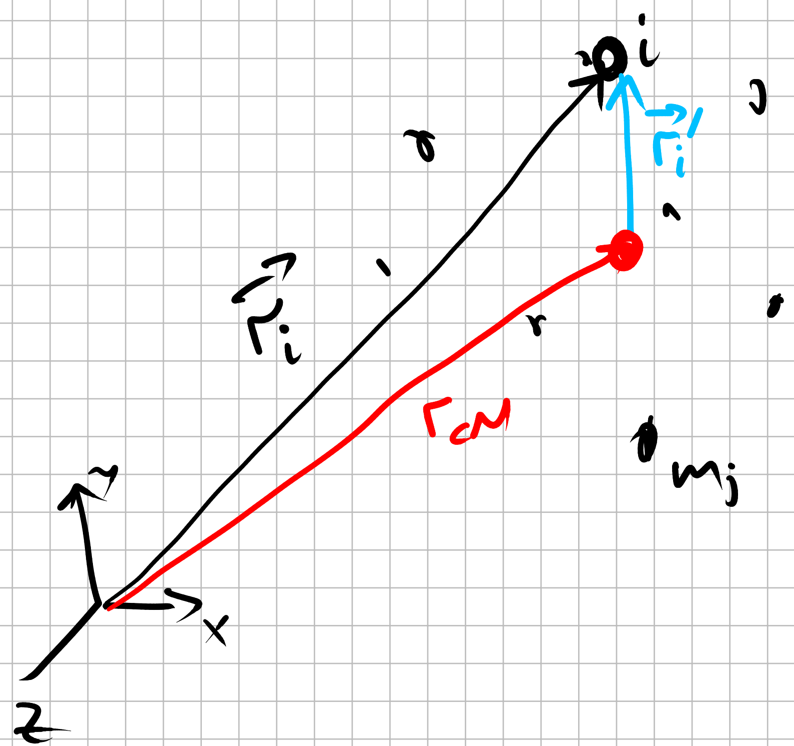
Center of mass

$$\boxed{\vec{F}_{\substack{ext\\net}} = \frac{d\vec{p}}{dt} = M_{tot} \cdot \frac{d^2}{dt^2}\vec{r}_{cm}} = M_{TOT} \cdot \vec{a}_{cm}$$

$$M_{TOT} = \sum_i m_i$$

$$\vec{r}_{cm} = \frac{\sum_i m_i \cdot \vec{r}_i}{M_{TOT}}$$



$$\vec{r}_i = \vec{r}_{cM} + \vec{r}_i'$$

$$\vec{F}_{\substack{ext\\net}} = \frac{d\vec{P}}{dt} =$$

$$= \sum_i \frac{d m_i v_i}{dt} =$$

$$= \sum_i m_i \frac{d^2 \vec{r}_i}{dt^2} = \sum_i m_i \frac{d^2}{dt^2}\left(\vec{r}_{cM} + \vec{r}_i'\right)$$

$$= \underbrace{\sum_i m_i}_{M_{Tot}} \underbrace{\frac{d^2}{dt^2}\vec{r}_{cM}}_{\vec{a}_{cM}} + \underbrace{\frac{d}{dt^2}\sum m_i \cdot \vec{r}_i'}_{\frac{d}{dt}\sum \vec{p}_i'}$$

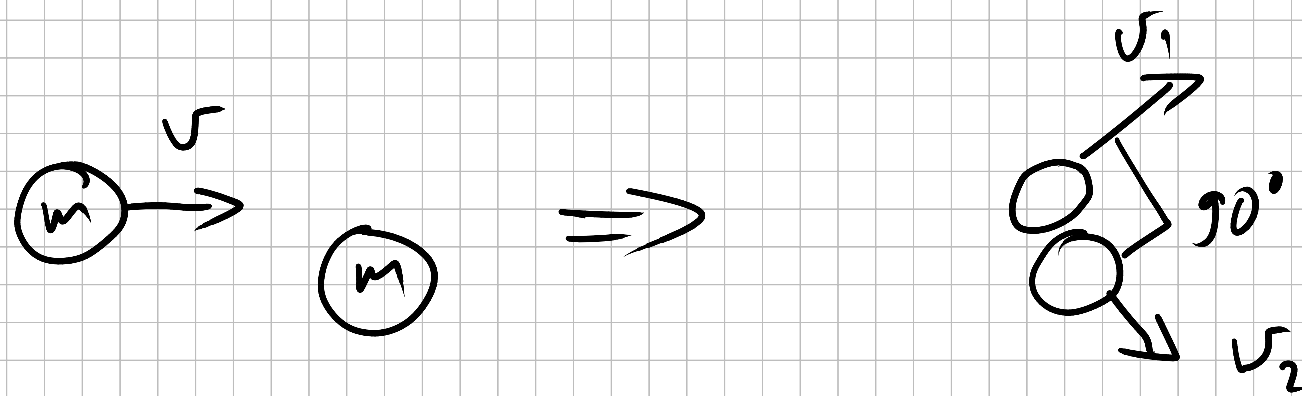
$$\vec{F}_{ext} = \underbrace{M_{Tot}\cdot \vec{a}_{cM}}_{=\vec{F}_{ext}} + \frac{d}{dt}\sum_i \vec{p}_i' = 0$$

$$\boxed{\frac{d}{dt}\sum_i \vec{p}_i\,' = 0}$$

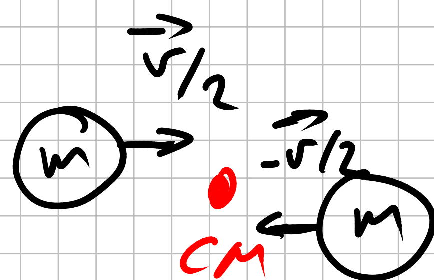
$$\Rightarrow \boxed{\sum_i \vec{p}_i\,' = 0}$$

in C.M. system of coordinates

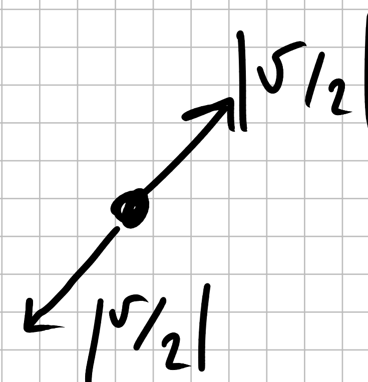
ex:



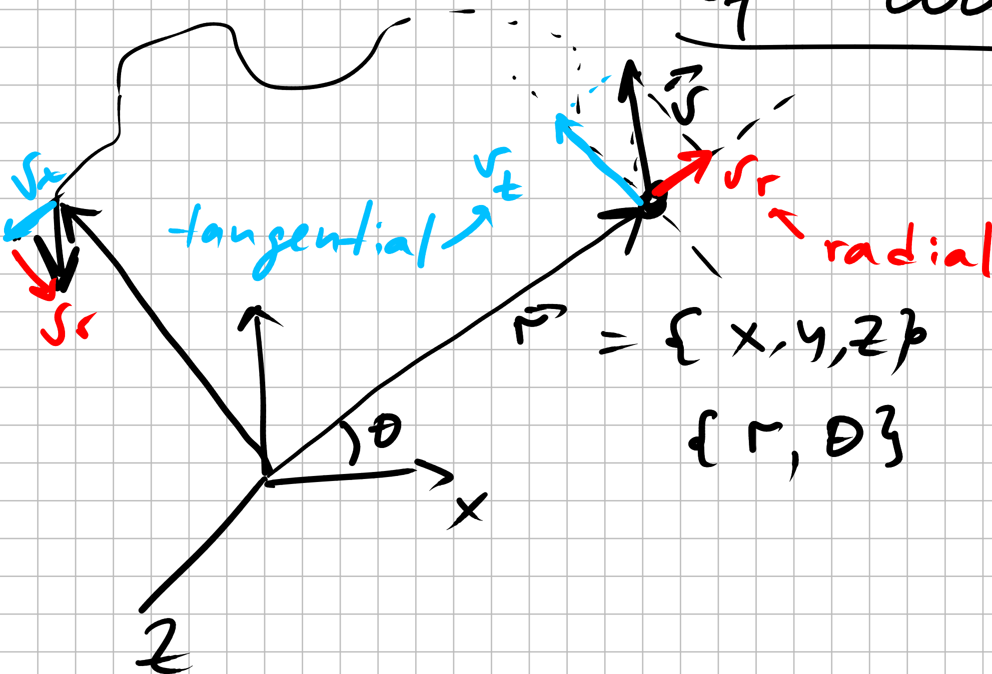
C.M.



CM: $\sum_i \vec{P}_i' = -m\frac{\vec{v}}{2} + m\cdot\vec{v}/2$



Polar/spherical system of coordinates



$\vec r = \{x, y, z\}$

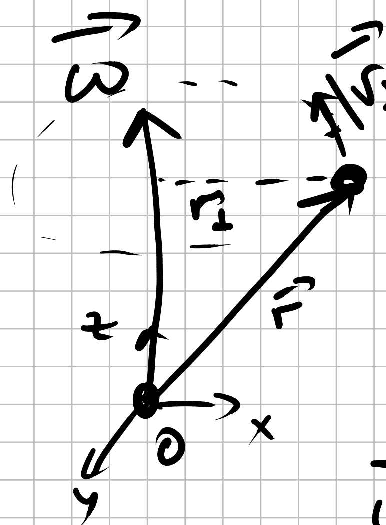
$\{r, \theta\}$

$\vec v = \{v_x, v_y, v_z\}$

$= \{v_r, v_t\}$

$\{v_r, \omega \cdot r\}$

$|\vec v_t| = \omega \cdot r_\perp$



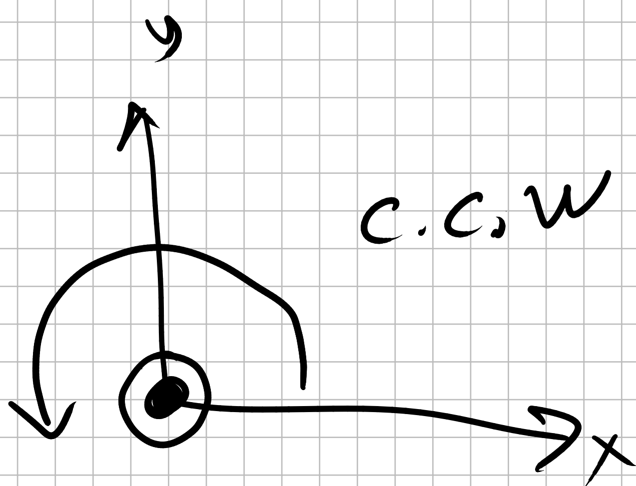
$\vec v_t = \vec\omega \times \vec r$

$\vec v_t = \vec\omega \times \vec r$

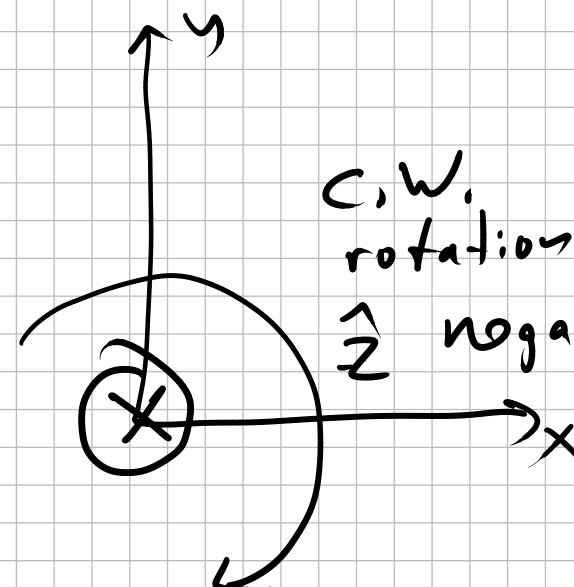
$$\begin{array}{cccc} & \hat i & \hat j & \hat k \\ \vec\omega = & \{\omega_x, & \omega_y, & \omega_z\} \\ \vec r = & \{x, & y, & z\} \end{array}$$

vector product

$$\vec{v}_t = \vec{\omega} \times \vec{r} = \hat{i}\,(\omega_y \cdot z - \omega_z \cdot y) + \hat{j}\,(\omega_z \cdot x - \omega_x z) + \hat{k}\,(\omega_x \cdot \cancel{z}\,y - \omega_{\cancel{z}\,y}\, x)$$



c.c.w $\Rightarrow$ $\hat{z}$ positive



c.w. rotation $\hat{z}$ negative