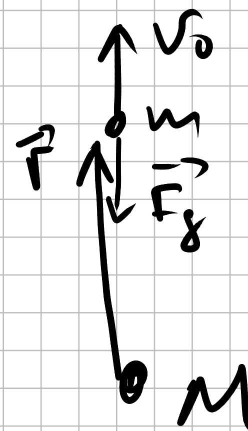
Gravitational pull



$$F_g = G\frac{M_E m}{r^2}$$

$$v = \int a\,dt$$

$\cancel{mgh}$

$$\underline{E = K_i + U_i = K_f + U_f}$$

unit vector along $\vec{r}$

$$W_g = \int_i^f \vec{F}_g \cdot d\vec{r} = \int_i^f G\frac{Mm}{r^2}\underbrace{(-\hat{r})\,d\vec{r}}_{-dr} =$$

$$= \int_i^f -G\frac{Mm}{r^2}\,dr = +G\frac{Mm}{r}\Big|_{r_i}^{r_f}$$

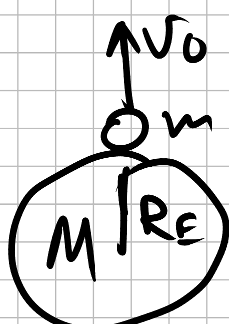
$$W_g = \frac{GMm}{r_f} - \frac{GMm}{r_i} = -(U_f - U_i)$$

$$\boxed{U(r) = -\frac{GMm}{r}}$$ Potential energy

$$E = U_i + K_i = U_f + K_f$$ Energy conservation

$$-\frac{GMm}{R_E} + \frac{mv_0^2}{2} = -\underbrace{\cancel{\frac{GMm}{r_f}}}_{0} + \frac{mv_f^2}{2} \quad v_f = 0 \quad = 0$$

$$r_f \to \infty$$



$$-\frac{GM\cancel{m}}{R_E} + \frac{\cancel{m}\,v_0^2}{2} = 0$$

$$v_0 = \sqrt{2\underbrace{\frac{GM}{R_E}\cdot\frac{R_E}{R_E}}_{=g}}$$

$$v_0 = \sqrt{2gR_E} = \sqrt{2 \cdot 10 \frac{m}{s^2} \cdot 6.4 \cdot 10^6 m}$$

$$= \sqrt{2 \cdot 64 \cdot 10^6} = \sqrt{2} \cdot 8 \cdot 10^3 \frac{m}{s}$$

$$\approx 12 \text{ km/s} \quad \text{Escape velocity}$$

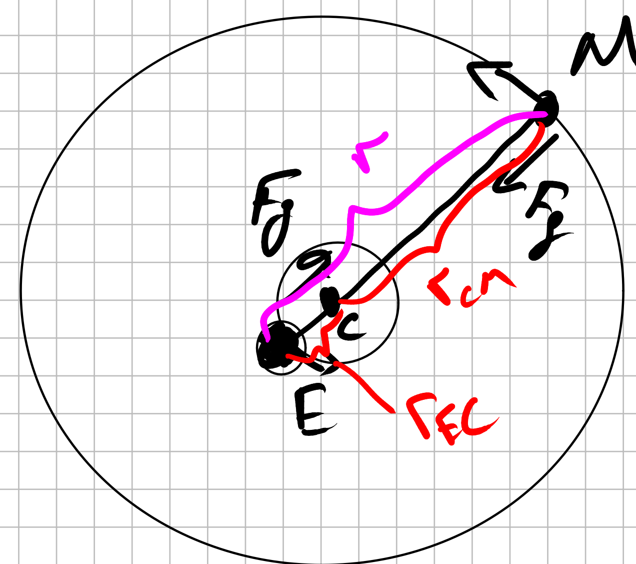
Earth - Moon system

$$m_E \cdot a_{c_E} = F_g = m_M \cdot a_{c_M}$$

$$\parallel$$

$$\frac{v^2}{r_{EC}} = \omega^2 r_{EC}$$

$$v = \frac{2\pi r_{EC}}{T}$$



$$m_E \cancel{\omega^2} r_{EC} = m_M \cancel{\omega^2} r_{cM}$$

$$r_{EC} = \frac{m_M}{m_E} \cdot r_{cM}$$

$$r_{cM} = (r_{EM} - r_{EC})$$

$$r_{EC} = \frac{m_M}{M_E}\left(r_{EM} - r_{EC}\right)$$

$$r_{EC}\left(1 + \frac{m_M}{M_E}\right) = \frac{m_M}{M_E}\, r_{EM}$$

$$r_{EC} = \frac{m_M}{M_E}\,\frac{1}{1+\frac{m_M}{M_E}}\, r_{EM} \qquad \frac{m_M}{M_E} \ll 1$$

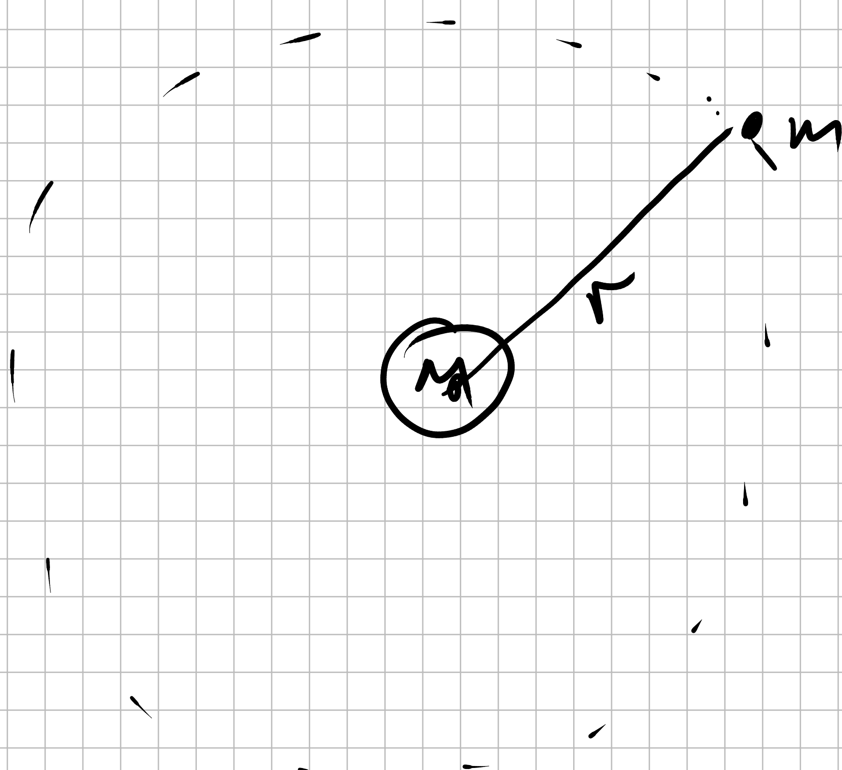
$$\approx \frac{m_M}{M_E}\, r_{EM}\left(1 - \frac{m_M}{M_E}\right)$$

$$\approx \frac{7.35\cdot 10^{22}\,\mathrm{kg}}{5.98\cdot 10^{24}\,\mathrm{kg}} \cdot 384\cdot 10^{6}\,\mathrm{m}\,\left(1 - \cancel{\ldots}\right)$$

$$\approx \frac{7}{6}\cdot 3.84\cdot 10^{6} \approx 4.1\cdot 10^{6}\,\mathrm{m} = r_{EC}$$

$$R_E = 6.4\cdot 10^{6}\,\mathrm{m}$$

Mass of a heavy (compared to a satelete) star or planet. Connection to between period of the orbit (T), distance from the central object (r), and mass of the heavy object (M)



$$\cancel{m} a_c = G\frac{\cancel{m}M}{r^2}$$

$$\frac{v^2}{r} = \left(\frac{2\pi r}{T}\right)^2 \frac{1}{r} = G\frac{M}{r^2}$$

$$\frac{(2\pi)^2}{T^2} r = G\frac{M}{r^2}$$

$$\boxed{M = \frac{(2\pi)^2}{G}\frac{r^3}{T^2}}$$