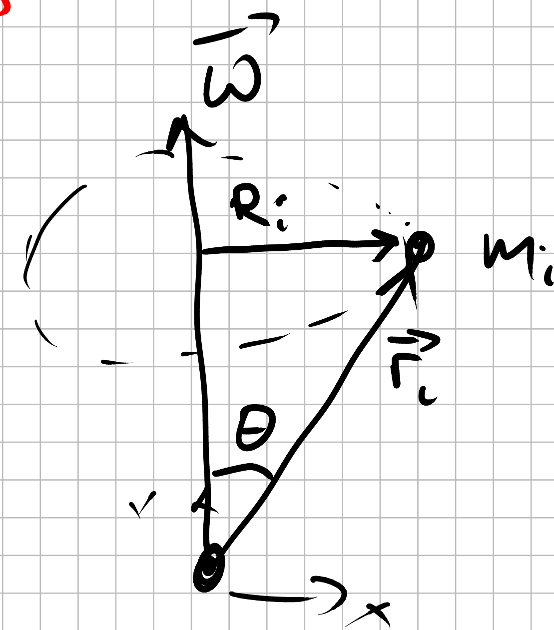
$$= \sum_i \frac{m_i}{2}\left(\vec{\omega} \times \vec{r}_i\right)^2 =$$

$$= \sum_i \frac{m_i}{2}\left(\omega \cdot r \cdot \sin\theta\right)^2$$

$$= \frac{1}{2}\underbrace{\left[\sum_i \left(m_i R_i^2\right)\right]}_{I_A} \omega^2 = \frac{1}{2} I_A \cdot \omega^2$$

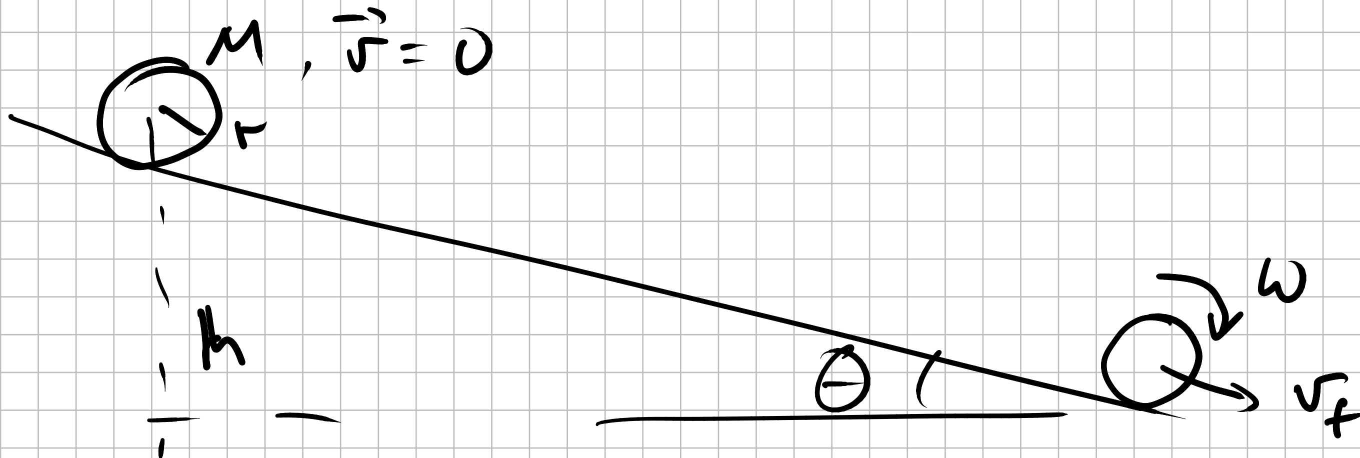
moment of inertia

resembles $\frac{1}{2} m v^2$



$$\boxed{K = \frac{M_{tot}}{2} v_{cm}^2 + \underbrace{\frac{I_A}{2}\omega^2}_{\text{Rotational K. energy}}}$$

For rigid objects



$$K = \frac{Mv^2}{2} + \frac{I_A}{2}\omega^2 = \frac{M}{2} r^2 \omega^2 + \frac{I_A}{2}\omega^2$$

$$r \cdot \omega = v \qquad Mgh = K = \left(\frac{M}{2} r^2 + \frac{I_A}{2}\right)\omega^2$$

$$I_A = M \cdot r^2 \cdot C_{shape}$$