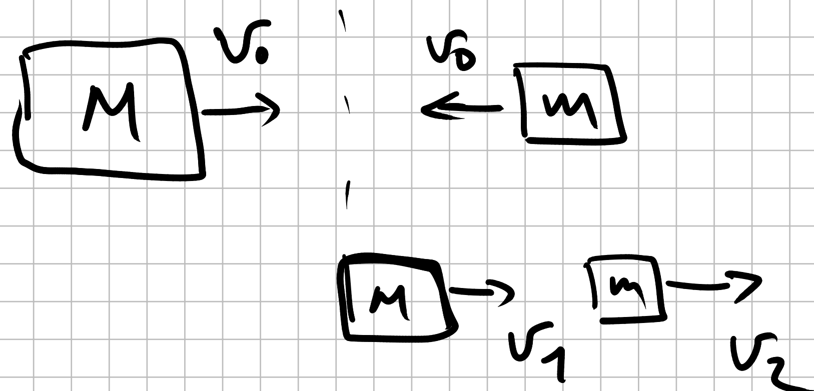
Elastic collision

Before

after

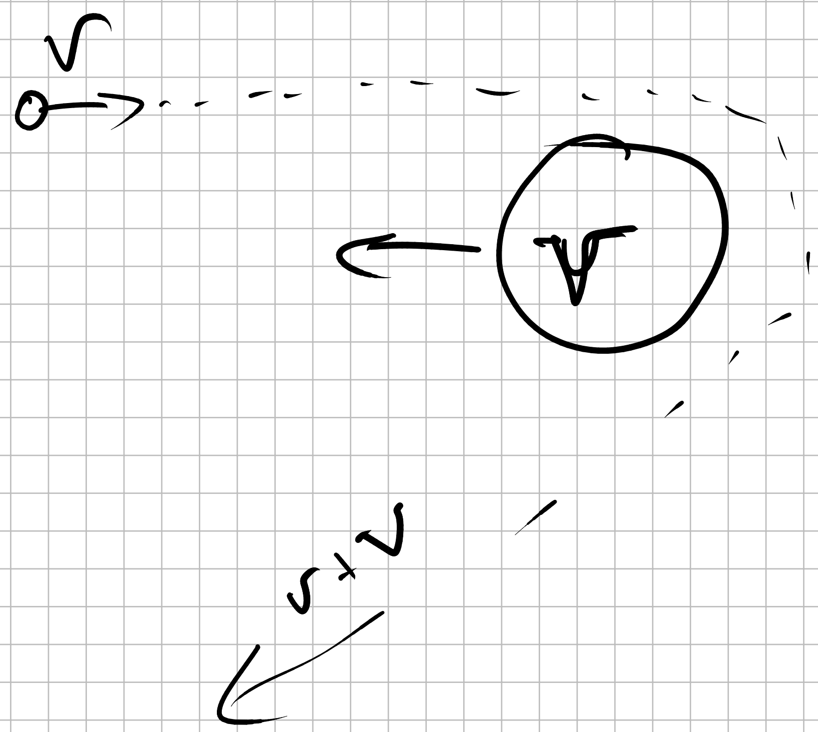


$m \ll M$

$$\boxed{v_2 = 3 v_0}, \quad -v_0$$

Boring
no collision

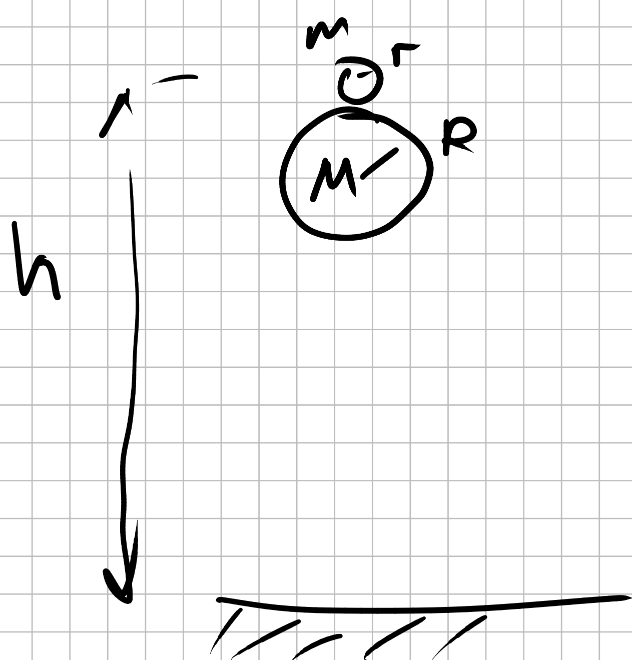
$$\boxed{v_1} = \frac{M - m}{M} v_0 - \frac{m}{M} v_2 \approx \boxed{v_0}$$

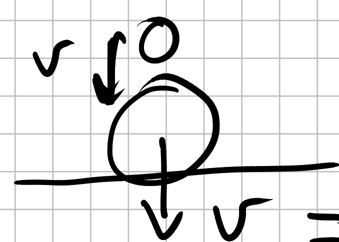

v
V
v+V

Galileo canon

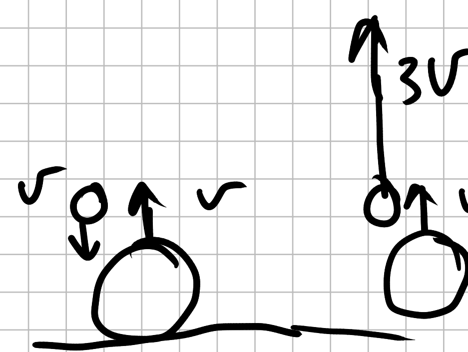
$h \gg R, r$

$M \gg m$





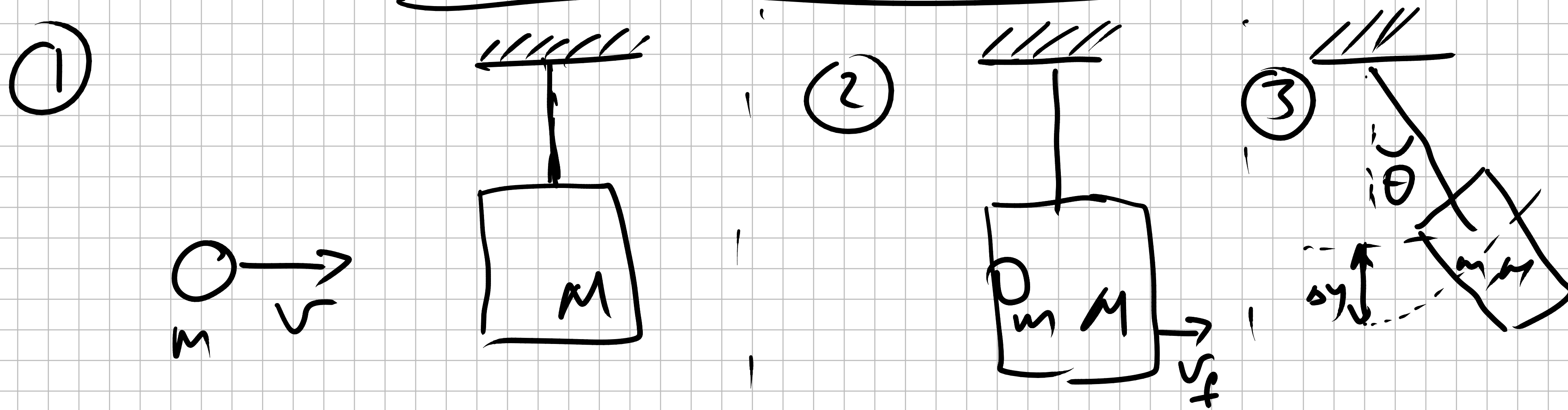
$v = \sqrt{2gh}$



$$m\frac{v_f^2}{2} = mgH_f$$

$$\frac{m(3v)^2}{2} = \frac{\cancel{m}\, 3^2\, \cancel{2}gh}{\cancel{2}} = \cancel{m}gH_f \Rightarrow \boxed{H_f = 9h}$$

Ballistic pendulum



$$P = m\vec{v} + M\cdot\vec{0} = (m+M)\vec{v}_f$$

$$E_i = m\frac{v^2}{2} + M\frac{0^2}{2} \neq \text{not conserved}$$

$$E_f = \frac{(m+M)}{2} v_f^2 = (m+M) g \Delta y$$

Kinetic Potential

$$x: \; v_f = \frac{mv}{m+M} \longrightarrow K_f = \frac{(m+M)}{2} v_f^2 = \frac{(m+M)}{2}\left(\frac{m}{(m+M)}\right)^2 v^2$$

$$= \frac{m^2}{2}\frac{v^2}{M+m} \neq \frac{mv^2}{2}$$

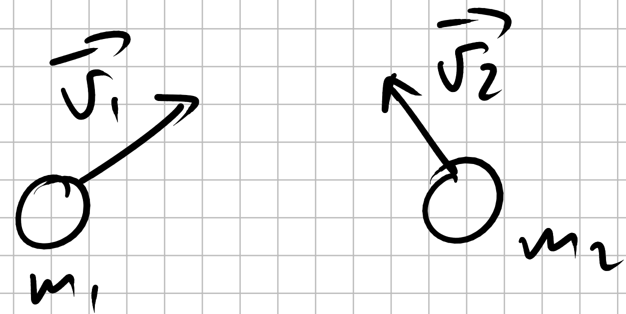
$$K_f = U_f$$

$$\frac{m^2}{2}\frac{v^2}{M+m} = (M+m)g\,\Delta y$$

$$\boxed{\Delta y = \left(\frac{m}{M+m}\right)^2 \frac{v^2}{2}\,\frac{1}{g}}$$

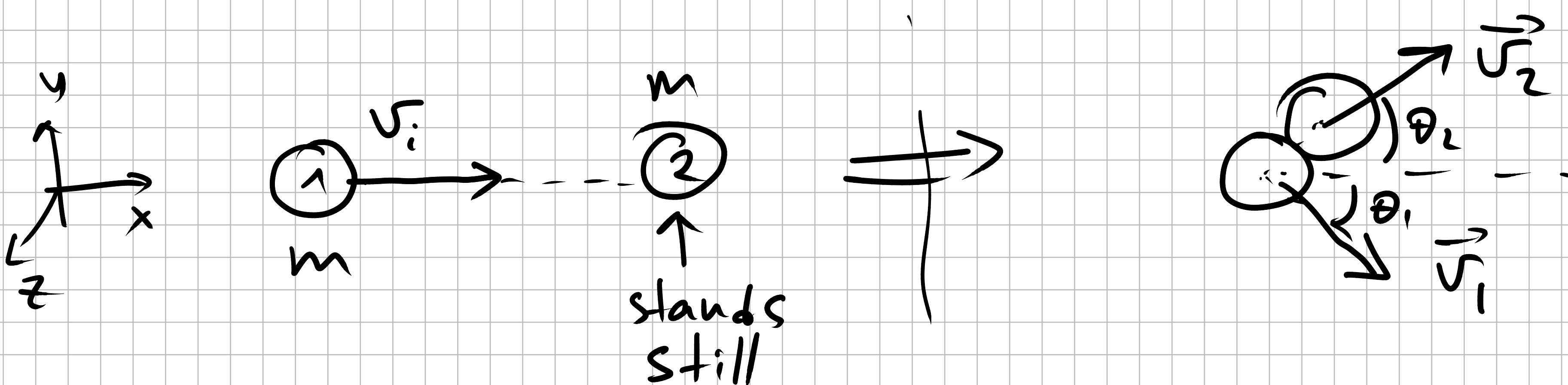
General case in 3D

$$\sum_i \vec{p}_i = \text{constant}$$, if thera no total external forces



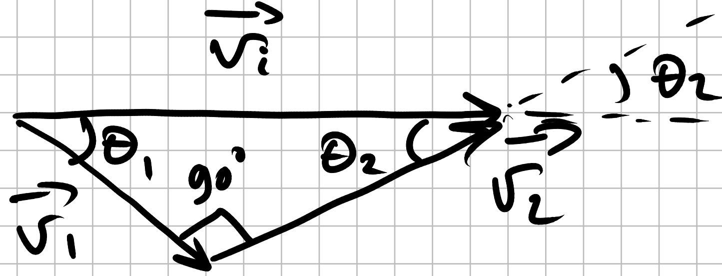
$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = \text{constant}$$

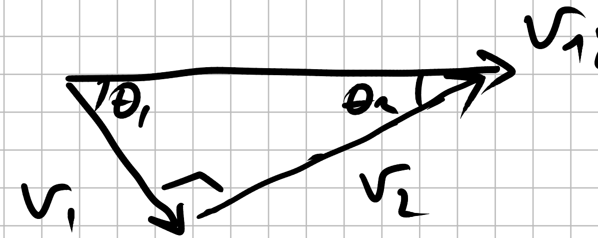
Game of Billiard : $m_1 = m_2 = m$



$$\vec{p} = m\vec{v_i} + m\cdot\vec{0} = m\vec{v_1} + m\vec{v_2}$$

$$\vec{v_i} = \vec{v_1} + \vec{v_2}$$





Energy $$\frac{m v_i^2}{2} + \frac{m\, 0^2}{2} = \frac{m v_1^2}{2} + \frac{m v_2^2}{2}$$

$$\boxed{v_i^2 = v_1^2 + v_2^2}$$

Right angle triangle

$$\theta_1 + \theta_2 = 90°$$

$$\boxed{\begin{aligned} v_1 &= v_i \cos\theta_1 \\ v_2 &= v_i \cos\theta_2 = v_i \sin\theta_1 \end{aligned}}$$

$$\theta_1 \in (-90°, 90°)$$

$$\vec{F}_{ext} = \frac{d\vec{P}_{tot}}{dt} = \frac{d}{dt}\sum_i m_i \vec{v}_i =$$

$$= \frac{d}{dt}\sum_i m_i \frac{d\vec{r}_i}{dt} =$$

$$= \frac{d^2}{dt^2}\left(\sum_i m_i \vec{r}_i\right) \cdot \frac{\sum_i m_i}{\sum_i m_i}$$

$$= \boxed{M_{Tot} = \sum m_i} = \left(\sum_i m_i\right)\frac{d}{dt^2}\frac{\sum m_i \vec{r}_i}{M_{TOT}}$$

$$\boxed{\vec{F}_{ext} = M_{Tot}\frac{d^2}{dt^2}\vec{r}_{cM}}, \quad \boxed{\vec{r}_{cM} = \frac{\sum_i m_i \vec{r}_i}{M_{TOT}}}$$

Center of mass position