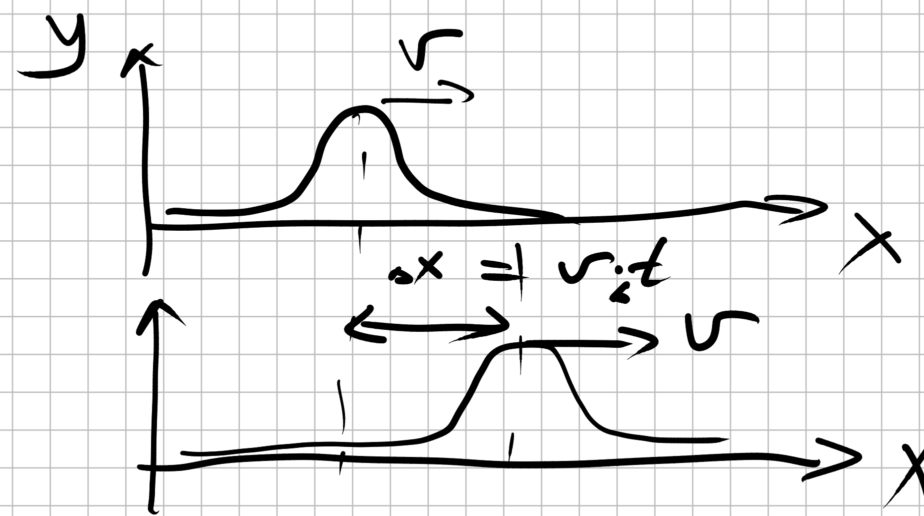
Traveling waves (pulses)

$$y(x,t) =$$

$$= y(x - vt)$$



$$y(x,t) = y(x + vt)$$

Wave equation

$y(x \pm vt)$

$$\frac{\partial y}{\partial x} = \frac{\partial y(x \pm vt)}{\partial (x \pm vt)} \cdot \frac{\partial (x \pm vt)}{\partial x}$$

$x \pm vt \Longleftrightarrow u$

$$\frac{\partial y}{\partial x} = \underbrace{\left(\frac{\partial y}{\partial u}\right)}_{y'} \cdot \underbrace{\frac{\partial u}{\partial x}}_{1} = y'$$

$$\frac{\partial y}{\partial t} = \underbrace{\left(\frac{\partial y}{\partial u}\right)}_{y'} \frac{\partial u}{\partial t} = y' \cdot (\pm v)$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial (y')}{\partial x} = y'' \cdot \underbrace{\left(\frac{\partial u}{\partial x}\right)}_{1} = y''$$

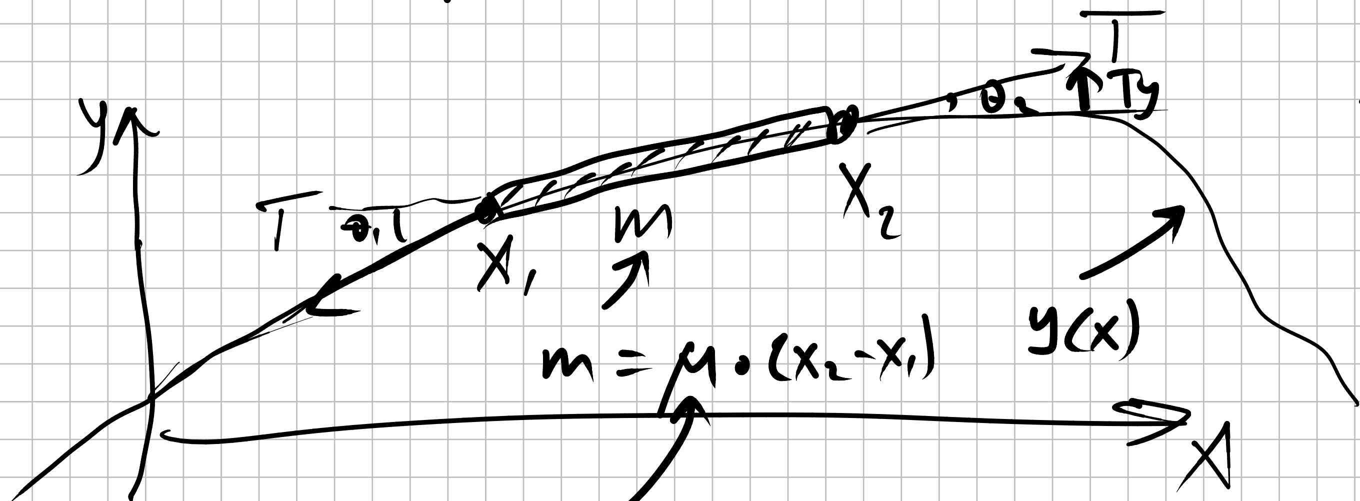
v.r. to u

$$\frac{\partial^2 y}{\partial t^2} = y'' \cdot (\pm v)^2$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}} \quad , \quad y(x \pm vt)$$

wave equation

Rope



$m = \mu \cdot (x_2 - x_1)$

linear mass density

$F_y = T \cdot \sin(\theta_2) - T\sin(\theta_1)$

$\theta_1, \theta_2 \ll 1$

$\sin(\theta) \longrightarrow \theta$

$\tan(\theta) \longrightarrow \theta$

$$F_y = T(\underbrace{\theta_2}_{\left.\frac{dy}{dx}\right|_{x_2}} - \underbrace{\theta_1}_{\left.\frac{dy}{dx}\right|_{x_1}})$$

$$F_y = T\cdot\left(\frac{dy}{dx}\Big|_{x_2} - \frac{dy}{dx}\Big|_{x_1}\right) = m \cdot \underbrace{\frac{d^2y}{dt^2}}_{a_y} = \mu \underbrace{(x_2 - x_1)}_{\Delta x} \cdot \ddot{y}$$

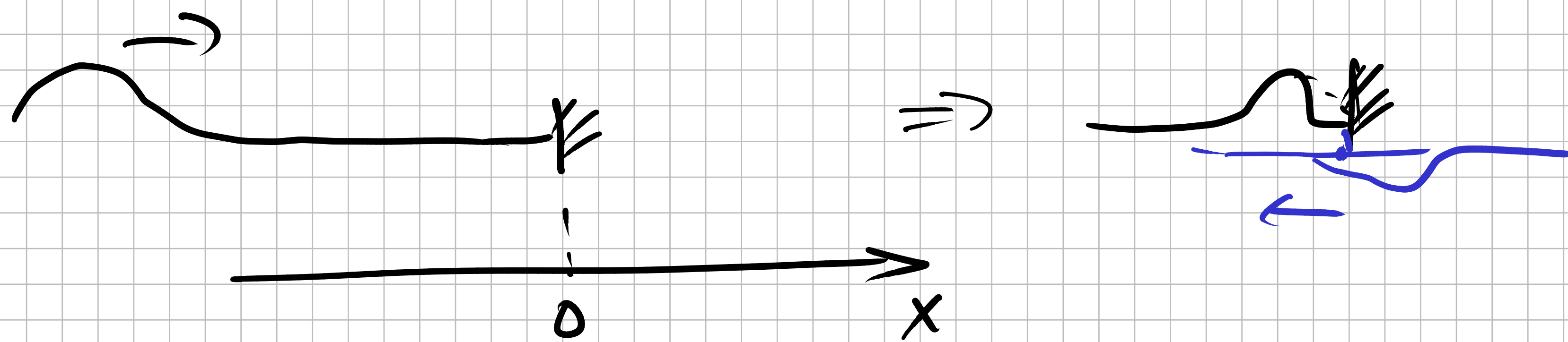
$$T \; \frac{\left(\frac{dy}{dx}\Big|_{x_2} - \frac{dy}{dx}\Big|_{x_1}\right)}{\Delta x} = \mu \cdot \ddot{y}$$

$$T \frac{\partial^2 y}{\partial x^2} = \mu \frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T} \frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

$$\Rightarrow \quad v^2 = \frac{T}{\mu} \quad \Rightarrow \quad \boxed{v = \sqrt{\frac{T}{\mu}}}$$

Fixed boundary condition



$$y(x,t) = \overset{\rightarrow v}{y_1(x-vt)} + \overset{\leftarrow v}{y_2(x+vt)}$$

Boundary condition $y(x=0,t) = 0 \Rightarrow y_1(-vt) + y_2(vt) = 0$

$$y_1 = -y_2$$

sign flip

reflected downword