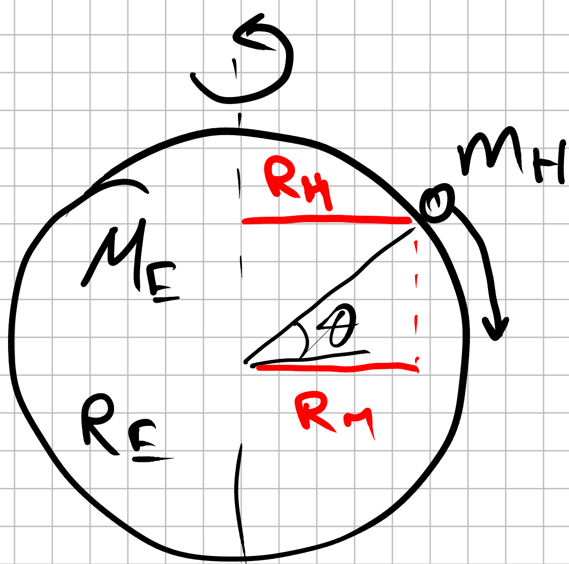
Angular momentum conservation

if $\vec{\tau}_{ext,net} = 0 \Rightarrow \vec{L} = const$



$R_H = R_E \cdot \cos\theta$

$$I = \sum_i m_i r_i^2 = $$

$$= \underbrace{\sum_{i \in E} m_i r_i^2}_{I_E} + \underbrace{\sum_{i \in H} m_i r_i^2}_{I_H}$$

$$I_E = \frac{2}{5} M_E \cdot R_E^2$$

$$\vec{L} = const = (I_E + I_H) \cdot \omega$$

$$(I_E + I_{M_i})\,\omega_i = (I_E + I_{M_f})\,\omega_f$$

$$\omega_i = \frac{2\pi}{T_i} \qquad \omega_f = \frac{2\pi}{T_f}$$

$$\frac{T_i}{\cancel{2\pi}} \frac{1}{I_E + I_{M_i}} = \frac{T_f}{\cancel{2\pi}} \frac{1}{I_E + I_{M_f}}$$

$$T_f = \frac{I_E + I_{M_f}}{I_E + I_{M_i}} \cdot T_i$$

$$\Delta T = T_f - T_i = \left(\frac{I_E + I_{M_f}}{I_E + I_{M_i}} - 1 \right) T_i$$

$$\Delta T = \left[\frac{I_E \left(1 + \frac{I_{Mf}}{I_E}\right)}{I_E \left(1 + \frac{I_{Mi}}{I_E}\right)} - 1 \right] \cdot T_i$$

$$\frac{1}{1 \mp X} \xrightarrow[X \to 0]{} \approx 1 \pm X \quad (*)$$

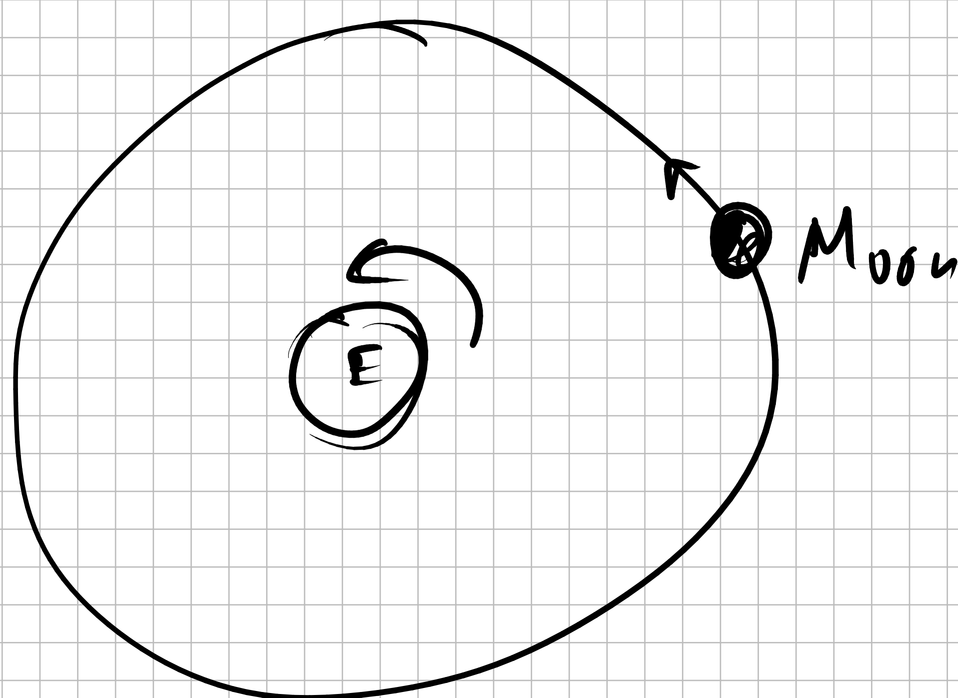
$$\Delta T = \left[\left(1 + \frac{I_{Mf}}{I_E}\right)\left(1 - \frac{I_{Mi}}{I_E}\right) - 1 \right] T_i$$

$$= \left[1 + \frac{I_{Mf}}{I_E} - \frac{I_{Mi}}{I_E} + \frac{I_{Mi} I_{Mf}}{I_E^2} - 1 \right] T_i \quad \left(\frac{I_{Mi} I_{Mf}}{I_E^2} \to 0\right)$$

$$= \frac{I_{Mf} - I_{Mi}}{I_E} \cdot T_i = \frac{m_M R_E^2 - m_M R_E^2 \cos^2\theta}{\frac{2}{5} M_E R_E^2} \cdot T_i$$

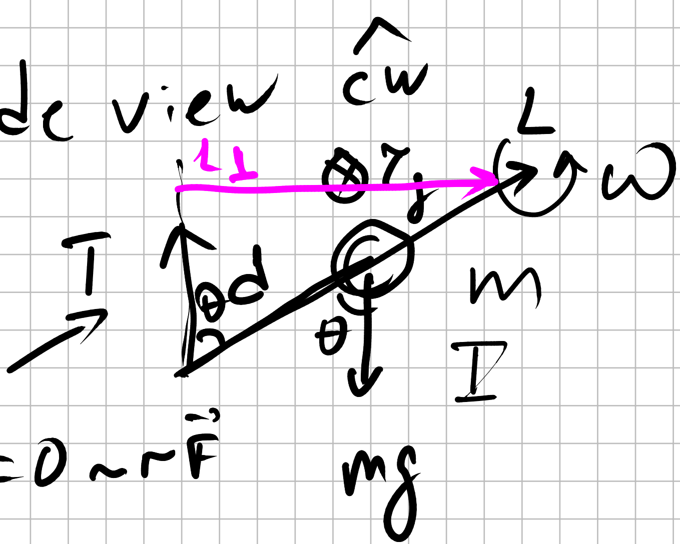
$$2T_i \cdot \frac{5}{2} \frac{m_M}{M_E} \sin^2\theta \approx 3.6 \cdot 10^{-17}\,s$$

Earth - Moon



Gyroscope precession

side view

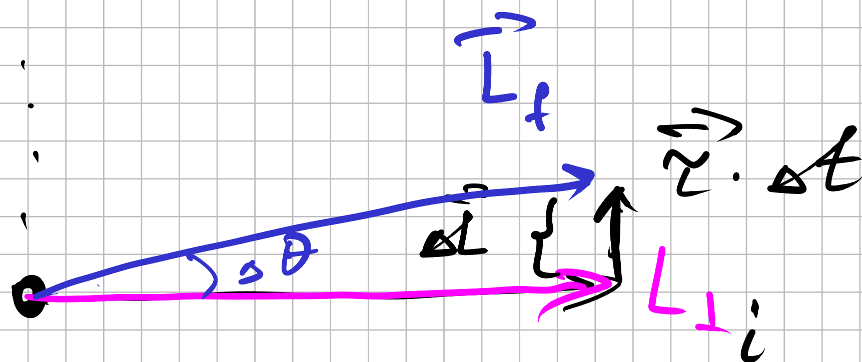


$$\tau_T = 0 \sim r\vec{F}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{ext} = \vec{\tau}_g =$$

$$= d \cdot mg \cdot \sin\theta \cdot \hat{cw}$$

Top view



$$\omega_p = \frac{\Delta\theta}{\Delta t} = \frac{\tau \Delta t}{\Delta t \cdot L_\perp}$$

$$= \frac{d \cdot mg \cdot \sin\theta}{I \cdot \omega \cdot \sin\theta}$$

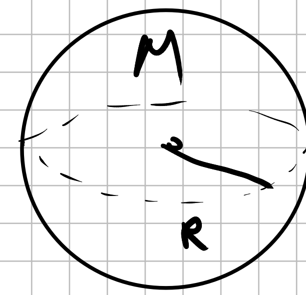
$$\tan \Delta\theta = \frac{\vec{\tau}\Delta t}{L_\perp}$$

$\Delta\theta \ll 1$

$$\boxed{\omega_p = \frac{d \cdot mg}{I \cdot \omega}}$$

$M_e = 5.9 \cdot 10^{24}$ kg

$R_e = 6.378 \cdot 10^{6}$ m



$I = \frac{2}{5} M R^2$

↑ sphere