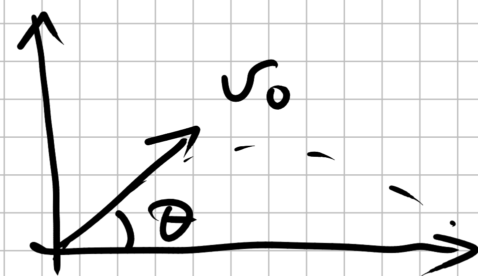
Lecture 4: projectile motion and its applications

$$X_f = -\frac{v_0^2}{a_y} \sin(2\theta)$$



to get maximum distance

$\sin(2\theta) = 1 \Rightarrow 2\theta = 90°$

$\Rightarrow \boxed{\theta = 45°}$

$\vec{a} = \{0, -9.8\frac{m}{s^2}, 0\}$

$-9.8\frac{m}{s^2} = -g$

$$X_{max} = -\frac{v_0^2}{a_y} \cdot 1 = \frac{v_0^2}{g}$$

$v_0 = 1000\ m/s = 1\ km/s = 10^3\ m/s$

$$X_{max} = \frac{1000^2}{9.8} \simeq \frac{(10^3)^2}{10} \simeq \frac{10^6}{10} \simeq 10^5 m = 100 km$$

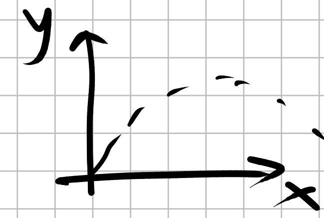
How far can a human jump?

Long jump is about 9m.

$$v_0 \simeq 10\ \frac{m}{s}$$

$$x_{max} \simeq \frac{10^2\ m/s}{10\ m/s^2} \simeq 10\ m$$

What is the maximum hiegth of a projectile.



$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{\vec{a} \cdot t^2}{2}$$

$$\vec{v}(t) = \vec{v}_0 + \vec{a} \cdot t$$

$$x(t) = \cancel{x_0} + v_{0x} t + 0 \qquad (x_0 \to 0)$$

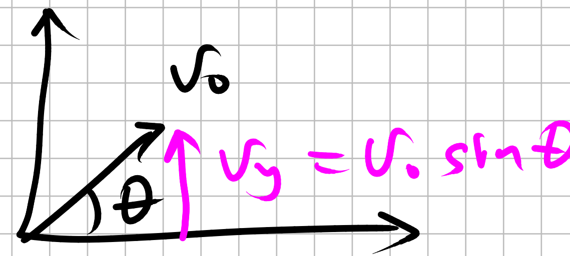
$$y(t) = \cancel{y_0} + v_{0y} t + \frac{a_y t^2}{2} \qquad (y_0 \to 0)$$

at the top $\quad v_y(t_{Top}) = 0 = v_{y0} + a_y \cdot t_T$

$$t_T = -\frac{v_{y0}}{a_y}$$

$$y(t_T) = v_{0y} t_T + \frac{a_y t_T^2}{2} = -v_{0y} \frac{v_{0y}}{a_y} + \frac{a_y}{2} \left(\frac{v_{y0}}{a_y}\right)^2$$

$$y_{max} = y(t_T) = -\frac{v_{0y}^2}{2a_y} \quad a_y = -g$$



$$\boxed{y_{max} = \frac{v_0^2 \sin^2\theta}{2g}}$$

$$\theta = 90^\circ$$

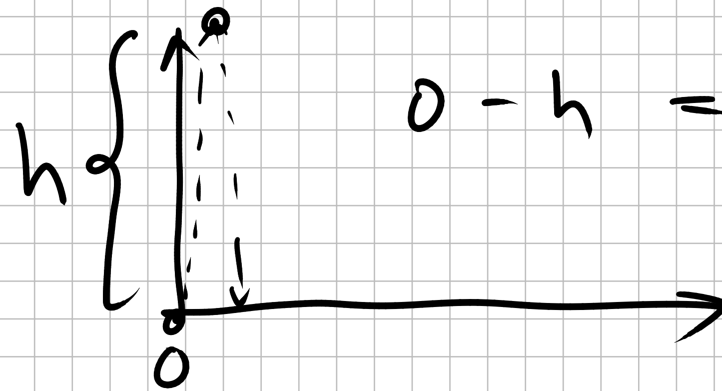
$$y_{max_{human}} = \frac{10^2 (m/s)^2}{2 \cdot 10\, m/s} = 5\, m$$

How high can a human jump?

Why pole vault can be as high as 6m?

What is equivalent hight for a collison at a given speed?

$$y_f - y_0 = \frac{v_{f_y}^2 - v_{0_y}^2}{2 a_y} \qquad (v_{0_y} = 0)$$



$$0 - h = \frac{v_{f_y}^2 - 0^2}{2 a_y}$$

$$= \frac{v_{f_y}^2}{2(-g)}$$

$$h = \frac{v_{f_y}^2}{2g} = \frac{(70\ mi/h)^2}{2 \cdot 10\ m/s^2}$$

$$70 \frac{mi}{h} = \frac{70\ mi \cdot 1600 \frac{m}{mi}}{1h \cdot 3600 \frac{s}{h}} =$$

$$= \frac{7 \cdot 1.6 \cdot 10^4}{3.6 \cdot 10^3} \approx 3.5 \cdot 10 \frac{m}{s}$$

$$h = \frac{(35\ m/s)^2}{2 \cdot 10} \simeq \frac{1000\ \frac{m^2}{s^2}}{20\ m/s^2} \simeq 50\ m$$

$$25\ \frac{mi}{h} \simeq 12\ m/s$$

$$\hookrightarrow \quad h = \frac{12^2}{2 \cdot 10} \simeq \frac{144}{20} \simeq 7\ m$$