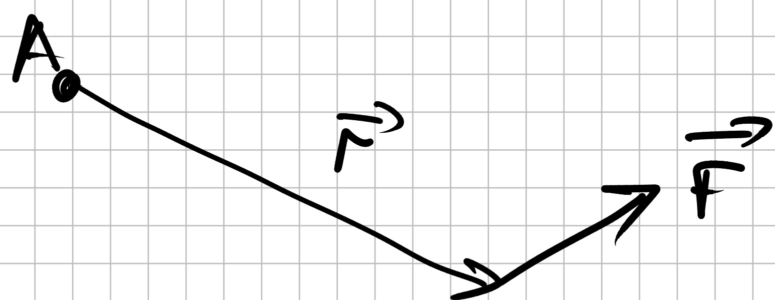
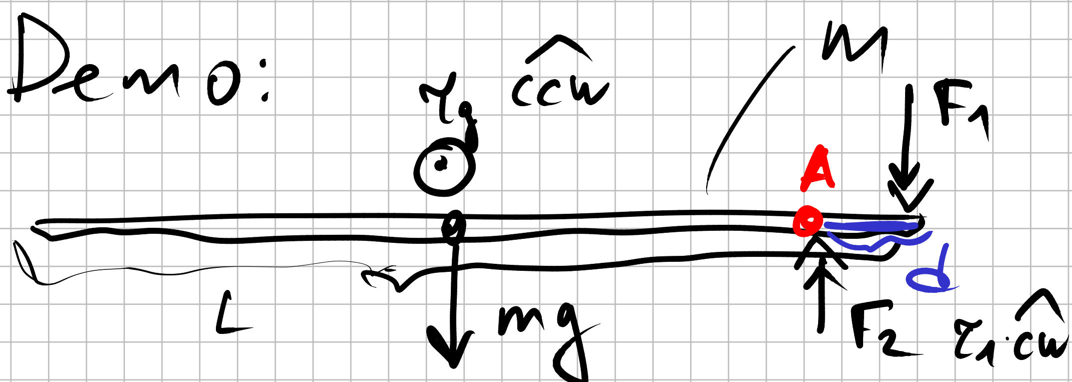
Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$I \cdot \vec{\alpha} = \sum_i \vec{\tau}_{ext}$$



Demo:



$$m\vec{a} = m\vec{g} + \vec{F_1} + \vec{F_2} = 0$$

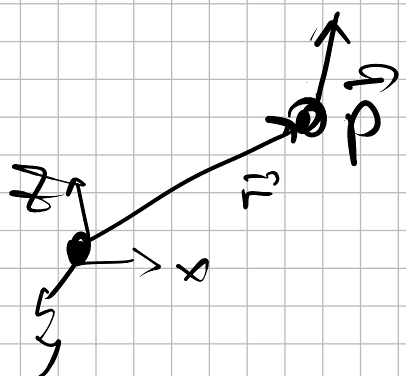
$$\vec{\alpha} = 0 = \sum \vec{\tau} = \vec{\tau}_g + \vec{\tau}_1 + \vec{\tau}_2$$

$$\vec{\alpha} = 0 = \left(\frac{L}{2} - d\right) mg\, \widehat{ccw} + (d) \cdot F_1\, \underbrace{\widehat{cw}}_{= -\widehat{ccw}} + \overset{\vec{\tau}_2}{\overset{\parallel}{0}}$$

$$0 = \left[\left(\frac{L}{2} - d\right) mg - d \cdot F_1\right] \widehat{ccw}$$

$$F_1 = \frac{\left(\frac{L}{2} - d\right)}{d} mg$$

Angular momentum



$$\vec L = \vec r \times m\vec v = \vec r \times \vec p$$

$$\frac{d\vec L}{dt} = \frac{d\vec r}{dt} \times \vec p + \vec r \times \frac{d\vec p}{dt}$$

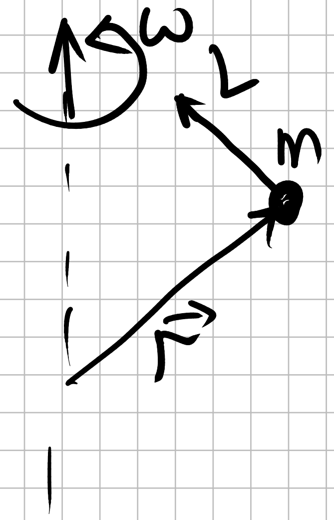
$$= \vec v \times \vec p + \underbrace{\vec r \times \vec F}_{\vec \tau}$$

$$= \vec v \times m\vec v + \vec \tau$$

$$= m \cdot (\underbrace{\vec v \times \vec v}_{=0}) + \vec \tau$$

$$\boxed{\frac{d\vec L}{dt} = \vec \tau = I\vec \alpha} \longleftrightarrow \frac{d\vec p}{dt} = \vec F = m\vec a$$

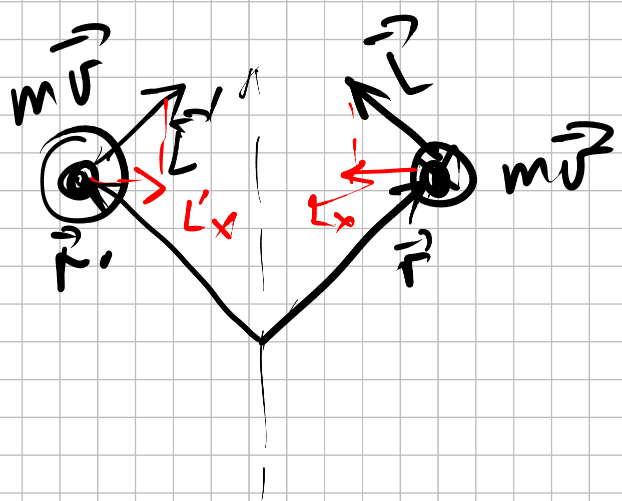
not always true
but for symmetrical objects



$\vec{v} = v \cdot \hat{\otimes}$

$\vec{L} = \vec{r} \times m\vec{v}$

$\vec{L} = I\vec{\omega}$ not always true



$\vec{L}_{tot} \parallel \vec{\omega}$

If object is symmetric with respect to axis of rotation then $\vec{L} \parallel \vec{\omega}$ and $\boxed{\vec{L} = I \cdot \vec{\omega}}$

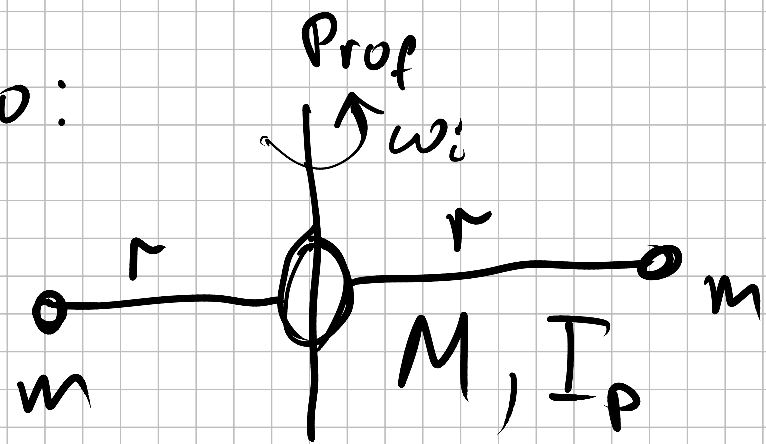
$$\frac{d\vec{L}}{dt} = \frac{d}{dt}\left(I \cdot \vec{\omega}\right) = I \cdot \frac{d\vec{\omega}}{dt} = I \cdot \vec{\alpha}$$

$$\boxed{\frac{d\vec{L}}{dt} = I\vec{\alpha} = \sum \vec{\tau}_{ext}}$$

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}\left(I\vec{\omega}\right) = \sum \vec{\tau}_{ext}$$

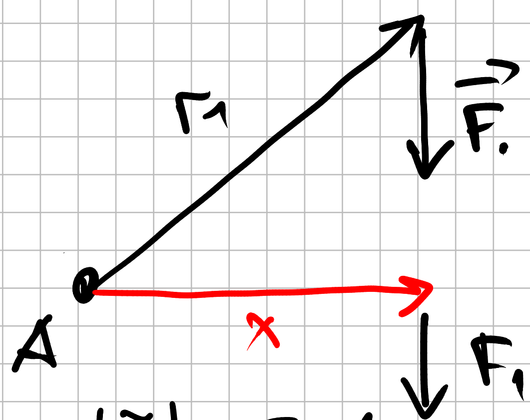
$$\boxed{\vec{\tau}_{ext} = 0 \Rightarrow I\vec{\omega} = \text{const}}$$

demo:

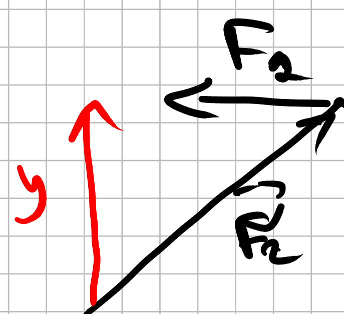


$$L_{tot} = (I_p + 2mr^2)\omega_i = (I_p + 0)\,\omega_f$$

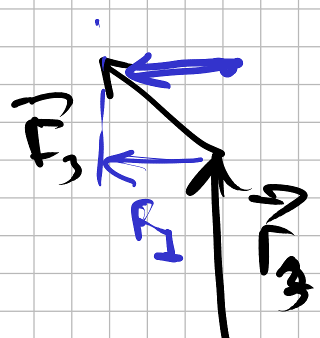
$$I = \sum_i m_i r_i^2 \qquad \omega_f = \frac{I_p + 2mr^2}{I_p}\,\omega_i$$

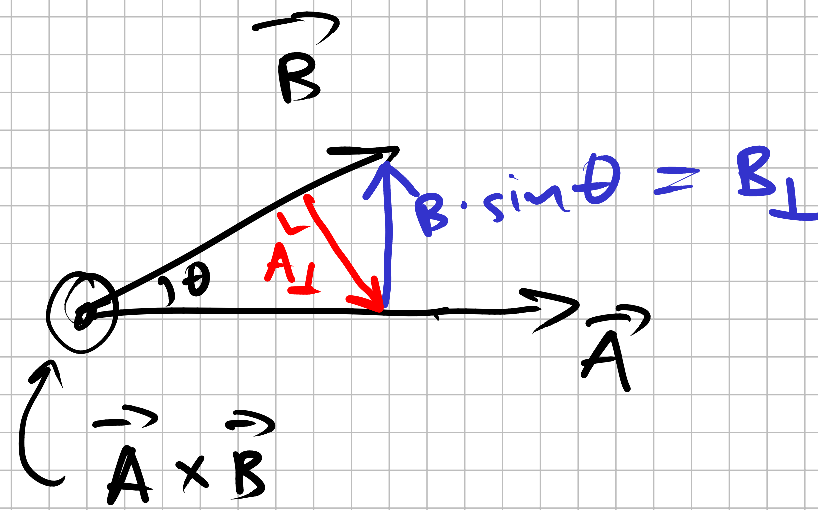


$$|\tau| = F_1 \cdot x$$



$$|\tau| = F_2 \cdot y$$





$$|\vec{A} \times \vec{B}| = A \cdot \underbrace{B \cdot \sin\theta}_{B_\perp}$$

$$= B \cdot \underbrace{A \cdot \sin\theta}_{A_\perp}$$