Homework 04 Name: ____________________

Attention! Wording of problem 3, 4, and 5 is changed.

Problem 1 (10 points)
Consider the infinite one dimensional square well, with

$$U(x) = \begin{cases} 0 & \text{if} |x| \leq a; \\ \infty & \text{if} |x| > a \end{cases}$$

Use the variational principle and guess/trial/anzatz function $\Psi(x) = C(a - |x|)^b$, where $b > 0$ is a variable parameter and C is the normalization constant, to find the estimate of the ground level energy.

Problem 2 (10 points)
Consider a harmonic oscillator

$$\hat{\mathbf{H}}_0 = \frac{\hat{\mathbf{p}}^2}{2m} + \frac{1}{2} m\omega^2 \hat{\mathbf{x}}^2 \tag{1}$$

Apply the variational principle to find the ground state energy estimate. Use the guess/trial/anzatz function

$$\Psi(x) = \frac{C}{x^2 + b^2}$$

where C is normalization constant and b is variable parameter.

Problem 3 (10 points)
Consider a particle of mass m and charge q (sign is unknown) is moving in one dimension ('x') and influenced by the electric field

$$\vec{E}(x) = \begin{cases} E_0 \vec{x} & \text{if} x > 0; \\ -E_0 \vec{x} & \text{if} x < 0; \end{cases}$$

Here, $\vec{x}$ stands for unit vector along 'x' direction.
Consider a trial $\Psi(x) = C \exp(-a|x|)$ and estimate the ground-state energy by minimizing the expectation value of the energy with respect to parameter a.
For which sign of charge q the bound state can exist.

Problem 4 (10 points)
Generally, we do not know the ground level eigen state $|\psi_g\rangle$. However, if we have some information about $|\psi_g\rangle$ and can guess a function $|\psi\rangle$ such that $\langle\psi|\psi_g\rangle = 0$. Show that in this case $\langle\psi| \hat{\mathbf{H}} |\psi\rangle \geq E_{fe}$, where E_{fe} is the energy of the first exited state.

Problem 5 (10 points)
If the potential is even then the ground state function is even too. For example, ground state of the harmonic oscillator (Eq. 1) is even. Thus based on the result of the previous problem, we can use an odd trial function to estimate the energy of the first exited state. Apply this idea to the Hamiltonian from the problem 2 and its anzatz function for the ground state. This time use the trial function

$$\Psi(x) = x\frac{C}{x^2 + b^2}$$

and estimate E_{fe}. You should arrive to somewhat useless result, which will teach you about one more condition required for the use the theorem outlined in Problem 4. Discuss this condition.
Bonus 10 points: modify the anzatz function, and get a better estimate of the first exited state energy.