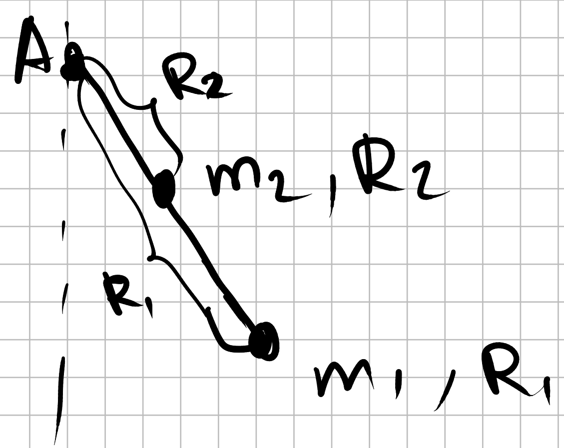
Kinetic energy of moving and rotating of a rigid body. (without derivation)

$$\boxed{K = M_{total} \frac{v^2_{translation}}{2} + I_{CM} \frac{\omega^2}{2}}$$

important
calculated with respect to CM

Fixed axis rotation



$$E = \underbrace{\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2}}_{K} + \underbrace{m_1 g y_1 + m_2 g y_2}_{U}$$

$$= \frac{m_1}{2}(R_1\omega)^2 + \frac{m_2 (R_2\omega)^2}{2} +$$

$$g\,\underbrace{(m_1 y_1 + m_2 y_2)}_{m_1 \vec{r}_{1y} + m_2 \vec{r}_{2y}}$$

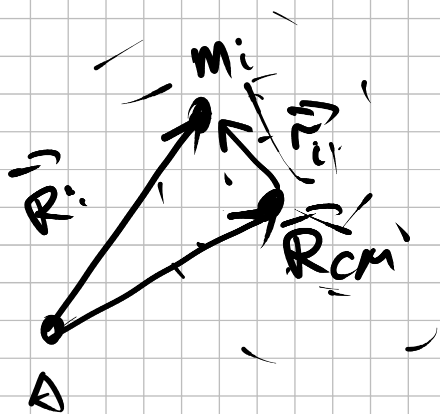
$$= \frac{1}{2}\underbrace{(m_1 R_1^2 + m_2 R_2^2)}_{I_A}\omega^2 + g\left(\frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{M_{tot}}\right)_y \cdot M_{tot}$$

I_A moment of inertia

$= \vec{R}_{CM}$

$$\boxed{E = \frac{1}{2} I_A \omega^2 + M_{tot} \cdot g \cdot (\vec{R}_{cm})_y}$$

total energy for a rigid Body
when there are no external forces



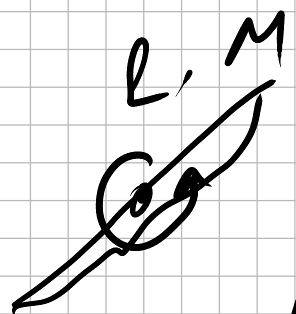
$$\vec{R}_i = \vec{R}_{CM} + \vec{r}_i$$

$$I_A = \sum_i m_i (\vec{R}_{CM} + \vec{r}_i)^2 =$$

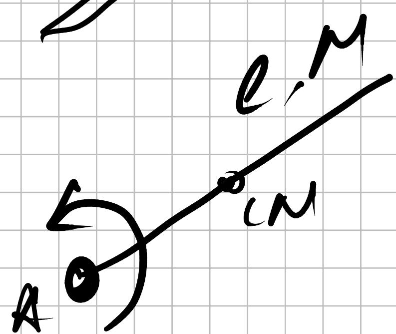
$$= \underbrace{\sum_i m_i}_{= M_{tot}} \vec{R}_{CM}^{\,2} + \cancel{\underbrace{\Big(\sum_i m_i \vec{r}_i\Big)}_{\sim \vec{r}_{CM} = \vec{0}} \cdot \vec{R}_{CM} \cdot 2} \to 0$$

$$+ \underbrace{\sum_i m_i \vec{r}_i^{\,2}}_{I_{CM}}$$

$$\boxed{I_A = M_{tot} \cdot R_{CM}^2 + I_{CM}}$$ Parallel axis theorem



$$I_{CM} = \frac{M \cdot L^2}{12}$$

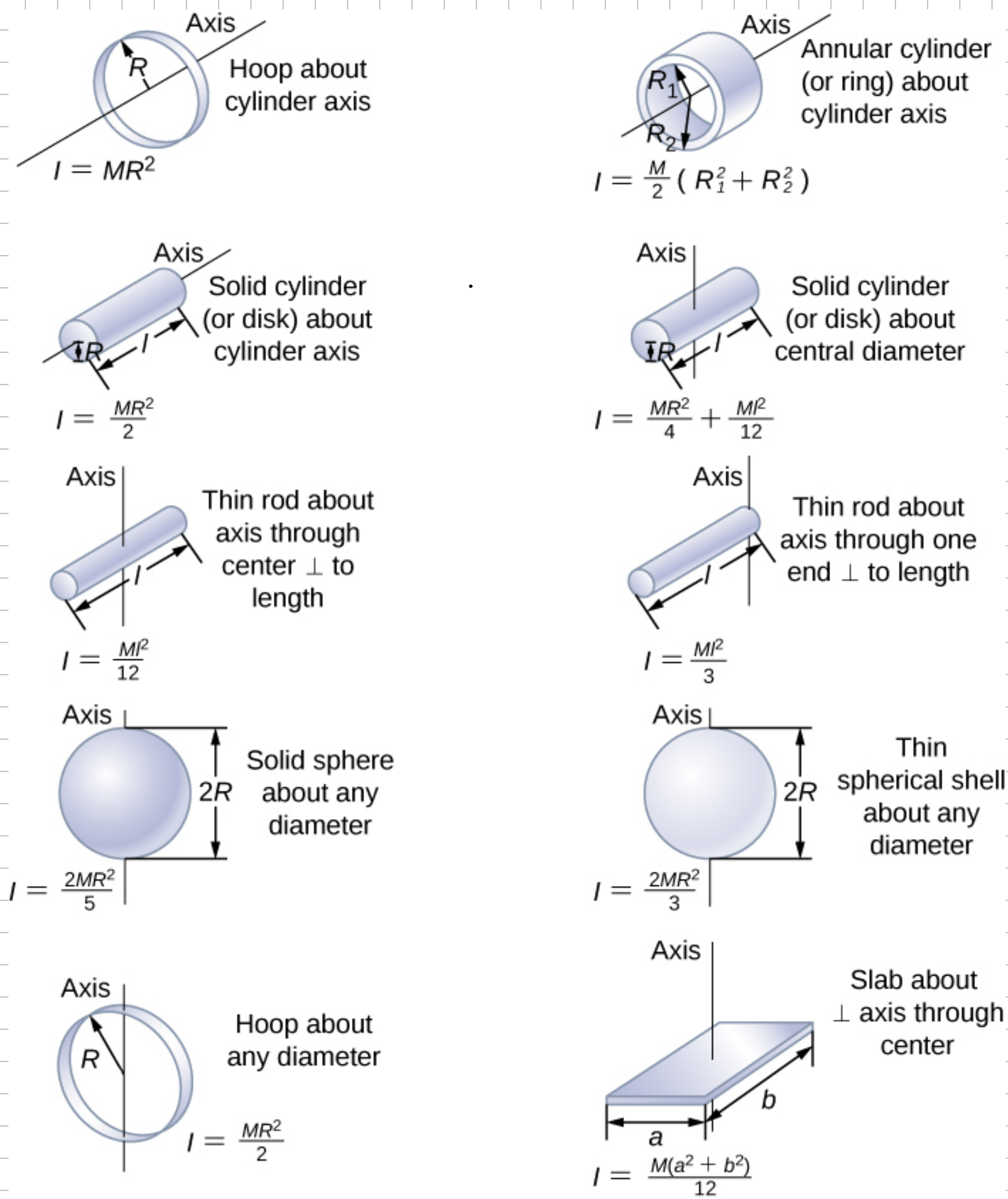


$$I_A = M \cdot \left(\frac{L}{2}\right)^2 + \frac{ML^2}{12} =$$

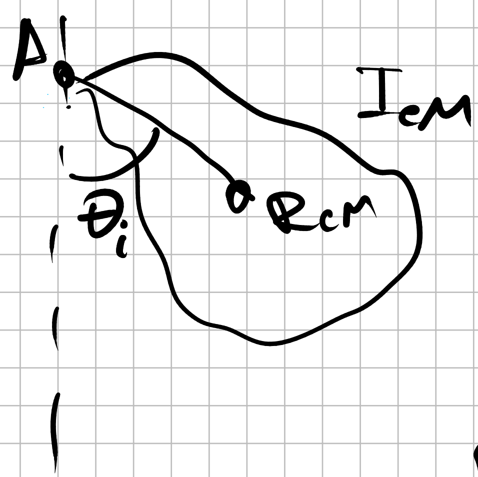
$$= ML^2\left(\frac{1}{4} + \frac{1}{12}\right) =$$

$$= ML^2\left(\frac{3}{12} + \frac{1}{12}\right) = \frac{ML^2}{3}$$

Moments of Inertia



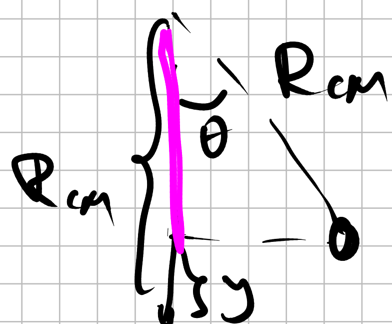
Swinging rod



$$E = I_A \cdot \frac{\omega^2}{2} + M_{cm} \cdot g \cdot y_{cm}$$

$$E_i = E_f \quad , \quad \omega_i = 0 \quad , \quad y_f = 0$$

$$E_i = M_{cm} \cdot g \cdot y_{cm} = E_f = I_A \frac{\omega^2}{2}$$



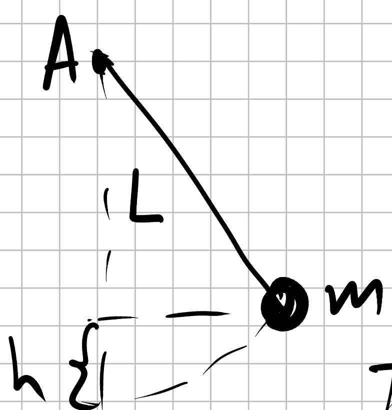
$$y_{cm} = R_{cm} - R_{cm} \cdot \cos\theta =$$

$$= R_{cm} (1 - \cos\theta)$$

$$I_A \frac{\omega^2}{2} = M_{cm} \cdot g \, R_{cm} (1 - \cos\theta)$$

$$\omega^2 = \frac{2\, M_{cm} \cdot g\, R_{cm}\,(1-\cos\theta)}{I_A}$$

1st case



m, $R_{cm} = L$,

$I_A = mL^2$

2nd case

$R_{cm} = \frac{L}{2}$

$I_A = \frac{mL^2}{3}$

1st case

$$\frac{v^2}{L^2} = \omega^2 = \frac{2mgL(1-\cos\theta)}{mL^2} \Rightarrow v^2 = 2gL(1-\cos\theta)$$

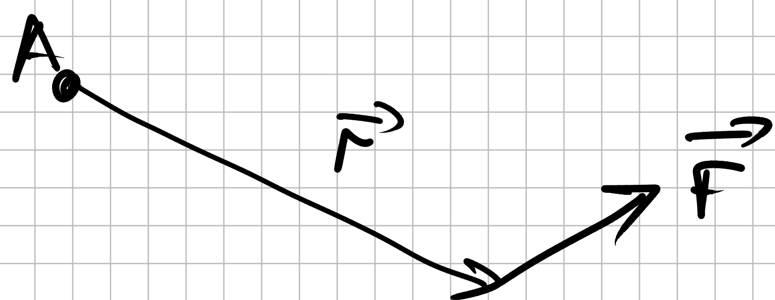
$$= 2gh$$

2nd case

$$\omega^2 = \frac{2mg(1-\cos\theta)\frac{L}{2}}{mL^2/3} = 3\frac{mg(1-\cos\theta)L}{mL^2}$$

Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$



$$I \cdot \vec{\alpha} = \sum_i \vec{\tau}_{ext}$$