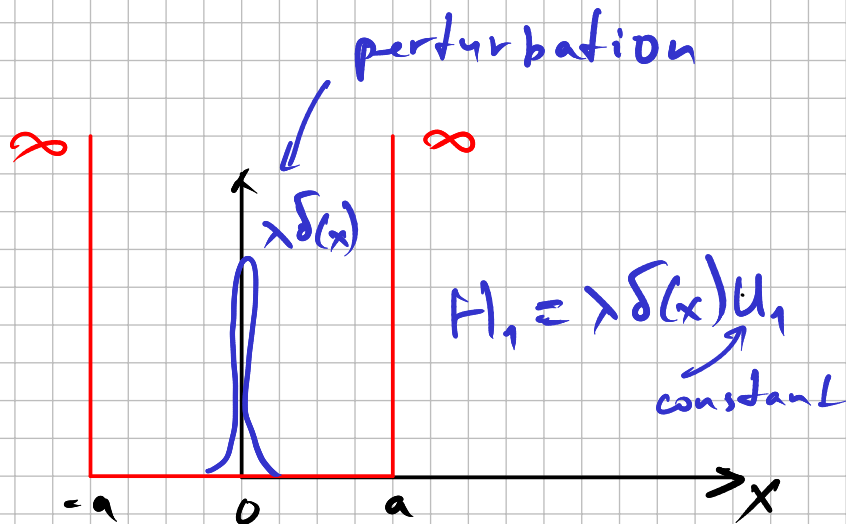
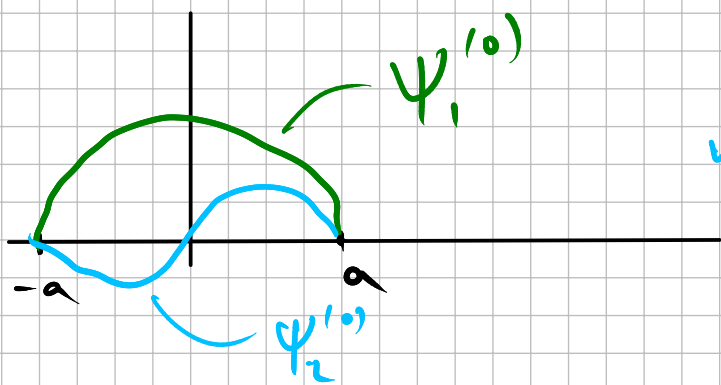


# Example: Square well + $\delta$ in the middle



$\psi_n^{(0)}(|x| > a) = 0$   
Boundary condition



$$\hat{H}_0 = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + U_0(x)$$

$$U_0 = \begin{cases} 0, & |x| < a \\ \infty, & |x| > a \end{cases}$$

$$\psi_n^{(0)} = \sqrt{\frac{1}{a}} \times \begin{cases} \cos\left(\frac{\pi}{2a} n x\right), & n=1, 3, 5, \dots \\ \sin\left(\frac{\pi}{2a} n x\right), & n=2, 4, 6, \dots \end{cases} \text{ for } |x| < a$$

$$E_n^{(0)} = \frac{\hbar^2}{2m} \left(\frac{\pi \cdot n}{2a}\right)^2$$

note that  $\psi_n^{(0)}$  with even 'n' has zero value at  $x=0$

i.e. seems to not interact with  $H_1 = \lambda \delta(x)$

let's confirm that  $E_n^{(1)}$  for  $n=2, 4, 6 \dots$  (even) is zero.

$$E_n^{(1)} = \langle \psi_n^{(0)} | H_1 | \psi_n^{(0)} \rangle =$$

$$= \int_{-a}^a \sqrt{\frac{1}{a}} \cos\left(\frac{\pi}{2a} n x\right) \lambda \delta(x) U_1 \sqrt{\frac{1}{a}} \cos\left(\frac{\pi}{2a} n x\right) dx =$$

$$= \lambda \frac{1}{a} \times \frac{\cos^2\left(\frac{\pi}{2a} n \cdot 0\right)}{\sin^2\left(\frac{\pi}{2a} n \cdot 0\right)} = \lambda \frac{1}{a} \times \begin{pmatrix} U_1 \\ 0 \end{pmatrix}$$

given by  
cos  $n=1, 3, 5$

given by sin,  $n=2, 4, 6 \dots$

$$E_n^{(1)} = \begin{cases} \frac{\lambda U_1}{a}, & n=1, 3, 5 \dots \\ 0, & n=2, 4, 6 \dots \end{cases}$$

What about  $c_{kn} = \frac{\langle \psi_k | H_1 | \psi_n \rangle}{E_n^{(1)} - E_k^{(0)}}$ , since  
 sin-like  $\psi$  do not overlap with  $H_1$   
 we expect no change in  $\psi_{n=2,4,6}$

$$\begin{aligned} \langle \psi_k | H_1 | \psi_n \rangle &= \int_{-a}^a dx \sqrt{\frac{1}{a}} \frac{\cos\left(\frac{\pi}{2a} nx\right)}{\sin\left(\frac{\pi}{2a} nx\right)} \cdot \lambda U_1 \delta(x) \cdot \sqrt{\frac{1}{a}} \frac{\cos\left(\frac{\pi}{2a} kx\right)}{\sin\left(\frac{\pi}{2a} kx\right)} \\ &= \frac{1}{a} \lambda U_1 \frac{\cos(0)}{\sin(0)} \cdot \frac{\cos(0)}{\sin(0)} = \begin{cases} \frac{1}{a} \lambda U_1, & n \in 1, 3, 5, \dots \text{ and } k \in 1, 3, 5, \dots \\ 0 & \text{if } k \text{ or } n \text{ is even} \end{cases} \end{aligned}$$

Note that even  $E_n^{(2)}$  for even  $n$  is zero