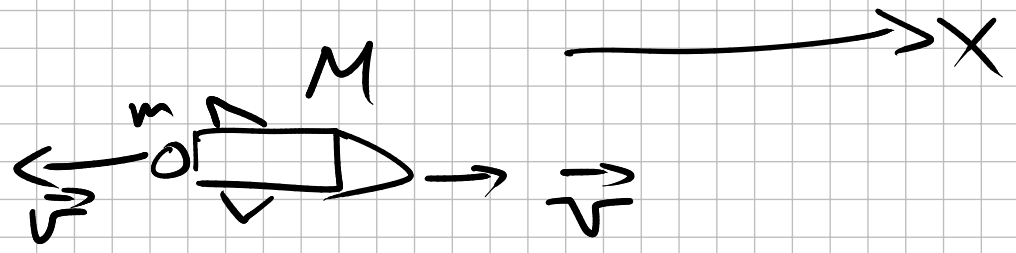
Propulsion

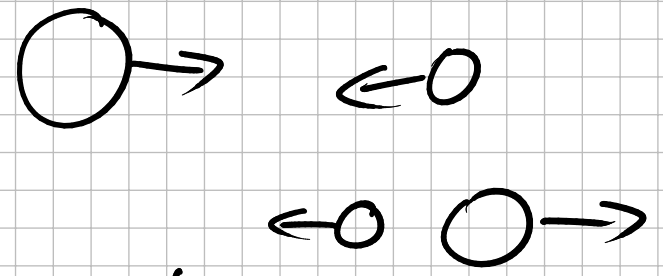


$$0 = m\underbrace{v_i}_{0} + M\underbrace{v_i}_{0} = -m v_f + M v_f$$

v_f = exost velocity

$$v_f = \frac{m}{M} v_f$$

Collisions

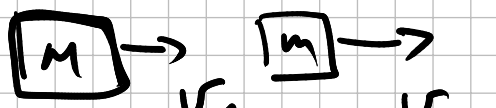


energy conserved

- elastic
- not conserved
  - in elastic

Before

after



Energy

$$\frac{Mv_0^2}{2} + \frac{mv_0^2}{2} = \frac{Mv_1^2}{2} + \frac{mv_2^2}{2}$$

Momentum

$$Mv_0 - mv_0 = Mv_1 + mv_2$$

$$Mv_1 = (M-m)v_0 - mv_2$$

$$v_1 = \frac{M-m}{M} v_0 - \frac{m}{M} v_2$$

$$= v_0 - \frac{m}{M}(v_0 + v_2), \quad \text{if } \frac{m}{M} \ll 1$$

$$= v_0\left(1 - \frac{m}{M}\frac{v_0+v_2}{v_0}\right) \Rightarrow v_1 \approx v_0$$

$$m v_0^2 + M v_0^2 = M v_1^2 + m v_2^2$$

$$m v_0^2 + M v_0^2 = M v_0^2 \left(1 - \underbrace{\frac{m}{M}\frac{v_0+v_2}{v_0}}_{\ll 1}\right)^2 + m v_2^2$$

$$(1-x)^2 \approx 1 - 2x + \cdots \quad (x \ll 1)$$

$$m v_0^2 + M v_0^2 = M v_0^2 \left(1 - 2\frac{m}{M}\frac{v_0+v_2}{v_0}\right) + m v_2^2$$

$$m v_0^2 = -2m(v_0+v_2)v_0 + m v_2^2$$

$$1 = -2 - 2\frac{v_2}{v_0} + \left(\frac{v_2}{v_0}\right)^2$$

$$0 = \left(\frac{v_2}{v_0}\right)^2 - 2\left(\frac{v_2}{v_0}\right) - 3 \quad , \quad \frac{v_2}{v_0} = x$$

$$0 = x^2 - 2x - 3 \quad , \quad 0 = ax^2 + bx + c$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{2 \pm \sqrt{2^2 - 4 \cdot (-3) \cdot 1}}{2 \cdot 1} = \frac{2 \pm \sqrt{16}}{2}$$

$$= 1 \pm 2 = +3, -1$$

$$x = \frac{v_2}{v_0} \Rightarrow \boxed{v_2 = 3v_0 \, , \, -v_0}$$

Boring
no collision