Homework 07 Name: ____________________

Problem 1 (10 points)
In class we derived the scattering amplitude $f(0)$ for Yukawa potential

$$V(r) = \beta \frac{e^{-\mu r}}{r}$$

in the Born approximation case.
Calculate the total cross-section σ for this potential. Do not forget that change of momentum vector $\hbar\vec{\varkappa} = \hbar(\vec{k'} - \vec{k})$ contains dependence on θ.

Problem 2 (10 points)
Use result of the previous problem and show that Rutherford scattering (scattering on the Coulomb potential) has infinite cross-section. Hint: think what should you change to convert Yukawa potential to Coulomb one.
Note: Technically we should not use results of the Born approximation here since our potential does not drop fast enough with distance. But we get the correct result anyway.

Problem 3 (10 points)
Consider scattering of the following potential

$$V(r) = \begin{cases} V_0, r \leq a \\ 0, r > a \end{cases}$$

Using the Born approximation find the total cross-section σ.

Problem 4 (10 points)
Using the Born approximation find the total cross-section σ for the spherical shell potential

$$V(r) = \alpha\delta(r - a)$$

Problem 5 (10 points)
Consider 1D infinite well

$$V(x) = \begin{cases} 0, & 0 \leq x \leq L \\ \infty, & \text{otherwise} \end{cases}$$

Assume that initially a particle with mass (m) sits at the ground level. Then the time dependent Hamiltonian $\hat{\mathbf{H}}_1(t)$ acts on the system

$$\hat{\mathbf{H}}_1(t) = \begin{cases} \alpha\delta(x - L/2), & 0 \leq t \leq \tau \\ 0, & \text{otherwise} \end{cases}$$

- Find expression for transition probabilities to other energy levels as function of time.
- Show that transition probabilities do not change after time $t \geq \tau$.
- Find τ which maximizes the transition probability to the n_{th} energy level.