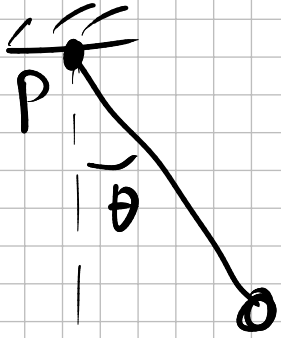


# Pendulums

$$\omega = \sqrt{\frac{Lmg}{I_P}}$$



for simple pendulum

$$\omega = \sqrt{\frac{g}{L}}$$

angular frequency  
not velocity

$$\ddot{x} = -\omega^2 x$$

$$\ddot{\theta} = -\omega^2 \theta \Rightarrow \theta(t) = A \cdot \cos(\omega t + \varphi)$$

$\swarrow \theta_A$

$$\dot{\theta} = \frac{d}{dt} \theta = \frac{d}{dt} (\theta_A \cos(\omega t + \varphi))$$

$$= -\theta_A \cdot \omega \sin(\omega t + \varphi)$$

$$= \underbrace{-\omega \theta_A}_{\text{angular velocity}} \sin(\omega t + \varphi) = \omega \psi(t)$$

angular  
velocity

$$\ddot{\theta} = \alpha = \ddot{\theta} = \dot{\omega} = -\omega^2 \theta_A \cos(\omega t + \varphi) = -\omega^2 \theta_A \cos(\omega t + \varphi)$$

↑ angular acceleration

sample problem:

at  $t=0$  →  $\begin{cases} \theta_0 = \theta(t=0) \\ \omega_0 = \omega(t=0) \end{cases}$

$$\theta_A \cos(\omega t + \varphi)$$

↑ given  $\omega$

$$\begin{aligned} \theta_0 &= \theta_A \cos(\omega t + \varphi) = \\ &= \theta_A \cos(\varphi) \end{aligned}$$

$$\begin{aligned} \omega_0 &= -\theta_A \omega \sin(\omega t + \varphi) = \\ &= -\theta_A \omega \sin(\varphi) \end{aligned}$$

$$\frac{z\omega_0}{\theta_0} = -\omega \frac{\sin\varphi}{\cos\varphi} = -\omega \tan(\varphi) \Rightarrow \varphi$$

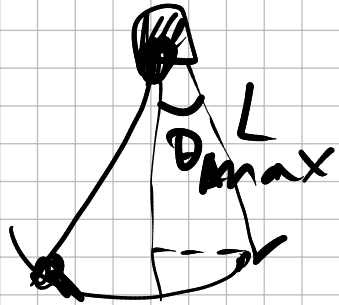
$$\begin{aligned} (\theta_0 \omega)^2 + z\omega_0^2 &= (\theta_A \omega \cos(\dots))^2 \\ &+ (\theta_A \omega \sin(\dots))^2 = \end{aligned}$$

$$= (\theta_A \omega)^2 \underbrace{(\cos^2(\dots) + \sin^2(\dots))}_1$$

$$\theta_A = \sqrt{\theta_0^2 + \left(\frac{z\omega_0}{\omega}\right)^2}$$

$$\omega = \sqrt{\frac{g}{L}} \quad \rightarrow \quad T = 2\pi \sqrt{\frac{L}{g}}$$

People



$$d = L \cdot \sin \theta_{max}$$

$$v \approx \frac{2d}{T/2} \approx \frac{d}{T}$$

$$v \approx \frac{L \cdot \sin \theta_{max}}{\sqrt{\frac{L}{g}}}$$

$$v \approx \sqrt{Lg}$$