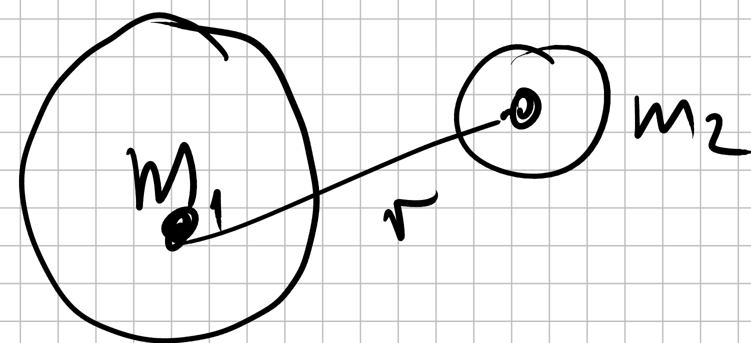
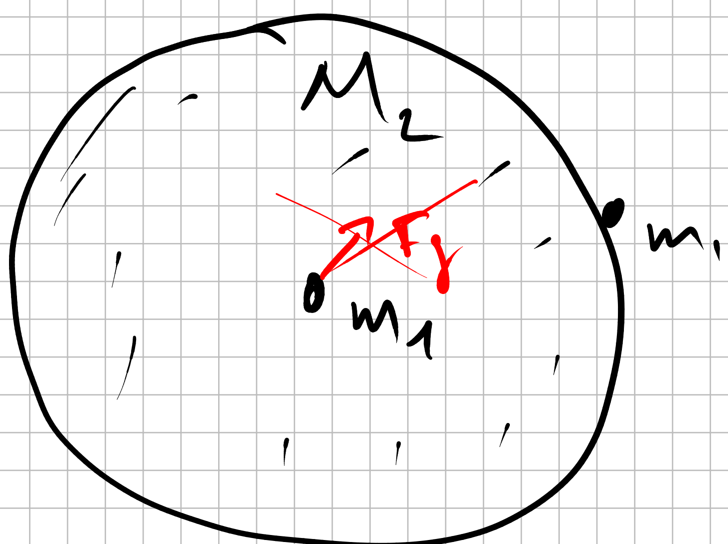
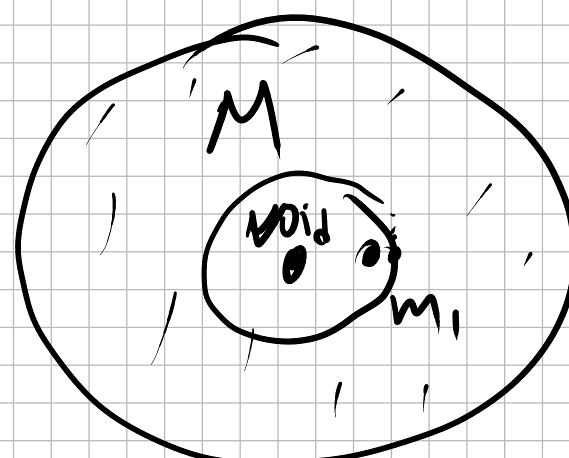
Gravity Force

$$\vec{F}_g = G \frac{m_1 m_2}{r^2} \hat{r}$$





F_g in the center $= 0$



F on m_1 inside a massive shell $= 0$

Linear momentum

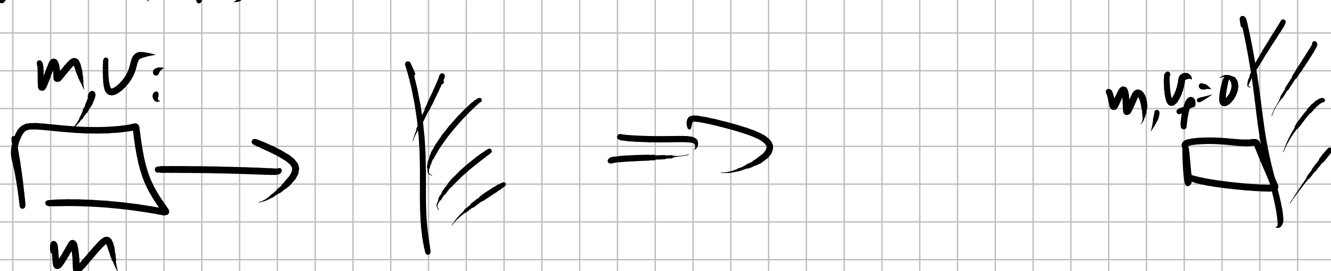
$$\boxed{\vec p = m\vec v}$$

$$\frac{d\vec p}{dt} = m\frac{d\vec v}{dt} = m\vec a = \vec F$$

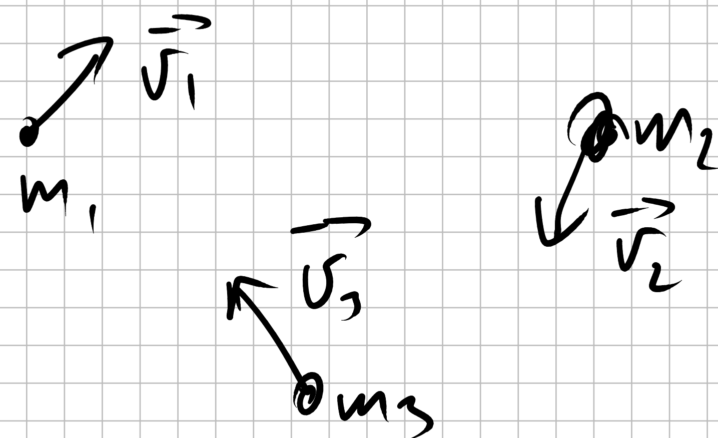
$$d\vec p = \vec F \cdot dt \Rightarrow \underbrace{\Delta\vec P = \vec F\,\Delta t} = \vec J$$

impulse

Car hits wall



$$\boxed{\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}}$$



Total linear momentum

$$\boxed{\vec{P} = \sum_i m_i \vec{v}_i}$$

$$= m_1\vec{v}_1 + m_2\vec{v}_2 + \dots$$

Ex. 2 point

$$\frac{d}{dt}\left(\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2\right)$$

$$\frac{d\vec{p}}{dt} = m_1 \frac{d}{dt}\vec{v_1} + m_2 \frac{d\vec{v_2}}{dt}$$

$$= m_1 \vec{a_1} + m_2 \vec{a_2}$$

$$= \vec{F_1} + \vec{F_2} =$$

$$= \left(\vec{F}_{1ext} + \vec{F}_{12}\right) + \left(\vec{F}_{2ext} + \vec{F}_{21}\right)$$

force on 1st
from 2nd

$$= \vec{F}_{1ext} + \vec{F}_{2ext} + \cancel{\vec{F}_{12}} + \cancel{\vec{F}_{21}}$$

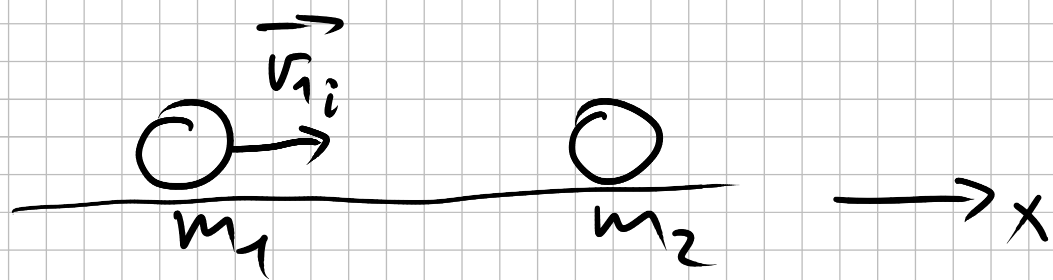
$\vec{F}_{21} = -\vec{F}_{12}$ 3rd Newton Law

$$\boxed{\frac{d\vec{p}}{dt} = \vec{F}_{1ext} + \vec{F}_{2ext} = \vec{F}_{Net\,ext}}$$

Iff $\vec{F}_{\substack{net \\ ext}} = 0 \Rightarrow \frac{d\vec{p}}{dt} = 0$

$\Rightarrow \vec{p}$ is constant

ex.



$\vec{p} = \text{const} \Rightarrow m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$

$\vec{v}_{2i} = \vec{0}$

x: $m_1 v_{1i_x} + m_2 0 = m_1 v_{1f} + m_2 v_{2f}$

implied

$\boxed{m_1 = m_2 = m}$ an assumption

Assume that energy is conserved

$$\frac{\cancel{m}\, v_{1i}^2}{\cancel{2}} + \frac{\cancel{m}\, 0^2}{\cancel{2}} = \frac{\cancel{m}\, v_{1f}^2}{\cancel{2}} + \frac{\cancel{m}\, v_{2f}^2}{\cancel{2}}$$

Energy : $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$

Momentum : $(v_{1i} = v_{1f} + v_{2f})^2$

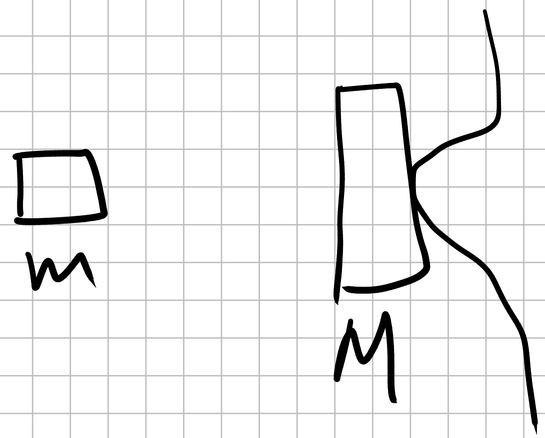
$$v_{1i}^2 = \underline{\underline{v_{1f}^2 + v_{2f}^2}} + \underbrace{2 v_{1f} \cdot v_{2f}}_{=0} = \underline{\underline{v_{1f}^2 + v_{2f}^2}}$$

must be zero

if either $\boxed{v_{1f} = 0}$ not boring

$v_{2f} = 0$

$$\Longrightarrow \boxed{v_{2f} = v_{1i}}$$



$$m v_i + M \cdot 0 = m v_f + M v_f$$

$$v_f = \frac{m}{m+M} v_i$$

$$K = \frac{M \cdot v_f^2}{2} = \frac{M}{2}\left(\frac{m}{m+M}\right)^2 v_i^2 =$$

$$= \frac{M \cdot m}{(\cancel{m}+M)^2} \cdot \underbrace{\left(\frac{m}{2} v_i^2\right)}_{K_i}$$

$m \ll M$

$$K = \frac{\cancel{M} \cdot m}{M^{\cancel{2}}} K_i = \frac{m}{M} K_i$$

$$= \frac{k x^2}{2}$$