Homework 06 Name: ____________________

Problem 1 (10 points)
In class we covered non relativistic fermions. That derivation used "classical" connection between momentum and energy of the particle $E_k = p^2/(2m) = (\hbar k)^2/(2m)$. However, in dense enough stars, fermions move with velocities close to the speed of light.
For such relativistic (fast) fermions $E_k = cp = c\hbar k$.
Express, total energy of fermions via number density of them $n_f = N/V$ for a star. The resulting formula may use mass of a fermion m_f, mass of the star (M), and fundamental constants.

Problem 2 (10 points)
Using above results, find the degeneracy pressure of the relativistic fermions.

Problem 3 (10 points)
Use the result of the previous problem and find "true" limit for a stable white dwarf mass which still can be supported by the relativistic degeneracy pressure of electrons (The Chandrasekhar Limit).

Problem 4 (10 points)
Degeneracy energy in 2D. Assume that N ($N \gg 1$) fermions were put into a 2D square well with area A. Find the total energy of them in the ground state. Treat fermions as non relativistic (slow) particles.

Problem 5 (10 points)
In class we discussed the relationship which governs allowed wave number k and wave function phase shift ϕ for the periodic 1D chain of atoms spaced by a:

$$\cos(\phi) = \cos(z) + \beta\frac{\sin(z)}{z} = f(z)$$

(a, 5 points) Make a computer generated plot of $f(z)$ for $\beta = 5$. On this plot sketch allowed bands (all of them). It is ok to use hand drawn overlay for allowed bands on this plot.
(b, 5 points) Find numerically the lowest value of z. Note: you need to solve the transcendental equation, i.e. there is no analytical closed form for the answer. So do you best to estimate it or use numerical methods (Phys256).