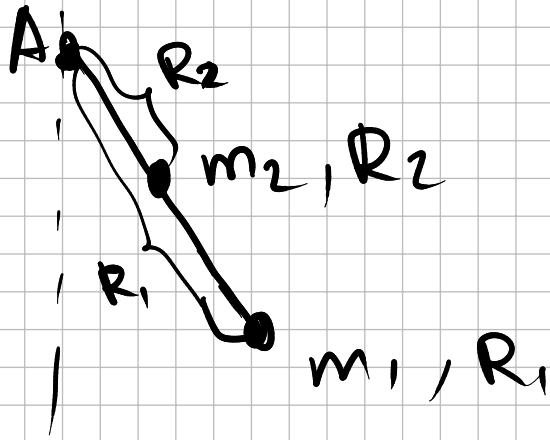


Kinetic energy of moving and rotating of a rigid body. (without derivation)

$$K = M_{\text{total}} \frac{v_{\text{translation}}^2}{2} + \frac{I_{\text{CM}}}{2} \omega^2$$

important
calculated with respect
to CM

Fixed axis rotation



$$E = \underbrace{\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2}}_K + \underbrace{m_1 g y_1 + m_2 g y_2}_U$$

$$= \frac{m_1 (R_1 \omega)^2}{2} + \frac{m_2 (R_2 \omega)^2}{2} +$$

$$g (m_1 y_1 + m_2 y_2)$$

$$m_1 \vec{r}_1 y + m_2 \vec{r}_2 y$$

$$= \frac{1}{2} (m_1 R_1^2 + m_2 R_2^2) \omega^2 +$$

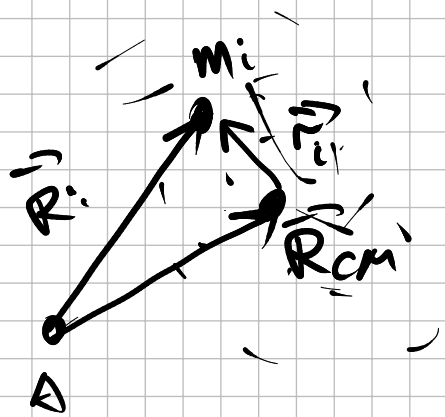
I_A moment of inertia

$$g \left(\frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{M_{tot}} \right) y \cdot M_{tot}$$

$$= \vec{R}_{CM}$$

$$E = \frac{1}{2} I_A \omega^2 + M_{\text{tot}} \cdot g \cdot (\vec{R}_{\text{cm}})_y$$

total energy for a rigid body
 when there are no external forces



$$\vec{R}_i = \vec{R}_{\text{cm}} + \vec{r}_i$$

$$I_A = \sum_i m_i (\vec{R}_{\text{cm}} + \vec{r}_i)^2 =$$

$$= \sum_i m_i R_{\text{cm}}^2 + \left(\sum_i m_i \vec{r}_i \right) \cdot \vec{R}_{\text{cm}} \cdot 2$$

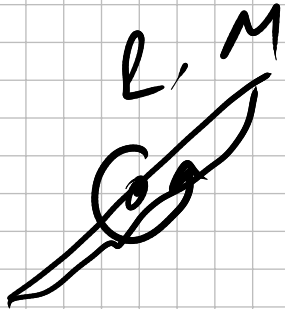
~~$\sum_i m_i \vec{r}_i = 0$~~

$$+ \sum_i m_i r_i^2$$

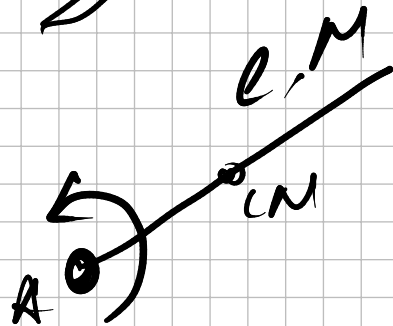
I_{cm}

$$I_A = M_{tot} \cdot R_{cm}^2 + I_{cm}$$

Parallel axis theorem



$$I_{cm} = \frac{M \cdot L^2}{12}$$



$$I_A = M \cdot \left(\frac{L}{2}\right)^2 + \frac{M L^2}{12} =$$

$$= M L^2 \left(\frac{1}{4} + \frac{1}{12} \right) =$$

$$= M L^2 \left(\frac{3}{12} + \frac{1}{12} \right) = \frac{M L^2}{3}$$

Moments of Inertia

Hoop about cylinder axis

$$I = MR^2$$

Annular cylinder (or ring) about cylinder axis

$$I = \frac{M}{2} (R_1^2 + R_2^2)$$

Solid cylinder (or disk) about cylinder axis

$$I = \frac{MR^2}{2}$$

Solid cylinder (or disk) about central diameter

$$I = \frac{MR^2}{4} + \frac{MR^2}{12}$$

Thin rod about axis through center \perp to length

$$I = \frac{Ml^2}{12}$$

Thin rod about axis through one end \perp to length

$$I = \frac{Ml^2}{3}$$

Solid sphere about any diameter

$$I = \frac{2MR^2}{5}$$

Thin spherical shell about any diameter

$$I = \frac{2MR^2}{3}$$

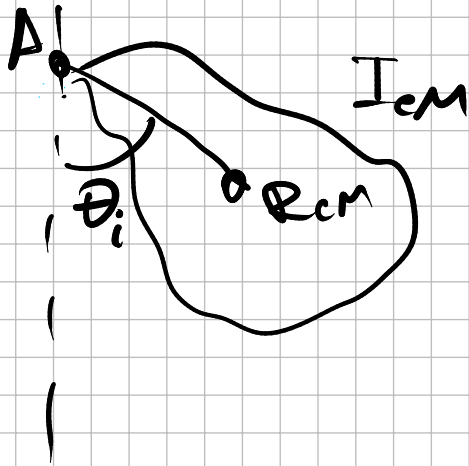
Hoop about any diameter

$$I = \frac{MR^2}{2}$$

Slab about \perp axis through center

$$I = \frac{M(a^2 + b^2)}{12}$$

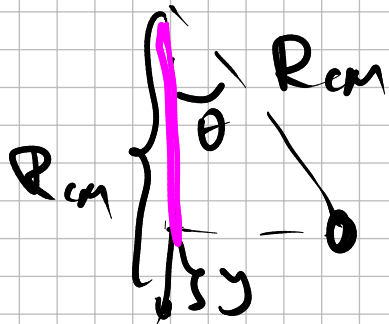
Swinging rod



$$E = I_A \cdot \frac{\omega^2}{2} + M_{cm} \cdot g \cdot y_{cm}$$

$$E_i = E_f, \quad \omega_i = 0, \quad y_f = 0$$

$$E_i = M_{cm} \cdot g \cdot y_{cm} = E_f = I_A \frac{\omega^2}{2}$$



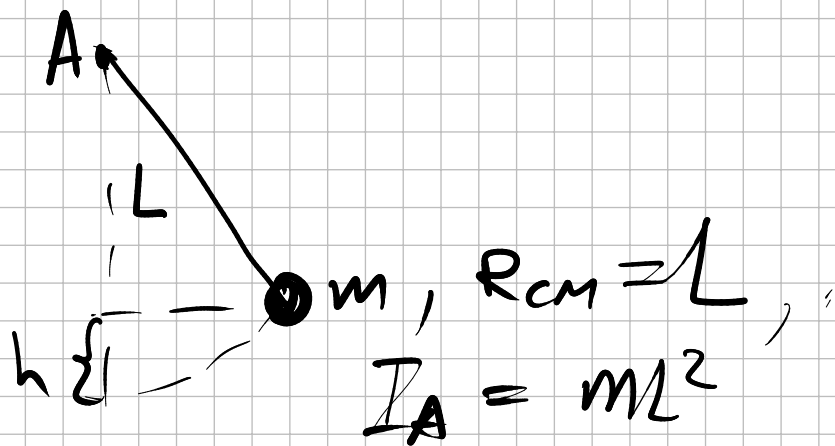
$$y_{cm} = R_{cm} - R_{cm} \cos \theta =$$

$$= R_{cm} (1 - \cos \theta)$$

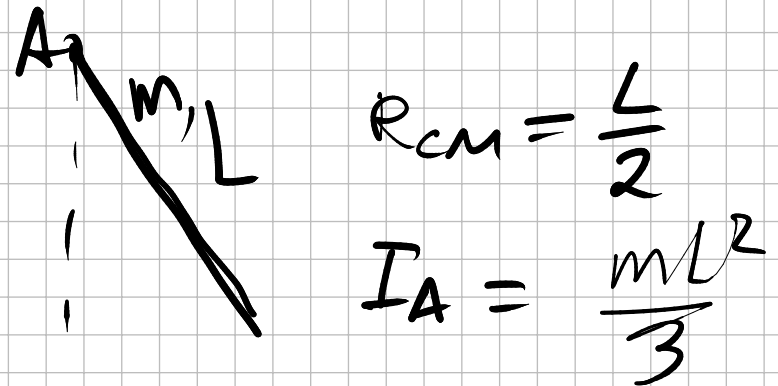
$$I_A \frac{\omega^2}{2} = M_{cm} \cdot g \cdot R_{cm} (1 - \cos \theta)$$

$$\omega^2 = \frac{2 M_{cm} \cdot g R_{cm} (1 - \cos\theta)}{I_A}$$

1st case



2nd case



1st case

$$\frac{v^2}{L^2} = \omega^2 = \frac{2mgL(1 - \cos\theta)}{mL^2} \Rightarrow v^2 = 2gL(1 - \cos\theta) = 2gh$$

2nd case

$$\omega^2 \frac{2mg(1-\cos\theta)\frac{L}{2}}{mL^2/3} = 3 \frac{mg(1-\cos\theta)L}{mL^2}$$

Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{L} = \sum_i \vec{r}_i \times \vec{F}_i$$

