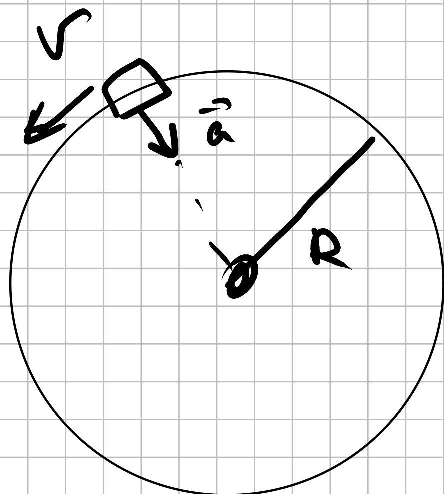
Motion on a circle



$$|\vec{v}| = v = \text{const}$$

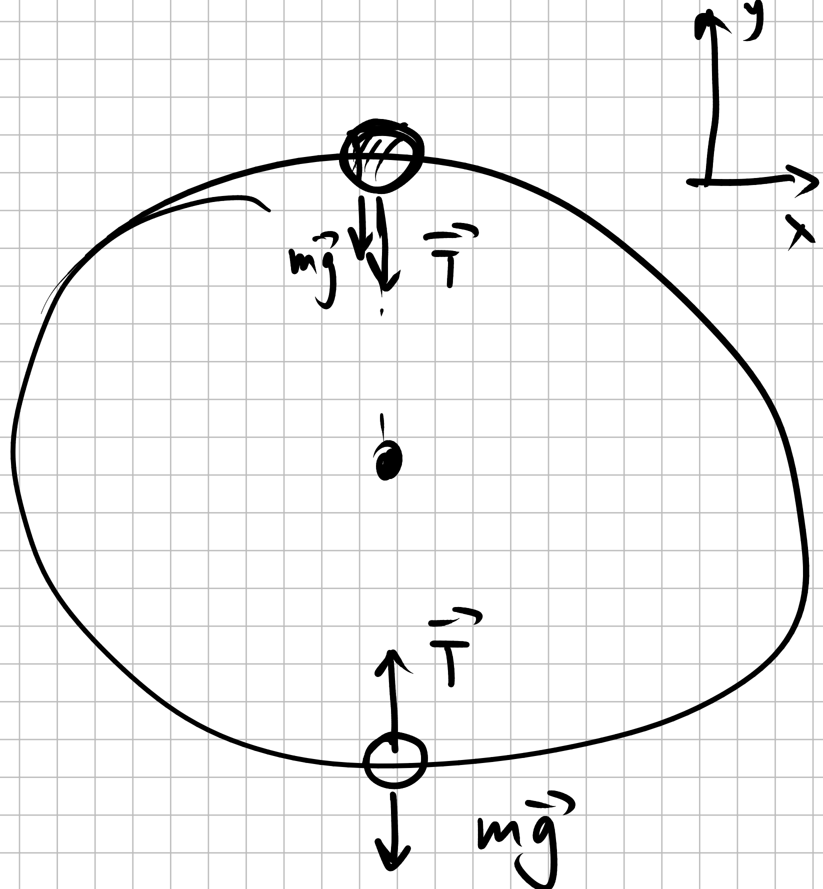
$$|\vec{a}| = a = \frac{v^2}{R} = \omega^2 R$$

centripetal acceleration

angular velocity $\omega = \frac{v}{R} = \left[\frac{\text{radians}}{s}\right]$

$$\omega = \frac{2\pi}{T}$$

period



$$m\vec{a} = \sum_i \vec{F}_i = m\vec{g} + \vec{T}$$

$$y: \quad ma_y = mg_y + T_y =$$

$$= m(-g) - T_{Top}$$

$$T_{Top} \leq 0 \Rightarrow ma_{y_{Top}} \leq -mg$$

$$ma_{y_{Top}} = -ma_{Top} \leq -mg$$

$$a_{Top} = g$$

$$m\vec{a} = m\vec{g} + \vec{T}$$

$$ma_{y_{Bot}} = m(-g) + T_{Bot}$$

$$+m\,a_{Bot} = -mg + T_{Bot}$$

let's assume that v is the same

thus 'a' is the same

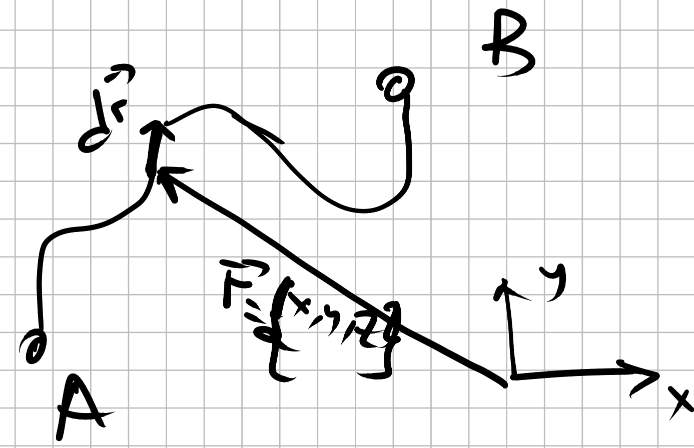
$\uparrow$ acceleration

$$a_{Top} = a_{Bot}$$

$$m a_{Bot} = m a_{Top} = mg = -mg + T_{Bot}$$

$$\Rightarrow \boxed{T_{Bot} = 2mg}$$

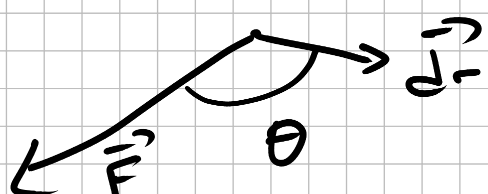
Work



$$W_{A\to B} = \int_A^B \vec{F}(\vec{r})\cdot d\vec{r} =$$

$$\int_A^B \left(F_x(\vec{r})\,dx + F_y(\vec{r})\,dy + F_z(\vec{r})\,dz \right)$$

$$\vec{F}\cdot d\vec{r} = F\cdot dr\cdot\cos\theta$$



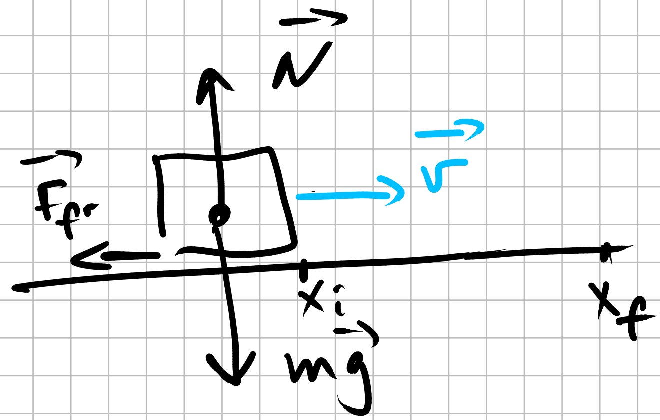
$$W_{A\to B} = \int_A^B \vec{F}_{Total}\, d\vec{r} = \int_A^B m\vec{a}\cdot d\vec{r} =$$

$$= \int_A^B m\frac{d\vec{v}}{dt}\cdot d\vec{r} = \int m\, d\vec{v}\cdot\frac{d\vec{r}}{dt} \quad = \vec{v}$$

$$= m\int d\vec{v}\cdot\vec{v} = m\int_A^B \vec{v}\cdot d\vec{v} = m\frac{\vec{v}^2}{2}\Big|_A^B$$

$$= \frac{m\vec{v}_B^2}{2} - \frac{m\vec{v}_A^2}{2} = \boxed{K_B - K_A = W_{A\to B}}$$

Kinetic energy



$$\vec F_{tot} = \vec F_{fr} + \cancel{\vec N + m\vec g} =$$

$$= \vec F_{fr}$$

$$W_{fr} = \int_{x_i}^{x_B} \vec F_{fr} \cdot d\vec r = \int_{x_i}^{x_f} \left(F_{fr_x}\,dx + \overset{0\cdot 0}{F_{fr_y}\,dy} + \overset{0\cdot 0}{F_{fr_z}\,dz} \right)$$

$$= \int_{x_i}^{x_f} F_{fr_x} \cdot dx = F_{fr_x} \cdot x \Big|_{x_i}^{x_f} =$$

$$= \underset{-\mu m g}{\underset{\uparrow}{F_{fr_x}}} \cdot \underset{\displaystyle \frac{v_{fx}^2 - v_{ix}^2}{2a_x}}{\underset{\|}{(x_f - x_i)}} = -\mu m g \cdot \frac{v_{fx}^2 - v_{ix}^2}{-\mu g \cdot 2}$$

$$= \frac{F_{fr}}{m} = -\frac{\mu m g}{m}$$

$$W_{fr} = m\frac{v_f^2}{2} - \frac{m v_i^2}{2}$$