

Homework 05

Name: _____

Problem 1 (30 points, each item 5 points)

Disregard the spin coordinate in this problem.

Suppose we have N identical bosons of mass m in a one dimensional box of length $L = Na$ (the allowed x is between 0 and L) with the Hamiltonian

$$\hat{H} = \sum_{i=1}^N \hat{p}_i^2 / (2m) + \lambda/2 \sum_{i \neq j} \delta(x_i - x_j)$$

- Set $\lambda = 0$. How should the ground state wave function look? Explicitly show it for $N = 3$. **Hint:** Google about Slater determinant (for fermions) and permanents (for bosons), I suggest to read about Slater determinant first.
- How should it look for N particles, how many summed products should it have? Do not express it explicitly this time, just call it $\Psi(x_1, x_2, \dots, x_N)$.
- Now assume that λ is not zero but small. Find the ground state energy. Hint: recall perturbation methods.
- Replace all bosons with fermions. Assume that $\lambda = 0$. Explicitly, show the ground state wave function for $N = 3$.
- What is the ground state energy for $\lambda = 0$ and $N \gg 1$?
- Allow $\lambda \neq 0$ and $N \gg 1$, prove that 1st order correction is independent of λ . Hint: what should be the value of the fermions wave function if two arbitrary coordinates are the same?

Problem 2 (10 points)

In class we discussed white dwarfs supported by electrons degeneracy pressure. Once gravitational pull becomes larger than electron degeneracy pressure, the electrons are "pressed" into a proton, and together they form a neutron (which is also a fermion). Assume that all electrons and protons replaced with neutron. Also, assume that initially the star was made of heavy elements for which the number of neutrons equal to number of protons.

- Find the maximum mass of this neutron star which can be supported by the neutron degeneracy pressure. Express it in the solar mass (M_\odot) units.
- What is the maximum radius of such neutron star? Compare to the size of the Williamsburg.