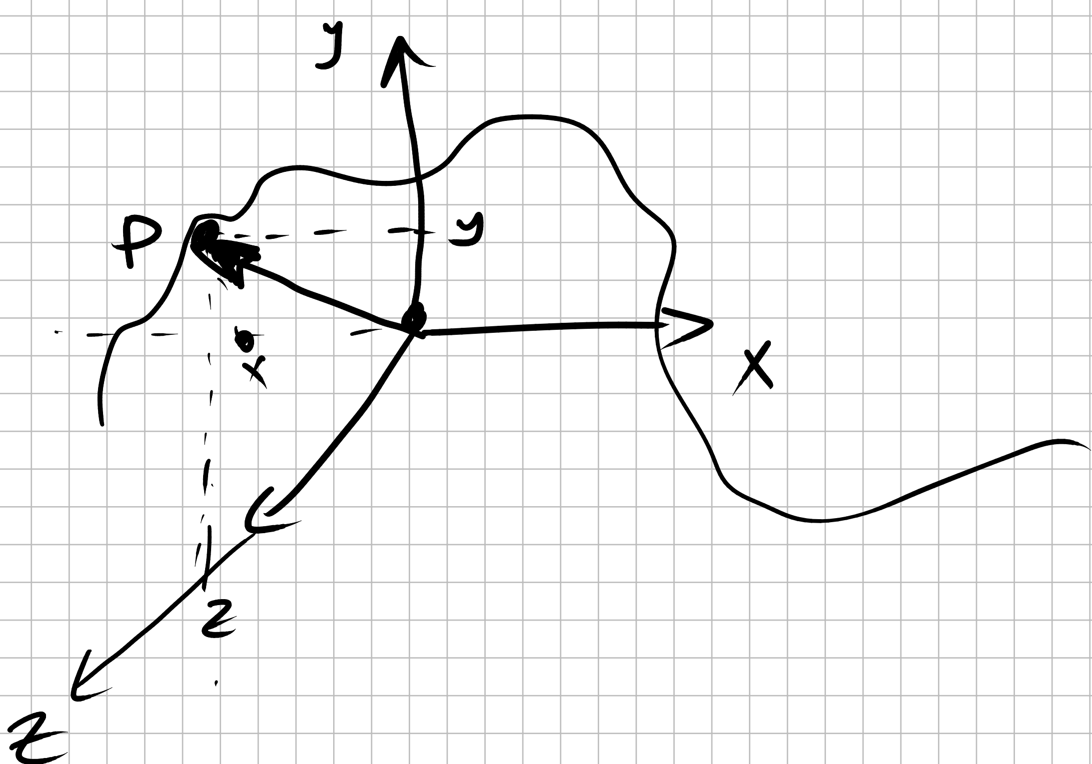
Motion in 3 D

$$P = \{x, y, z\} = \vec{r}$$

radius vector



$$\vec{r}(t) = \{x(t), y(t), z(t)\}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} =$$

$$= \left\{ \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\}$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

$$= \{x'', y'', z''\}$$

We focus on cases $\vec{a}$ is constant

acceleration

same amount and same direction

$$\vec{a} = \{ \underset{\text{const}}{a_x}, \underset{\text{const}}{a_y}, \underset{\text{const}}{a_z} \}$$

$$\Rightarrow \quad \vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{\vec{a} t^2}{2}, \quad \vec{a} = \text{const}$$

at $t=0$

$\Updownarrow$

y direction

$$y(t) = y_0 + v_{0y} \cdot t + \frac{a_y t^2}{2}$$

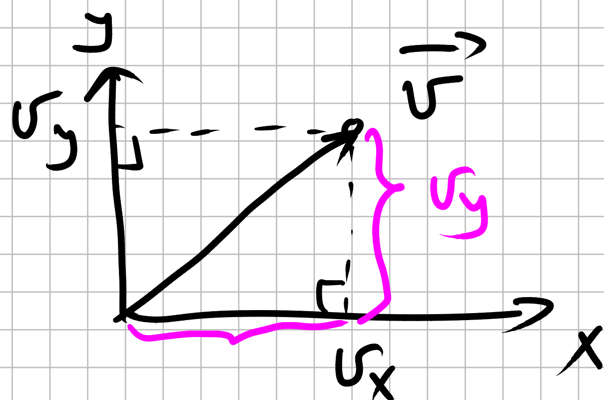
$$y(t) - y_0 = \frac{v_y^2 - v_{y0}^2}{2 a_y}$$

$|\vec{v}|$ length of the vector

$$= \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{|\vec{v} \cdot \vec{v}|}$$

scalar product

$$\vec{A} \cdot \vec{B} = A_x \cdot B_x + A_y \cdot B_y + A_z \cdot B_z$$



Projectile motion

z part is alway 0, assumes $\vec{a}$ is const

$$\vec{a} = \{a_x, a_y, a_z\} = \{0, a_y, 0\}$$

$$\vec{v} = \{v_x, v_y, 0\}$$

$$\vec{r} = \{x, y, 0\}$$

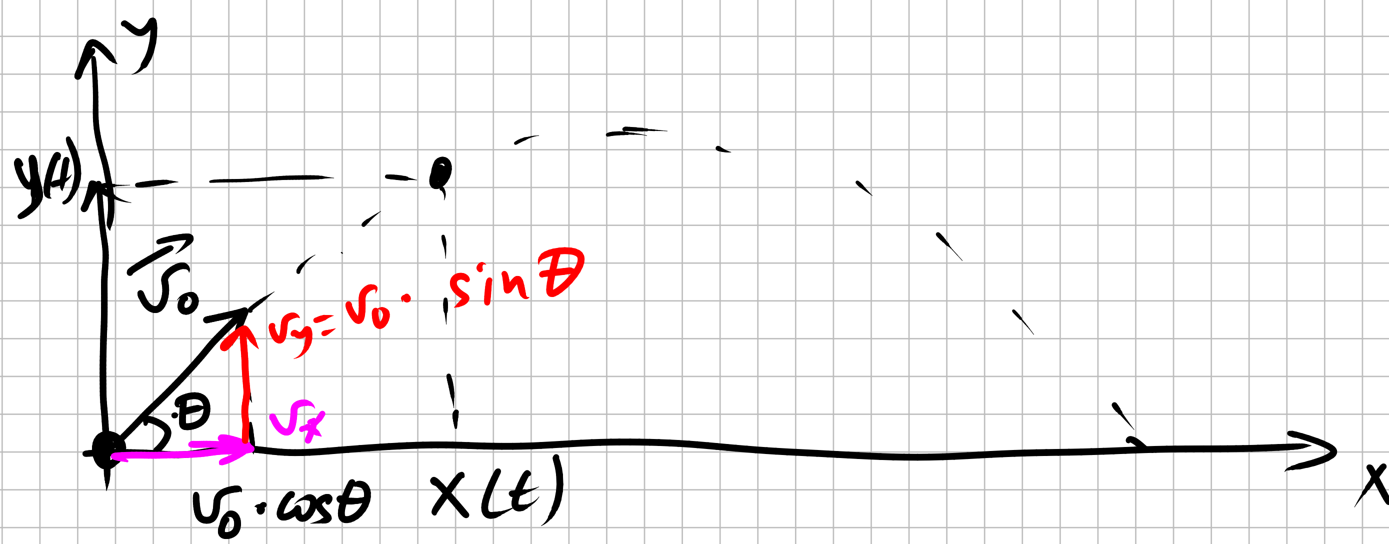
$$\boxed{\vec{r}(t) = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{\vec{a} \cdot t^2}{2}}$$

$$\begin{cases} x(t) = x_0 + v_{0x} t \\ y(t) = y_0 + v_{0y} t + \dfrac{a_y t^2}{2} \end{cases}$$

by choice of the reference frame $x_0 = 0$, $y_0 = 0$

$$x = v_{0x} t \quad ; \quad v_x(t) = v_{x0}$$

$$y = v_{0y} t + \frac{a_y t^2}{2} \quad) \quad v_y(t) = v_{y0} + a_{y0} \cdot t$$



$$y = 0 = v_{0y} t_f + \frac{a_y t_f^2}{2}$$

at which time we hit the ground
$y = 0$

$$t_f = 0 \ , \ -\frac{2 v_{0y}}{a_y}$$

$$x_f = v_{0x} \cdot t_f = v_{0x} \cdot \left(-\frac{2 v_{0y}}{a_y}\right) =$$

$$x_f = v_0 \cos\theta \left(\frac{-2 v_0 \sin\theta}{a_y} \right) = -\frac{v_0^2}{a_y} \underbrace{2\cos\theta \cdot \sin\theta}_{\sin(2\theta)}$$

$$\boxed{x_f = -\frac{v_0^2}{a_y} \sin(2\theta) = \frac{v_0^2}{g} \sin(2\theta)}$$

$$a_y = -\text{number} = -9.8 \ \frac{m}{s^2} = -g$$

$$\boxed{g = 9.8 \ m/s^2}$$