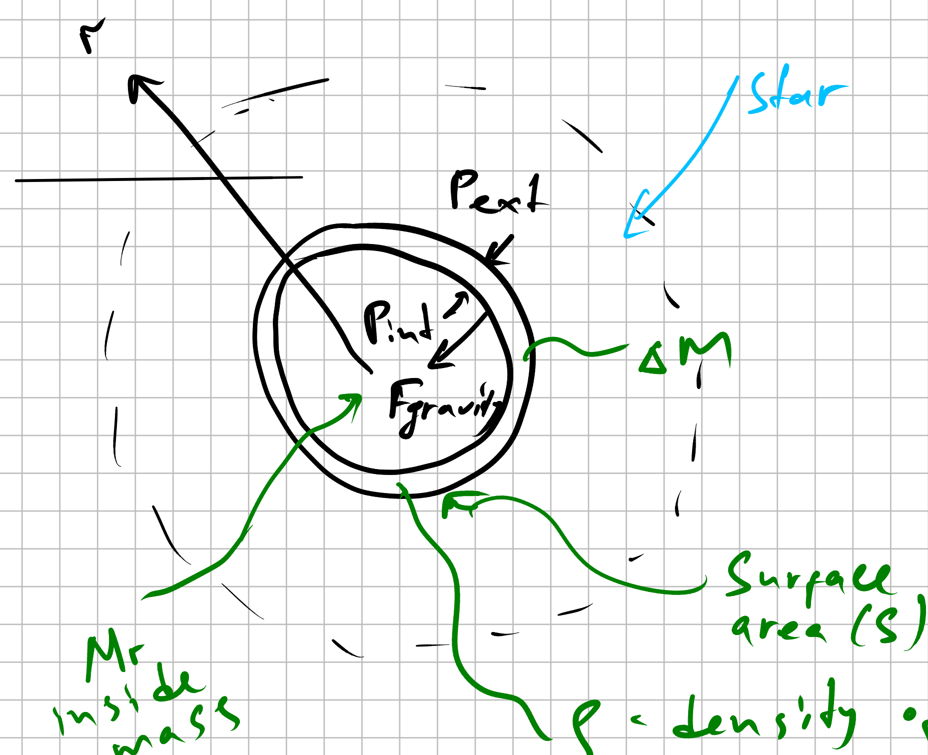
White dwarf stars

While a star sustain nuclear reaction it stays hot, so gravitational pull is counter acted by the light pressure and thermal pressure.

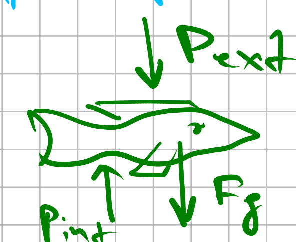
Once star cools the above mechanism is no good and we are left with the degeneracy pressure.



ρ - density of star

Hydrostatic Equilibrium condition

same as for fish



$$\Delta m\, a = (P_{int} - P_{ext})\cdot S - G\frac{\Delta m \cdot M_r}{r^2} = 0$$

$$0 = -\Delta P \cdot S - G\,\frac{\rho \cdot \Delta V}{r^2} M_r$$

$$-\Delta P \cdot S = G\, \rho \cdot \frac{S \cdot \Delta r}{r^2} M_r$$

$\frac{4\pi r^3}{3} \rho$

assuming $\rho = \text{const}$

$$\frac{\Delta P}{\Delta r} \xrightarrow[\Delta r \to 0]{} \frac{dP}{dr} = -G \rho \frac{M_r}{r^2}$$

$$\frac{dP}{dr} = -G \rho^2 \frac{4\pi}{3} r \implies dP = -G \rho^2 \frac{4\pi}{3} r\, dr$$

center of the star

$$P_c = P(r=0) = \int dP = -\int_R^0 G \rho^2 \frac{4\pi}{3} r\, dr + P(R)$$

$P(R) = 0$

outside of star radius R

$$\boxed{P_c = G \rho^2 \frac{2\pi}{3} R^2}$$

$$\rho = \frac{M_s}{V_s} = \frac{M_s}{\frac{4\pi}{3} R^3}$$

mass of the star

$$P_c = G \cdot \frac{M_s^2}{\left(\frac{4\pi}{3} R^3\right)^2} \cdot \frac{2\pi}{3} R^2 = G\, 3 \frac{M_s^2}{8\pi}$$

$$\boxed{P_c = \frac{3}{8\pi} G \frac{M_s^2}{R^4}}$$

For star to be stable (non collapsing)

$$P_c \leq P_f = \frac{2}{3} \frac{\hbar^2 (3\pi^2 N)^{5/3}}{10\pi^2 m_f} V^{-5/3}$$

$$= \frac{2}{3} \hbar^2 \frac{1}{10\pi^2} \left(3\pi^2 \frac{N}{V} \right)^{5/3} \frac{1}{m_f} =$$

$$= \Big/ \quad N = \frac{1}{2} \frac{M_s}{m_{proton}} \quad \Big| =$$

number of electrons

$\frac{1}{2}$ - for every proton there is a neutron

$$= \frac{2}{3} \frac{\hbar^2}{10\pi^2} (3\pi^2)^{5/3} \left(\frac{1}{2} \frac{M_s}{m_p} \frac{1}{\frac{4\pi}{3} R^3} \right)^{5/3} \frac{1}{m_{electron}} \geq P_c = \frac{3}{8\pi} G \frac{M_s^2}{R^4}$$

$$\frac{2}{3} \frac{\hbar^2}{10\pi^2} (3\pi^2)^{5/3} \left(\frac{3}{8\pi} \frac{M_s}{m_p} \right)^{5/3} \frac{1}{R^5} \frac{1}{m_{electron}} \geq \frac{3}{8\pi} G \frac{M_s^2}{R^4}$$

$$\boxed{R \leq \frac{16}{9}\,\frac{1}{10\pi}\,\frac{\hbar^2}{G}\,\frac{1}{M_s^{1/3}}\,(3\pi^2)^{5/3}\left(\frac{3}{8\pi}\,\frac{1}{m_p}\right)^{5/3}\frac{1}{m_e}}$$

let's estimate maximum radius for a sun like star:

$M_s = M_\odot = 2 \cdot 10^{30}\,kg$

$m_p = 1.67 \cdot 10^{-27}\,kg$, $m_e = 9.1 \cdot 10^{-31}\,kg$

$\hbar = 1.05 \cdot 10^{-34}\,J \cdot s$

$G = 6.674 \cdot 10^{-11}\,\frac{m^3}{kg \cdot s^2}$

$\boxed{R = 2.84 \cdot 10^{6}\,m}$, compare to Earth radius $R_e = 6.4 \cdot 10^{6}\,m$

white dwarf Radius

electron degeneracy supported radius

Note if star mass is larger $\Rightarrow$ R is smaller but pressure is higher

at some high pressure electrons are "pressed into" proton $\Rightarrow$ $p + e = n$ and become neutron $\Rightarrow$ neutron star with very different $R_n < R_e$